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Building Code Requirements for Structural Concrete (ACI 318-95) and Commentary (ACI 318R-95)

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BUILDING CODE REQUIREMENTS FOR STRUCTURAL CONCRETE (ACI 318-95) AND COMMENTARY (ACI 318R-95)

REPORTED BY ACI COMMITTEE 318

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BUILDING CODE REQUIREMENTS FOR STRUCTURAL CONCRETE (ACI 318-95)* AND COMMENTARY (ACI 318R-95)†

REPORTED BY ACI COMMITTEE 318

The code portion of this document covers the proper design and construction of buildings of structural concrete. The code has been written in such form that it may be adopted by reference in a general building code and earlier editions have been widely used in this manner.

Among the subjects covered are: drawings and specifications; inspection; materials; durability requirements; concrete quality, mixing, and placing; formwork; embedded pipes and construction joints; reinforcement details; analysis and design; strength and serviceability; flexural and axial loads; shear and torsion; development and splices of reinforcement; slab systems; walls; footings; precast concrete; composite flexural members; prestressed concrete; shells and folded plate members; strength evaluation of existing structures; special provisions for seismic design; structural plain concrete; an alternate design method in Appendix A; unified design provisions in Appendix B; and alternative load and strength reduction factors in Appendix C.

The quality and testing of materials used in the construction are covered by reference to the appropriate ASTM standard specifications. Welding of reinforcement is covered by reference to the appropriate ANSI/AWS standard.

Because the ACI Building Code is written as a legal document so that it may be adopted by reference in a general building code, it cannot present background details or suggestions for carrying out its requirements or intent. It is the function of this commentary to fill this need.

The commentary discusses some of the considerations of the committee in developing the code with emphasis given to the explanation of new or revised provisions that may be unfamiliar to code users.

References to much of the research data referred to in preparing the code are cited for the user desiring to study individual questions in greater detail. Other documents that provide suggestions for carrying out the requirements of the code are also cited.

The chapter and section numbering of the code are followed throughout.

Keywords: admixtures; aggregates; anchorage (structural); beam-column frame; beams (supports); **building codes;** cements; cold weather construction; columns (supports); combined stress; composite construction (concrete and steel); composite construction (concrete to concrete); compressive strength; **concrete construction; concretes;** concrete slabs; construction joints; continuity (structural); contraction joints; cover; curing; deep beams; deflections; drawings; earthquake resistant structures; embedded service ducts; flexural strength; floors; folded plates; footings; formwork (construction); frames; hot weather construction; inspection; isolation joints; joints (junctions); joists; lightweight concretes; loads (forces); load tests (structural); materials; mixing; mix proportioning; modulus of elasticity; moments; pipe columns; pipes (tubing); placing; plain concrete; precast concrete; prestressed concrete; prestressing steels; quality control; **reinforced concrete**; reinforcing steels; roofs; serviceability; shear strength; shearwalls; shells (structural forms); spans; specifications; splicing; strength; strength analysis; stresses; **structural analysis; structural concrete; structural design**; structural integrity; T-beams, torsion; walls; water; welded wire fabric.

ACI 318-95 was adopted as a standard of the American Concrete Institute July 1, 1995 to supersede ACI 318-89 (Revised 1992) in accordance with the Institute's standardization procedure.

Vertical lines in the margins indicate the 1995 code changes.

*A complete metric companion to ACI 318/318R has been developed, 318M/318RM; therefore no metric equivalents are included in this document.

†ACI Committee Reports, Guides, Standard Practices, and Commentaries are intended for guidance in designing, planning, executing, or inspecting construction, and in preparing specifications. Reference to these documents shall not be made in the Project Documents. If items found in these documents are desired to be part of the Project Documents they should be phrased in mandatory language and incorporated into the Project Documents.

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The 1995 ACI Building Code and Commentary are presented in a side-by-side column format, with code text placed in the left column and the corresponding commentary text aligned in the right column. To further distinguish the Code from the Commentary, the Code has been printed in Helvetica, the same type face in which this paragraph is set. Vertical lines in the margins indicate changes from 318-89 (Revised 1992).

This paragraph is set in Times Roman, all portions of the text exclusive to the Commentary are printed in this type face. Commentary section numbers are preceded by an "R" to further distinguish them from Code section numbers.

INTRODUCTION

This commentary discusses some of the considerations of Committee 318 in developing the provisions contained in "Building Code Requirements for Structural Concrete (ACI 318-95)," hereinafter called the code or the 1995 code. Emphasis is given to the explanation of new or revised provisions that may be unfamiliar to code users. In addition, comments are included for some items contained in previous editions of the code to make the present commentary independent of the commentary for ACI 318-89 (Revised 1992). Comments on specific provisions are made under the corresponding chapter and section numbers of the code.

The commentary is not intended to provide a complete historical background concerning the development of the ACI Building Code,* nor is it intended to provide a detailed résumé of the studies and research data reviewed by the committee in formulating the provisions of the code. However, references to some of the research data are provided for those who wish to study the background material in depth.

As the name implies, "Building Code Requirements for Structural Concrete (ACI 318-95)" is meant to be used as part of a legally adopted building code and as such must differ in form and substance from documents that provide detailed specifications, recommended practice, complete design procedures, or design aids.

The code is intended to cover all buildings of the usual types, both large and small. Requirements more stringent than the code provisions may be desirable for unusual construction. The code and commentary cannot replace sound engineering knowledge, experience, and judgment.

A building code states only the minimum requirements necessary to provide for public health and safety. The ACI Building Code is based on this principle. For any structure, the owner or the structural designer may require the quality of materials and construction to be higher than the minimum requirements necessary to protect the public as stated in the code. However, lower standards are not permitted.

The commentary directs attention to other documents that provide suggestions for carrying out the requirements and intent of the code. However, those documents and the commentary are not intended to be a part of the code.

The code has no legal status unless it is adopted by the government bodies having the police power to regulate building design and construction. Where the code has not been adopted, it may serve as a reference to good practice even though it has no legal status.

The code provides a means of establishing minimum standards for acceptance of designs and construction by a legally appointed building official or his designated representatives. The code and commentary are not intended for use in settling disputes between the owner, engineer, architect, contractor, or their agents, subcontractors, material suppliers, or testing agencies. Therefore, the code cannot define the contract responsibility of each of the parties in usual construction. General references requiring compliance with ACI 318 in the job specifications should be avoided since the contractor is rarely in a position to accept responsibility for design details or construction requirements that depend on a detailed knowledge of the design. Generally, the drawings, specifications and contract documents should contain all of the necessary requirements to insure compliance with the code. In part, this can be accomplished by reference to specific code sections in the job specifications. Other ACI publications, such as "Specifications for Structural Concrete for Buildings" (ACI 301) are written specifically for use as contract documents for construction.

Committee 318 recognizes the desirability of standards of performance for individual parties involved in the contract documents. Available for this purpose are the plant certification programs of the Precast/Prestressed Concrete Institute, the Post-Tensioning Institute, and the National Ready Mixed Concrete Association. In addition, "Recommended Practice for Inspection and Testing Agencies for Concrete, Steel, and Bituminous Materials As Used in Construction" (ASTM E 329-77) recommends performance requirements for inspection and testing agencies.

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* For a history of the ACI Building Code see Kerekes, Frank, and Reid, Harold B., Jr., "Fifty Years of Development in Building Code Requirements for Reinforced Concrete," ACI JOURNAL, *Proceedings* V. 50, No. 6, Feb. 1954, p. 441. For a discussion of code philosophy, see Siess, Chester P., "Research, Building Codes, and Engineering Practice," ACI JOURNAL, *Proceedings* V. 56, No. 5, May 1960, p. 1105.

Design reference material illustrating application of the code requirements may be found in the following documents. The design aids listed may be obtained from the sponsoring organization.

Design aids:

"Design Handbook in Accordance with the Strength Design Method of ACI 318-89," Volume 2 — Columns, ACI Committee 340, Publication SP-17A(90), American Concrete Institute, Detroit, 1990, 222 pp. (Provides tables and charts for design of eccentricity loaded columns by the Strength Design Method).

"Design Handbook in Accordance with the Strength Design Method of ACI 318-89," V. 3 — Two-Way Slabs, ACI Committee 340, Publication SP-17(91), American Concrete Institute, Detroit, 1991, 104 pp. (Provides design aids for use in the engineering design and analysis of reinforced concrete slab systems carrying loads by two-way action. Design aids are also provided for the selection of slab thickness and for reinforcement required to control deformation and assure adequate shear and flexural strengths.)

"ACI Detailing Manual—1994," ACI Committee 315, Publication SP-66(94), American Concrete Institute, Detroit, 1994, 244 pp. (Includes the standard, ACI 315-92, and report, ACI 315R-94. Provides recommended methods and standards for preparing design drawings, typical details, and drawings for fabrication and placing of reinforcing steel in reinforced concrete structures. Separate sections define responsibilities of both engineer and reinforcing bar detailer.)

CRSI Handbook, Concrete Reinforcing Steel Institute, Schaumburg, Ill., 7th Edition, 1992, 840 pp. (Provides tabulated designs for structural elements and major concrete slab systems. All designs are based on normal weight concrete and Grade 60 reinforcing bars. Design examples are provided to show the basis of, and use of the load tables. Tabulated designs are given for beams; square, round and rectangular columns; one-way slabs; and one-way joist construction. The design tables for two-way slab systems include flat plates, flat slabs and waffle slabs. The sections on foundations provide design tables for square footings, pile caps, drilled piers (caissons) and cantilever retaining walls. Other design aids are presented for crack control; and development of reinforcement and lap splices.)

"Reinforcement: Anchorages, Lap Splices and Connections," Concrete Reinforcing Steel Institute, Schaumburg, Ill., 3rd Edition, 1990, 37 pp. (Provides accepted practices in splicing reinforcement. The use of lap splices, mechanical connections, and welded splices are described. Simplified design data is presented for development of reinforcement and lap splice lengths.)

"Structural Welded Wire Fabric Manual of Standard Practice," Wire Reinforcement Institute, McLean, Va., 4th Edition, Apr. 1992, 31 pp. (Describes wire fabric material, gives nomenclature and wire size and weight tables. Lists specifications and properties and manufacturing limitations. Book emphasizes ACI 318 Building Code requirements as code affects welded wire fabric. Also gives development length and splice length tables. Manual contains customary units and soft metric units.)

"Structural Welded Wire Fabric Detailing Manual," Wire Reinforcement Institute, McLean Va., 1st Edition, 1983, 76 pp. (Provides information on detailing welded wire fabric reinforcement systems. Includes design aids for welded wire fabric in accordance with ACI 318 Building Code requirements for wire fabric.)

"Strength Design of Reinforced Concrete Columns," Portland Cement Association, Skokie, Ill., EB009D, 1978, 48 pp. (Provides design tables of column strength in terms of load in kips versus moment in ft-kips for concrete strength of 5000 psi and Grade 60 reinforcement. Design examples are included. Note that the PCA design tables do not include the strength reduction factor ϕ in the tabulated values; M_u/ϕ and P_u/ϕ must be used when designing with this aid.

"PCI Design Handbook—Precast and Prestressed Concrete," Precast/Prestressed Concrete Institute, Chicago, 4th Edition, 1992, 580 pp. (Provides load tables for common industry products, and procedures for design and analysis of precast and prestressed elements and structures composed of these elements. Provides design aids and examples.)

"Design and Typical Details of Connections for Precast and Prestressed Concrete," Precast/Prestressed Concrete Institute, Chicago, 2nd Edition, 1988, 270 pp. (Updates available information on design of connections for both structural and architectural products, and presents a full spectrum of typical details. Provides design aids and examples.)

"PTI Post-Tensioning Manual," Post-Tensioning Institute, Phoenix, 5th Edition, 1990, 406 pp. (Provides comprehensive coverage of post-tensioning systems, specifications, and design aid construction concepts.)

"PTI Design of Post-Tensioned Slabs," Post-Tensioning Institute, Phoenix, 2nd Edition, Apr. 1984, 56 pp. (Illustrates application of ACI 318 Building Code requirements for design of one-way and two-way post-tensioned slabs. Detailed design examples are presented.)

CONTENTS

PART 1—GENERAL

CHAPTER 1—GENERAL REQUIREMENTS 318-9

1.1—Scope
1.2—Drawings and specifications
1.3—Inspection
1.4—Approval of special systems of design or construction

CHAPTER 2—DEFINITIONS.. 318-17

PART 2—STANDARDS FOR TESTS AND MATERIALS

CHAPTER 3—MATERIALS 318-23

3.0—Notation
3.1—Tests of materials
3.2—Cements
3.3—Aggregates
3.4—Water
3.5—Steel reinforcement
3.6—Admixtures
3.7—Storage of materials
3.8—Standards cited in this code

PART 3—CONSTRUCTION REQUIREMENTS

CHAPTER 4—DURABILITY REQUIREMENTS 318-35

4.0—Notation
4.1—Water-cementitious materials ratio
4.2—Freezing and thawing exposures
4.3—Sulfate exposures
4.4—Corrosion protection of reinforcement

CHAPTER 5—CONCRETE QUALITY, MIXING, AND PLACING 318-41

5.0—Notation
5.1—General
5.2—Selection of concrete proportions
5.3—Proportioning on the basis of field experience and/or trial mixtures
5.4—Proportioning without field experience or trial mixtures
5.5—Average strength reduction
5.6 —Evaluation and acceptance of concrete
5.7 —Preparation of equipment and place of deposit
5.8 —Mixing
5.9 —Conveying
5.10—Depositing
5.11—Curing
5.12—Cold weather requirements
5.13—Hot weather requirements

CHAPTER 6—FORMWORK, EMBEDDED PIPES, AND CONSTRUCTION JOINTS 318-57

6.1—Design of formwork
6.2—Removal of forms, shores, and reshoring
6.3—Conduits and pipes embedded in concrete
6.4—Construction joints

CHAPTER 7—DETAILS OF REINFORCEMENT.. 318-63

7.0—Notation
7.1—Standard hooks
7.2—Minimum bend diameters
7.3—Bending
7.4—Surface conditions of reinforcement
7.5—Placing reinforcement
7.6—Spacing limits for reinforcement
7.7 —Concrete protection for reinforcement
7.8 —Special reinforcement details for columns
7.9 —Connections
7.10—Lateral reinforcement for compression members
7.11—Lateral reinforcement for flexural members
7.12—Shrinkage and temperature reinforcement
7.13—Requirements for structural integrity

PART 4—GENERAL REQUIREMENTS

CHAPTER 8—ANALYSIS AND DESIGN—GENERAL CONSIDERATIONS.......................... 318-77

8.0—Notation
8.1—Design methods
8.2—Loading
8.3—Methods of analysis
8.4—Redistribution of negative moments in continuous nonprestressed flexural members
8.5—Modulus of elasticity
8.6 —Stiffness
8.7 —Span length
8.8 —Columns
8.9 —Arrangement of live load
8.10—T-beam construction
8.11—Joist construction
8.12—Separate floor finish

CHAPTER 9—STRENGTH AND SERVICEABILITY REQUIREMENTS.. 318-87

9.0—Notation
9.1—General
9.2—Required strength
9.3—Design strength
9.4—Design strength for reinforcement
9.5—Control of deflections

CHAPTER 10—FLEXURE AND AXIAL LOADS......................................318-101

10.0—Notation
10.1—Scope
10.2—Design assumptions
10.3—General principles and requirements
10.4—Distance between lateral supports of flexural members
10.5—Minimum reinforcement of flexural members
10.6—Distribution of flexural reinforcement in beams and one-way slabs
10.7—Deep flexural members
10.8—Design dimensions for compression members
10.9 —Limits for reinforcement of compression members
10.10—Slenderness effects in compression members
10.11—Magnified moments—General
10.12—Magnified moments—Non-sway frames
10.13—Magnified moments—Sway frames
10.14—Axially loaded members supporting slab system
10.15—Transmission of column loads through floor system
10.16—Composite compression members
10.17—Bearing strength

CHAPTER 11—SHEAR AND TORSION...318-131

11.0—Notation
11.1—Shear strength
11.2—Lightweight concrete
11.3—Shear strength provided by concrete for nonprestressed members
11.4—Shear strength provided by concrete for prestressed members
11.5—Shear strength provided by shear reinforcement
11.6 —Design for torsion
11.7 —Shear-friction
11.8 —Special provisions for deep flexural members
11.9 —Special provisions for brackets and corbels
11.10—Special provisions for walls
11.11—Transfer of moments to columns
11.12—Special provisions for slabs and footings

CHAPTER 12—DEVELOPMENT AND SPLICES OF REINFORCEMENT...318-177

12.0—Notation
12.1—Development of reinforcement—General
12.2—Development of deformed bars and deformed wire in tension
12.3—Development of deformed bars in compression
12.4—Development of bundled bars
12.5—Development of standard hooks in tension
12.6—Mechanical anchorage
12.7—Development of welded deformed wire fabric in tension
12.8—Development of welded plain wire fabric in tension
12.9—Development of prestressing strand
12.10—Development of flexural reinforcement—General
12.11—Development of positive moment reinforcement
12.12—Development of negative moment reinforcement
12.13—Development of web reinforcement
12.14—Splices of reinforcement—General
12.15—Splices of deformed bars and deformed wire in tension
12.16—Splices of deformed bars in compression
12.17—Special splice requirements for columns
12.18—Splices of welded deformed wire fabric in tension
12.19—Splices of welded plain wire fabric in tension

PART 5—STRUCTURAL SYSTEMS OR ELEMENTS

CHAPTER 13—TWO-WAY SLAB SYSTEMS 318-207

13.0—Notation
13.1—Scope
13.2—Definitions
13.3—Slab reinforcement
13.4—Openings in slab systems
13.5—Design procedures
13.6—Direct design method
13.7—Equivalent frame method

CHAPTER 14—WALLS 318-227

14.0—Notation
14.1—Scope
14.2—General
14.3—Minimum reinforcement
14.4—Walls designed as compression members
14.5—Empirical design method
14.6—Nonbearing walls
14.7—Walls as grade beams

CHAPTER 15—FOOTINGS 318-231

15.0—Notation
15.1—Scope
15.2—Loads and reactions
15.3—Footings supporting circular or regular polygon shaped columns or pedestals
15.4—Moment in footings
15.5—Shear in footings
15.6 —Development of reinforcement in footings
15.7 —Minimum footing depth
15.8 —Transfer of force at base of column, wall, or reinforced pedestal
15.9 —Sloped or stepped footings
15.10—Combined footings and mats

CHAPTER 16—PRECAST CONCRETE 318-239

16.0—Notation
16.1—Scope
16.2—General
16.3—Distribution of forces among members
16.4—Member design
16.5—Structural integrity
16.6 —Connection and bearing design
16.7 —Items embedded after concrete placement
16.8 —Marking and identification
16.9 —Handling
16.10—Strength evaluation of precast construction

CHAPTER 17—COMPOSITE CONCRETE FLEXURAL MEMBERS.....318-247

17.0—Notation
17.1—Scope
17.2—General
17.3—Shoring
17.4—Vertical shear strength
17.5—Horizontal shear strength
17.6—Ties for horizontal shear

CHAPTER 18—PRESTRESSED CONCRETE 318-251

18.0 —Notation
18.1 —Scope
18.2 —General
18.3 —Design assumptions
18.4 —Permissible stresses in concrete—Flexural members
18.5 —Permissible stresses in prestressing tendons
18.6 —Loss of prestress
18.7 —Flexural strength
18.8 —Limits for reinforcement of flexural members
18.9 —Minimum bonded reinforcement
18.10—Statically indeterminate structures
18.11—Compression members—Combined flexure and axial loads
18.12—Slab systems
18.13—Tendon anchorage zones
18.14—Corrosion protection for unbonded prestressing tendons
18.15—Post-tensioning ducts
18.16—Grout for bonded prestressing tendons
18.17—Protection for prestressing tendons
18.18—Application and measurement of prestressing force
18.19—Post-tensioning anchorages and couplers

CHAPTER 19—SHELLS AND FOLDED PLATE MEMBERS 318-271

19.0—Notation
19.1—Scope and definitions
19.2—Analysis and design
19.3—Design strength of materials
19.4—Shell reinforcement
19.5—Construction

PART 6—SPECIAL CONSIDERATIONS

CHAPTER 20—STRENGTH EVALUATION OF EXISTING STRUCTURES 318-279

20.0—Notation
20.1—Strength evaluation—General
20.2—Determination of required dimensions and material properties
20.3—Load test procedure
20.4—Loading criteria
20.5—Acceptance criteria
20.6—Provision for lower load rating
20.7—Safety

CHAPTER 21—SPECIAL PROVISIONS FOR SEISMIC DESIGN......... 318-285

21.0—Notation
21.1—Definitions
21.2—General requirements
21.3—Flexural members of frames
21.4—Frame members subjected to bending and axial load
21.5—Joints of frames
21.6—Structural walls, diaphragms, and trusses
21.7—Frame members not proportioned to resist forces induced by earthquake motions
21.8—Requirements for frames in regions of moderate seismic risk

PART 7—STRUCTURAL PLAIN CONCRETE

CHAPTER 22—STRUCTURAL PLAIN CONCRETE 318-313

22.0—Notation
22.1—Scope
22.2—Limitations
22.3—Joints
22.4—Design method
22.5—Strength design
22.6—Walls
22.7—Footings
22.8—Pedestals
22.9—Precast members

COMMENTARY REFERENCES.. 318-323

APPENDICES

APPENDIX A—ALTERNATE DESIGN METHOD................................. 318-335

A.0—Notation
A.1—Scope
A.2—General
A.3—Permissible service load stresses
A.4—Development and splices of reinforcement
A.5—Flexure
A.6—Compression members with or without flexure
A.7—Shear and torsion

APPENDIX B—UNIFIED DESIGN PROVISIONS FOR REINFORCED AND PRESTRESSED CONCRETE FLEXURAL AND COMPRESSION MEMBERS.. 318-347

B.1—Scope

APPENDIX C—ALTERNATIVE LOAD AND STRENGTH REDUCTION FACTORS 318-355

C.1—General

APPENDIX D—NOTATION 318-357

APPENDIX E—STEEL REINFORCEMENT INFORMATION 318-363

INDEX 318-365

PART 1 — GENERAL

CHAPTER 1 — GENERAL REQUIREMENTS

CODE

1.1 — Scope

1.1.1 — This code provides minimum requirements for design and construction of structural concrete elements of any structure erected under requirements of the legally adopted general building code of which this code forms a part. In areas without a legally adopted building code, this code defines minimum acceptable standards of design and construction practice.

1.1.2 — This code supplements the general building code and shall govern in all matters pertaining to

COMMENTARY

R1.1 — Scope

The American Concrete Institute "**Building Code Requirements for Structural Concrete (ACI 318-95)**," hereinafter referred to as the code, provides minimum requirements for any structural concrete design or construction.

The 1995 edition of the ACI 318 Building Code both revised the previous standard **"Building Code Requirements for Reinforced Concrete (ACI 318-89) (Revised 1992)"** and revised and incorporated into the single new standard extensive material from **"Building Code Requirements for Structural Plain Concrete (ACI 318.1-89)(Revised 1992)."** Thus, this new standard includes in one document the rules for all concrete used for structural purposes including both plain and reinforced concrete. Title of the document has been changed in the 1995 edition to **"Building Code Requirements for Structural Concrete (ACI 318-95)."** The term "structural concrete" is used to refer to all plain or reinforced concrete used for structural purposes. This covers the spectrum of structural applications of concrete from nonreinforced concrete to concrete containing nonprestressed reinforcement, pretensioned or post-tensioned tendons, or composite steel shapes, pipe, or tubing.

Prestressed concrete is included under the definition of reinforced concrete. Provisions of the code apply to prestressed concrete except for those that are stated to apply specifically to nonprestressed concrete.

Chapter 21 of the code contains special provisions for design and detailing of earthquake resistant structures. See 1.1.8.

Appendix A of the code contains provisions for an alternate method of design for nonprestressed reinforced concrete members using service loads (without load factors) and permissible service load stresses. The Alternate Design Method is intended to give results that are slightly more conservative than designs by the Strength Design Method of the code.

Appendix B of the code contains new provisions for reinforcement limits, determination of the strength reduction factor ϕ, and moment redistribution. The new provisions are applicable to reinforced and prestressed concrete flexural and compression members. Designs made using the provisions of Appendix B are equally acceptable, provided the provisions of Appendix B are used in their entirety.

R1.1.2 — The American Concrete Institute recommends that the code be adopted in its entirety; however, it is recognized that when the code is made a part of a legally adopted

CODE

design and construction of structural concrete, except wherever this code is in conflict with requirements in the legally adopted general building code.

1.1.3 — This code shall govern in all matters pertaining to design, construction, and material properties wherever this code is in conflict with requirements contained in other standards referenced in this code.

1.1.4 — For special structures, such as arches, tanks, reservoirs, bins and silos, blast-resistant structures, and chimneys, provisions of this code shall govern where applicable.

COMMENTARY

general building code, that general building code may modify some provisions of this code.

R1.1.4 — Some special structures involve unique design and construction problems which are not covered by the code. However, many code provisions, such as the concrete quality and design principles, are applicable for these structures. Detailed recommendations for design and construction of some special structures are given in the following ACI publications:

"Standard Practice for the Design and Construction of Cast-in-Place Reinforced Concrete Chimneys" reported by ACI Committee 307.[1.1] (Gives material, construction, and design requirements for circular cast-in-place reinforced chimneys. It sets forth minimum loadings for the design of reinforced concrete chimneys and contains methods for determining the stresses in the concrete and reinforcement required as a result of these loadings.)

"Standard Practice for Design and Construction of Concrete Silos and Stacking Tubes for Storing Granular Materials" reported by ACI Committee 313.[1.2] (Gives material, design, and construction requirements for reinforced concrete bins, silos, and bunkers and stave silos for storing granular materials. It includes recommended design and construction criteria based on experimental and analytical studies plus worldwide experience in silo design and construction.)

(Bins, silos, and bunkers are special structures, posing special problems not encountered in normal building design. While this standard practice refers to **"Building Code Requirements for Structural Concrete"** (ACI 318) for many applicable requirements, it provides supplemental detail requirements and ways of considering the unique problems of static and dynamic loading of silo structures. Much of the method is empirical, but this standard practice does not preclude the use of more sophisticated methods which give equivalent or better safety and reliability.)

(This standard practice sets forth recommended loadings and methods for determining the stresses in the concrete and reinforcement resulting from these loadings. Methods are recommended for determining the thermal effects resulting from stored material and for determining crack width in concrete walls due to pressure exerted by the stored material. Appendices provide recommended minimum values of overpressure and impact factors.)

CODE

COMMENTARY

"Environmental Engineering Concrete Structures" reported by ACI Committee 350.[1.3] (Gives material, design and construction recommendations for concrete tanks, reservoirs, and other structures commonly used in water and waste treatment works where dense, impermeable concrete with high resistance to chemical attack is required. Special emphasis is placed on a structural design which minimizes the possibility of cracking and accommodates vibrating equipment and other special loads. Proportioning of concrete, placement, curing and protection against chemicals are also described. Design and spacing of joints receive special attention.)

"Code Requirements for Nuclear Safety Related Concrete Structures" reported by ACI Committee 349.[1.4] (Provides minimum requirements for design and construction of concrete structures which form part of a nuclear power plant and which have nuclear safety related functions. The code does not cover concrete reactor vessels and concrete containment structures which are covered by ACI 359.)

"Code for Concrete Reactor Vessels and Containments" reported by ACI-ASME Committee 359.[1.5] (Provides requirements for the design, construction, and use of concrete reactor vessels and concrete containment structures for nuclear power plants.)

1.1.5 — This code does not govern design and installation of portions of concrete piles and drilled piers embedded in ground.

R1.1.5 — The design and installation of piling fully embedded in the ground is regulated by the general building code. For portions of piling in air or water, or in soil not capable of providing adequate lateral restraint throughout the piling length to prevent buckling, the design provisions of this code govern where applicable.

Recommendations for concrete piles are given in detail in **"Recommendations for Design, Manufacture, and Installation of Concrete Piles"** reported by ACI Committee 543.[1.6] (Provides recommendations for the design and use of most types of concrete piles for many kinds of construction.)

Recommendations for drilled piers are given in detail in **"Design and Construction of Drilled Piers"** reported by ACI Committee 336.[1.7] (Provides recommendations for design and construction of foundation piers 2$^1/_2$ ft in diameter or larger made by excavating a hole in the soil and then filling it with concrete.)

1.1.6 — This code does not govern design and construction of soil-supported slabs, unless the slab transmits vertical loads from other portions of the structure to the soil.

1.1.7 — Concrete on steel form deck

R1.1.7 — Concrete on steel form deck

In steel framed structures, it is common practice to cast concrete floor slabs on stay-in-place steel form deck. In all cases, the deck serves as the form and may, in some cases, serve an additional structural function.

CODE

1.1.7.1 — Design and construction of structural concrete slabs cast on stay-in-place, noncomposite steel form deck are governed by this code.

1.1.7.2 — This code does not govern the design of structural concrete slabs cast on stay-in-place, composite steel form deck. Concrete used in the construction of such slabs shall be governed by Parts 1, 2, and 3 of this code, where applicable.

1.1.8 — Special provisions for earthquake resistance

1.1.8.1 — In regions of low seismic risk, provisions of Chapter 21 shall not apply.

1.1.8.2 — In regions of moderate or high seismic risk, provisions of Chapter 21 shall be satisfied. See 21.2.1.

COMMENTARY

R1.1.7.1 — In its most basic application, the steel form deck serves as a form, and the concrete serves a structural function and, therefore, must be designed to carry all superimposed loads.

R1.1.7.2 — Another type of steel form deck commonly used develops composite action between the concrete and steel deck. In this type of construction, the steel deck serves as the positive moment reinforcement. The design of composite slabs on steel deck is regulated by **"Standard for the Structural Design of Composite Slabs"** (ANSI/ASCE 3).$^{1.8}$ However, ANSI/ASCE 3 references the appropriate portions of ACI 318 for the design and construction of the concrete portion of the composite assembly. Guidelines for the construction of composite steel deck slabs are given in **"Standard Practice for the Construction and Inspection of Composite Slabs"** (ANSI/ASCE 9).$^{1.9}$

R1.1.8 — Special provisions for earthquake resistance

Special provisions for seismic design were first introduced in Appendix A of the 1971 ACI Building Code and were continued without revision in ACI 318-77. These provisions were originally intended to apply only to reinforced concrete structures located in regions of highest seismicity.

The special provisions were extensively revised in the 1983 code edition to include new requirements for certain earthquake-resisting systems located in regions of moderate seismicity. In the 1989 code, the special provisions were moved to Chapter 21.

R1.1.8.1 — For buildings located in regions of low seismic risk, no special design or detailing is required; the general requirements of the main body of the code apply for proportioning and detailing reinforced concrete buildings. It is the intent of Committee 318 that concrete structures proportioned by the main body of the code will provide a level of toughness adequate for low earthquake intensity.

R1.1.8.2 — For buildings in regions of moderate seismic risk, reinforced concrete moment frames proportioned to resist earthquake effects require some special reinforcement details, as specified in 21.8 of Chapter 21. The special details apply only to frames (beams, columns, and slabs) to which the earthquake-induced forces have been assigned in design. The special details are intended principally for unbraced concrete frames, where the frame is required to resist not only normal load effects, but also the lateral load effects of earthquake. The special reinforcement details will serve to provide a suitable level of inelastic behavior if the frame is subjected to an earthquake of such intensity as to require it to perform inelastically. There are no special requirements for structural walls provided to resist lateral effects of wind and earthquake, or nonstructural components of buildings located in regions of moderate seismic risk. Structural walls proportioned by the main body of the code are considered to have sufficient toughness at anticipated drift levels in regions of moderate seismicity.

CODE

1.1.8.3 — Seismic risk level of a region shall be regulated by the legally adopted general building code of which this code forms a part, or determined by local authority.

1.2 — Drawings and specifications

1.2.1 — Copies of design drawings, typical details, and specifications for all structural concrete construction shall bear the seal of a registered engineer or architect. These drawings, details, and specifications shall show:

(a) Name and date of issue of code and supplement to which design conforms

(b) Live load and other loads used in design

(c) Specified compressive strength of concrete at stated ages or stages of construction for which each part of structure is designed

(d) Specified strength or grade of reinforcement

(e) Size and location of all structural elements and reinforcement

(f) Provision for dimensional changes resulting from creep, shrinkage, and temperature

(g) Magnitude and location of prestressing forces

(h) Anchorage length of reinforcement and location and length of lap splices

(i) Type and location of welded splices and mechanical connections of reinforcement

COMMENTARY

For buildings located in regions of high seismic risk, all building components, structural and nonstructural, must satisfy requirements of 21.2 through 21.7 of Chapter 21. The special proportioning and detailing provisions of Chapter 21 are intended to provide a monolithic reinforced concrete structure with adequate "toughness" to respond inelastically under severe earthquake motions. See also R21.2.1

R1.1.8.3 — Definition of low, moderate, and high seismic risk as used by ACI 318 are not precise. Seismic risk level is usually designated by zones or areas of equal probability of risk of damage, related to the intensity of ground shaking, such as Zone 0—no damage; Zone 1—minor damage; Zone 2—moderate damage; and Zones 3 and 4—major damage. The tabulation is provided only as guide in interpreting the requirements of 1.1.8. The correlations implied are neither precise nor inflexible. Seismic risk levels (Seismic Zone Maps) are under the jurisdiction of a general building code rather than ACI 318. In the absence of a general building code that addresses earthquake loads and seismic zoning, it is the intent of Committee 318 that the local authorities (engineers, geologists, and building code officials) should decide on proper need and application of the special provisions for seismic design. Seismic zoning maps, such as recommended in References 1.10 and 1.11, are suitable for correlating seismic risk.

R1.2 — Drawings and specifications

R1.2.1 — The provisions for preparation of design drawings and specifications are, in general, consistent with those of most general building codes and are intended as supplements thereto.

The code lists some of the more important items of information that must be included in the design drawings, details, or specifications. The code does not imply an all inclusive list, and additional items may be required by the building official.

CODE

(j) Details and location of all contraction or isolation joints specified for plain concrete in Chapter 22.

1.2.2 — Calculations pertinent to design shall be filed with the drawings when required by the building official. Analyses and designs using computer programs shall be permitted provided design assumptions, user input, and computer-generated output are submitted. Model analysis shall be permitted to supplement calculations.

1.2.3 — Building official means the officer or other designated authority charged with the administration and enforcement of this code, or his duly authorized representative.

1.3 — Inspection

1.3.1 — As a minimum, concrete construction shall be inspected as required by the legally adopted general building code. In the absence of such requirements, concrete construction shall be inspected throughout the various work stages by an engineer or architect, or by a competent representative responsible to that engineer or architect.

COMMENTARY

R1.2.2 — Documented computer output is acceptable in lieu of manual calculations. The extent of input and output information required will vary, according to the specific requirements of individual building officials. However, when a computer program has been used by the designer, only skeleton data should normally be required. This should consist of sufficient input and output data and other information to allow the building official to perform a detailed review and make comparisons using another program or manual calculations. Input data should be identified as to member designation, applied loads, and span lengths. The related output data should include member designation and the shears, moments, and reactions at key points in the span. For column design, it is desirable to include moment magnification factors in the output where applicable.

The code permits model analysis to be used to supplement structural analysis and design calculations. Documentation of the model analysis should be provided with the related calculations. Model analysis should be performed by an engineer or architect having experience in this technique.

R1.2.3 — "Building official" is the term used by many general building codes to identify the person charged with administration and enforcement of the provisions of the building code. However, such terms as "building commissioner" or "building inspector" are variations of the title, and the term "building official" as used in this code is intended to include those variations as well as others which are used in the same sense.

R1.3 — Inspection

The quality of concrete structures depends largely on workmanship in construction. The best of materials and design practice will not be effective unless the construction is performed well. Inspection is provided to assure satisfactory work in accordance with the design drawings and specifications. Proper performance of the structure depends on construction which accurately represents the design and meets code requirements, within the tolerances allowed. In the public interest, local building ordinances should require the owner to provide inspections.

R1.3.1 — Inspection of construction by or under the supervision of the engineer or architect responsible for the design should be considered because the person in charge of the design is the best qualified to inspect for conformance with the design. When such an arrangement is not feasible, the owner may provide proper inspection of construction through his engineers or architects or through separate inspection organizations with demonstrated capability for performing the inspection.

CODE

1.3.2 — The inspector shall require compliance with design drawings and specifications. Unless specified otherwise in the legally adopted general building code, inspection records shall include:

(a) Quality and proportions of concrete materials and strength of concrete

(b) Construction and removal of forms and reshoring

(c) Placing of reinforcement

(d) Mixing, placing, and curing of concrete

(e) Sequence of erection and connection of precast members

(f) Tensioning of prestressing tendons

(g) Any significant construction loadings on completed floors, members, or walls

(h) General progress of work.

COMMENTARY

The building departments having jurisdiction over the construction may have the necessary expertise and capability to inspect structural concrete construction.

When inspection is done independently of the designer, it is recommended that the designer be employed to at least oversee inspection and observe the work to see that his design requirements are properly executed.

In some jurisdictions, legislation has established special registration or licensing procedures for persons performing certain inspection functions. A check should be made in the general building code or with the building official to ascertain if any such requirements exist within a specific jurisdiction.

Inspection responsibility and the degree of inspection required should be set forth in the contracts between the owner, architect, engineer, and contractor. Adequate fees should be provided consistent with the work and equipment necessary to properly perform the inspection.

R1.3.2 — By "inspection," the code does not mean that the inspector should supervise the construction. Rather it means that the one employed for inspection should visit the project with the frequency necessary to observe the various stages of work and ascertain that it is being done in compliance with contract documents and code requirements. The frequency should be at least enough to provide general knowledge of each operation, whether this be several times a day or once in several days.

Inspection in no way relieves the contractor from his obligation to follow the plans and specifications implicitly and to provide the designated quality and quantity of materials and workmanship for all job stages. The inspector should be present as frequently as he/she deems necessary to judge whether the quality and quantity of the work complies with the contract documents; to counsel on possible ways of obtaining the desired results; to see that the general system proposed for formwork appears proper (though it remains the contractor's responsibility to design and build adequate forms and to leave them in place until it is safe to remove them); to see that reinforcement is properly installed; to see that concrete is of the correct quality, properly placed, and cured; and to see that tests for quality control are being made as specified.

The code prescribes minimum requirements for inspection of all structures within its scope. It is not a construction specification and any user of the code may require higher standards of inspection than cited in the legal code if additional requirements are necessary.

Recommended procedures for organization and conduct of concrete inspection are given in detail in **"Guide for Concrete Inspection."**[1.12] (Sets forth procedures relating to concrete construction to serve as a guide to owners, architects, and engineers in planning an inspection program.)

CODE

1.3.3 — When the ambient temperature falls below 40 F or rises above 95 F, a record shall be kept of concrete temperatures and of protection given to concrete during placement and curing.

1.3.4 — Records of inspection required in 1.3.2 and 1.3.3 shall be preserved by the inspecting engineer or architect for 2 years after completion of the project.

1.3.5 — For moment frames resisting seismic loads in structures designed in conformance with Chapter 21 and located in regions of high seismic risk, a specially qualified inspector under the supervision of the person responsible for the structural design shall provide continuous inspection for the placement of the reinforcement and concrete.

1.4 — Approval of special systems of design or construction

Sponsors of any system of design or construction within the scope of this code, the adequacy of which has been shown by successful use or by analysis or test, but which does not conform to or is not covered by this code, shall have the right to present the data on which their design is based to the building official or to a board of examiners appointed by the building official. This board shall be composed of competent engineers and shall have authority to investigate the data so submitted, to require tests, and to formulate rules governing design and construction of such systems to meet the intent of this code. These rules when approved by the building official and promulgated shall be of the same force and effect as the provisions of this code.

COMMENTARY

Detailed methods of inspecting concrete construction are given in "**ACI Manual of Concrete Inspection**" (SP-2) reported by ACI Committee 311.[1.13] (Describes methods of inspecting concrete construction which are generally accepted as good practice. Intended as a supplement to specifications and as a guide in matters not covered by specifications.)

R1.3.3 — The term "ambient temperature" means the temperature of the environment to which the concrete is directly exposed. Concrete temperature as used in this section may be taken as the air temperature near the surface of the concrete; however, during mixing and placing it is practical to measure the temperature of the mixture.

R1.3.4 — A record of inspection in the form of a job diary is required in case questions subsequently arise concerning the performance or safety of the structure or members. Photographs documenting job progress may also be desirable.

Records of inspection must be preserved for at least 2 years after the completion of the project. The completion of the project is the date at which the owner accepts the project, or when a certificate of occupancy is issued, whichever date is later. The general building code or other legal requirements may require a longer preservation of such records.

R1.3.5 — The purpose of this section is to assure that the special detailing required in concrete ductile frames is properly executed through inspection by personnel who are qualified to do this work. Qualifications of inspectors should be determined by the jurisdiction enforcing the general building code.

R1.4 — Approval of special systems of design or construction

New methods of design, new materials, and new uses of materials must undergo a period of development before being specifically covered in a code. Hence, good systems or components might be excluded from use by implication if means were not available to obtain acceptance.

For special systems considered under this section, specific tests, load factors, deflection limits, and other pertinent requirements should be set by the board of examiners, and should be consistent with the intent of the code.

The provisions of this section do not apply to model tests used to supplement calculations under 1.2.2 or to strength evaluation of existing structures under Chapter 20.

CHAPTER 2 — DEFINITIONS

CODE

2.1 — The following terms are defined for general use in this code. Specialized definitions appear in individual chapters.

Admixture — Material other than water, aggregate, or hydraulic cement, used as an ingredient of concrete and added to concrete before or during its mixing to modify its properties.

Aggregate — Granular material, such as sand, gravel, crushed stone, and iron blast-furnace slag, used with a cementing medium to form a hydraulic cement concrete or mortar.

Aggregate, lightweight — Aggregate with a dry, loose weight of 70 lb/ft^3 or less.

Anchorage — In post-tensioning, a device used to anchor tendon to concrete member; in pretensioning, a device used to anchor tendon during hardening of concrete.

Bonded tendon — Prestressing tendon that is bonded to concrete either directly or through grouting.

Building official — See 1.2.3.

Cementitious materials — Materials as specified in Chapter 3, which have cementing value when used in concrete either by themselves, such as portland cement, blended hydraulic cements, and expansive cement, or such materials in combination with fly ash, other raw or calcined natural pozzolans, silica fume, and/or ground granulated blast-furnace slag.

Column — Member with a ratio of height-to-least lateral dimension exceeding 3 used primarily to support axial compressive load.

Composite concrete flexural members — Concrete flexural members of precast and/or cast-in-place concrete elements constructed in separate placements but so interconnected that all elements respond to loads as a unit.

Compression-controlled section — A cross section in which the net tensile strain in the extreme tension steel at nominal strength is less than or equal to the compression-controlled strain limit.

Compression-controlled strain limit — The net tensile strain at balanced strain conditions. See B10.3.2.

Concrete — Mixture of portland cement or any other

COMMENTARY

R2.1 — For consistent application of the code, it is necessary that terms be defined where they have particular meanings in the code. The definitions given are for use in application of this code only and do not always correspond to ordinary usage. A glossary of most used terms relating to cement manufacturing, concrete design and construction, and research in concrete is contained in **"Cement and Concrete Terminology"** reported by ACI Committee 116.[2.1]

By code definition, "sand-lightweight concrete" is structural lightweight concrete with *all* of the fine aggregate replaced by sand. This definition may not be in agreement with usage by some material suppliers or contractors where the majority, but not all, of the lightweight fines are replaced by sand. For proper application of the code provisions, the replacement limits must be stated, with interpolation when partial sand replacement is used.

Deformed reinforcement is defined as that meeting the deformed bar specifications of 3.5.3.1, or the specifications of 3.5.3.3, 3.5.3.4, 3.5.3.5, or 3.5.3.6. No other bar or fabric qualifies. This definition permits accurate statement of anchorage lengths. Bars or wire not meeting the deformation requirements or fabric not meeting the spacing requirements are "plain reinforcement," for code purposes, and may be used only for spirals.

A number of definitions for loads are given as the code contains requirements that must be met at various load levels. The terms "dead load" and "live load" refer to the unfactored loads (service loads) specified or defined by the general building code. Service loads (loads without load factors) are to be used where specified in the code to proportion or investigate members for adequate serviceability as in 9.5, Control of Deflections. Loads used to proportion a member for adequate strength are defined as "factored loads." Factored loads are service loads multiplied by the appropriate load factors specified in 9.2 for required strength. The term "design loads," as used in the 1971 code edition to refer to loads multiplied by appropriate load factors, was discontinued in the 1977 code to avoid confusion with the design load terminology used in general building codes to denote service loads, or posted loads in buildings. The factored load terminology, first adopted in the 1977 code, clarifies when the load factors are applied to a particular load, moment, or shear value as used in the code provisions.

Reinforced concrete is defined to include prestressed concrete. Although the behavior of a prestressed member with unbonded tendons may vary from that of members with

CODE

hydraulic cement, fine aggregate, coarse aggregate, and water, with or without admixtures.

Concrete, specified compressive strength of, **(f_c')** — Compressive strength of concrete used in design and evaluated in accordance with provisions of Chapter 5, expressed in pounds per square inch (psi). Whenever the quantity f_c' is under a radical sign, square root of numerical value only is intended, and result has units of pounds per square inch (psi).

Concrete, structural lightweight — Concrete containing lightweight aggregate that conforms to 3.3 and has an air-dry unit weight as determined by "Test Method for Unit Weight of Structural Lightweight Concrete" (ASTM C 567), not exceeding 115 lb/ft^3. In this code, a lightweight concrete without natural sand is termed "all-lightweight concrete" and lightweight concrete in which all of the fine aggregate consists of normal weight sand is termed "sand-lightweight concrete."

Contraction joint — Formed, sawed, or tooled groove in a concrete structure to create a weakened plane and regulate the location of cracking resulting from the dimensional change of different parts of the structure.

Curvature friction — Friction resulting from bends or curves in the specified prestressing tendon profile.

Deformed reinforcement — Deformed reinforcing bars, bar mats, deformed wire, welded plain wire fabric, and welded deformed wire fabric conforming to 3.5.3.

Development length — Length of embedded reinforcement required to develop the design strength of reinforcement at a critical section. See 9.3.3.

Effective depth of section **(d)** — Distance measured from extreme compression fiber to centroid of tension reinforcement.

Effective prestress — Stress remaining in prestressing tendons after all losses have occurred, excluding effects of dead load and superimposed load.

Embedment length — Length of embedded reinforcement provided beyond a critical section.

Extreme tension steel — The reinforcement (prestressed or nonprestressed) that is the farthest from the extreme compression fiber.

Isolation joint — A separation between adjoining parts of a concrete structure, usually a vertical plane, at a designed location such as to interfere least with performance of the structure, yet such as to allow rela-

COMMENTARY

continuously bonded tendons, bonded and unbonded prestressed concrete are combined with conventionally reinforced concrete under the generic term "reinforced concrete." Provisions common to both prestressed and conventionally reinforced concrete are integrated to avoid overlapping and conflicting provisions.

Strength of a member or cross section calculated using standard assumptions and strength equations, and nominal (specified) values of material strengths and dimensions is referred to as "nominal strength." The subscript n is used to denote the nominal strengths; nominal axial load strength P_n, nominal moment strength M_n, and nominal shear strength V_n. "Design strength" or usable strength of a member or cross section is the nominal strength reduced by the strength reduction factor ϕ.

The required axial load, moment, and shear strengths used to proportion members are referred to either as factored axial loads, factored moments, and factored shears, or required axial loads, moments, and shears. The factored load effects are calculated from the applied factored loads and forces in such load combinations as are stipulated in the code (see 9.2).

The subscript u is used only to denote the required strengths; required axial load strength P_u, required moment strength M_u, and required shear strength V_u, calculated from the applied factored loads and forces.

The basic requirement for strength design may be expressed as follows:

$$\text{Design strength} \geq \text{Required strength}$$

$$\phi P_n \geq P_u$$

$$\phi M_n \geq M_u$$

$$\phi V_n \geq V_u$$

For additional discussion on the concepts and nomenclature for strength design see commentary Chapter 9.

The term "compression member" is used in the code to define any member in which the primary stress is longitudinal compression. Such a member need not be vertical but may have any orientation in space. Bearing walls, columns, and pedestals qualify as compression members under this definition.

The differentiation between columns and walls in the code is based on the principal use rather than on arbitrary relationships of height and cross-sectional dimensions. The code, however, permits walls to be designed using the prin-

CODE

tive movement in three directions and avoid formation of cracks elsewhere in the concrete and through which all or part of the bonded reinforcement is interrupted.

Jacking force — In prestressed concrete, temporary force exerted by device that introduces tension into prestressing tendons.

Load, dead — Dead weight supported by a member, as defined by general building code of which this code forms a part (without load factors).

Load, factored — Load, multiplied by appropriate load factors, used to proportion members by the strength design method of this code. See 8.1.1 and 9.2.

Load, live — Live load specified by general building code of which this code forms a part (without load factors).

Load, service — Load specified by general building code of which this code forms a part (without load factors).

Modulus of elasticity — Ratio of normal stress to corresponding strain for tensile or compressive stresses below proportional limit of material. See 8.5.

Net tensile strain — The tensile strain at nominal strength exclusive of strains due to effective prestress, creep, shrinkage, and temperature.

Pedestal — Upright compression member with a ratio of unsupported height to average least lateral dimension of less than 3.

Plain concrete — Structural concrete with no reinforcement or with less reinforcement than the minimum amount specified for reinforced concrete.

Plain reinforcement — Reinforcement that does not conform to definition of deformed reinforcement. See 3.5.4.

Post-tensioning — Method of prestressing in which tendons are tensioned after concrete has hardened.

Precast concrete — Structural concrete element cast elsewhere than its final position in the structure.

Prestressed concrete — Structural concrete in which internal stresses have been introduced to reduce potential tensile stresses in concrete resulting from loads.

Pretensioning — Method of prestressing in which tendons are tensioned before concrete is placed.

Reinforced concrete — Structural concrete reinforced with no less than the minimum amounts of pre-

COMMENTARY

ciples stated for column design (see 14.4), as well as by the empirical method (see 14.5).

While a wall always encloses or separates spaces, it may also be used to resist horizontal or vertical forces or bending. For example, a retaining wall or a basement wall also supports various combinations of loads.

A column is normally used as a main vertical member carrying axial loads combined with bending and shear. It may, however, form a small part of an enclosure or separation.

CODE

COMMENTARY

stressing tendons or nonprestressed reinforcement specified in Chapters 1 through 21 and Appendices A through C.

Reinforcement — Material that conforms to 3.5, excluding prestressing tendons unless specifically included.

Reshores — Shores placed snugly under a concrete slab or other structural member after the original forms and shores have been removed from a larger area, thus requiring the new slab or structural member to deflect and support its own weight and existing construction loads applied prior to the installation of the reshores.

Shores — Vertical or inclined support members designed to carry the weight of the formwork, concrete, and construction loads above.

Span length — See 8.7.

Spiral reinforcement — Continuously wound reinforcement in the form of a cylindrical helix.

Splitting tensile strength (f_{ct}) — Tensile strength of concrete determined in accordance with ASTM C 496 as described in "Specification for Lightweight Aggregates for Structural Concrete" (ASTM C 330). See 5.1.4.

Stirrup — Reinforcement used to resist shear and torsion stresses in a structural member; typically bars, wires, or welded wire fabric (plain or deformed) either single leg or bent into L, U, or rectangular shapes and located perpendicular to or at an angle to longitudinal reinforcement. (The term "stirrups" is usually applied to lateral reinforcement in flexural members and the term "ties" to those in compression members.) See also *Tie.*

Strength, design — Nominal strength multiplied by a strength reduction factor ϕ. See 9.3.

Strength, nominal — Strength of a member or cross section calculated in accordance with provisions and assumptions of the strength design method of this code before application of any strength reduction factors. See 9.3.1.

Strength, required — Strength of a member or cross section required to resist factored loads or related internal moments and forces in such combinations as are stipulated in this code. See 9.1.1.

Stress — Intensity of force per unit area.

CODE

COMMENTARY

Structural concrete — All concrete used for structural purposes including plain and reinforced concrete.

Tendon — Steel element such as wire, cable, bar, rod, or strand, or a bundle of such elements, used to impart prestress to concrete.

Tension-controlled section — A cross section in which the net tensile strain in the extreme tension steel at nominal strength is greater than or equal to 0.005.

Tie — Loop of reinforcing bar or wire enclosing longitudinal reinforcement. A continuously wound bar or wire in the form of a circle, rectangle, or other polygon shape without re-entrant corners is acceptable. See also *Stirrup*.

Transfer — Act of transferring stress in prestressing tendons from jacks or pretensioning bed to concrete member.

Wall — Member, usually vertical, used to enclose or separate spaces.

Wobble friction — In prestressed concrete, friction caused by unintended deviation of prestressing sheath or duct from its specified profile.

Yield strength — Specified minimum yield strength or yield point of reinforcement in pounds per square inch. Yield strength or yield point shall be determined in tension according to applicable ASTM standards as modified by 3.5 of this code.

Notes

PART 2 — STANDARDS FOR TESTS AND MATERIALS

CHAPTER 3 — MATERIALS

CODE

3.0 — Notation

f_y = specified yield strength of nonprestressed reinforcement, psi

3.1 — Tests of materials

3.1.1 — Building official shall have the right to order testing of any materials used in concrete construction to determine if materials are of quality specified.

3.1.2 — Tests of materials and of concrete shall be made in accordance with standards listed in 3.8.

3.1.3 — A complete record of tests of materials and of concrete shall be available for inspection during progress of work and for 2 years after completion of the project, and shall be preserved by inspecting engineer or architect for that purpose.

3.2 — Cements

3.2.1 — Cement shall conform to one of the following specifications:

(a) "Specification for Portland Cement" (ASTM C 150).

(b) "Specification for Blended Hydraulic Cements" (ASTM C 595), excluding Types S and SA which are not intended as principal cementing constituents of structural concrete.

(c) "Specification for Expansive Hydraulic Cement" (ASTM C 845).

3.2.2 — Cement used in the work shall correspond to that on which selection of concrete proportions was based. See 5.2.

COMMENTARY

R3.1 — Tests of materials

R3.1.3 — The record of tests of materials and of concrete must be preserved for at least 2 years after completion of the project. Completion of the project is the date at which the owner accepts the project or when the certificate of occupancy is issued, whichever date is later. Local legal requirements may require longer preservation of such records.

R3.2 — Cements

R3.2.2 — Depending on the circumstances, the provision of 3.2.2 may require only the same type of cement or may require cement from the identical source. The latter would be the case if the standard deviation[3.1] of strength tests used in establishing the required strength margin was based on a cement from a particular source. If the standard deviation was based on tests involving a given type of cement obtained from several sources, the former interpretation would apply.

CODE

3.3 — Aggregates

3.3.1 — Concrete aggregates shall conform to one of the following specifications:

(a) "Specification for Concrete Aggregates" (ASTM C 33).

(b) "Specification for Lightweight Aggregates for Structural Concrete" (ASTM C 330).

Exception: Aggregates which have been shown by special test or actual service to produce concrete of adequate strength and durability and approved by the building official.

3.3.2 — Nominal maximum size of coarse aggregate shall be not larger than:

(a) $^{1}/_{5}$ the narrowest dimension between sides of forms, nor

(b) $^{1}/_{3}$ the depth of slabs, nor

(c) $^{3}/_{4}$ the minimum clear spacing between individual reinforcing bars or wires, bundles of bars, or prestressing tendons or ducts.

These limitations shall not apply if, in the judgment of the engineer, workability and methods of consolidation are such that concrete can be placed without honeycomb or voids.

3.4 — Water

3.4.1 — Water used in mixing concrete shall be clean and free from injurious amounts of oils, acids, alkalis, salts, organic materials, or other substances deleterious to concrete or reinforcement.

COMMENTARY

R3.3 — Aggregates

R3.3.1 — It is recognized that aggregates conforming to the ASTM specifications are not always economically available and that, in some instances, noncomplying materials have a long history of satisfactory performance. Such nonconforming materials are permitted with special approval when acceptable evidence of satisfactory performance is provided. It should be noted, however, that satisfactory performance in the past does not guarantee good performance under other conditions and in other localities. Whenever possible, aggregates conforming to the designated specifications should be used.

R3.3.2 — The size limitations on aggregates are provided to ensure proper encasement of reinforcement and to minimize honeycomb. Note that the limitations on maximum size of the aggregate may be waived if, in the judgment of the engineer, the workability and methods of consolidation of the concrete are such that the concrete can be placed without honeycomb or voids. In this instance, the engineer must decide whether or not the limitations on maximum size of aggregate may be waived.

R3.4 — Water

R3.4.1 — Almost any natural water that is drinkable (potable) and has no pronounced taste or odor is satisfactory as mixing water for making concrete. Impurities in mixing water, when excessive, may affect not only setting time, concrete strength, and volume stability (length change), but may also cause efflorescence or corrosion of reinforcement. Where possible, water with high concentrations of dissolved solids should be avoided.

Salts, or other deleterious substances contributed from the aggregate or admixtures are additive to the amount which might be contained in the mixing water. These additional amounts must be considered in evaluating the acceptability of the total impurities that may be deleterious to concrete or steel.

CODE

3.4.2 — Mixing water for prestressed concrete or for concrete that will contain aluminum embedments, including that portion of mixing water contributed in the form of free moisture on aggregates, shall not contain deleterious amounts of chloride ion. See 4.4.1.

3.4.3 — Nonpotable water shall not be used in concrete unless the following are satisfied:

3.4.3.1 — Selection of concrete proportions shall be based on concrete mixes using water from the same source.

3.4.3.2 — Mortar test cubes made with nonpotable mixing water shall have 7-day and 28-day strengths equal to at least 90 percent of strengths of similar specimens made with potable water. Strength test comparison shall be made on mortars, identical except for the mixing water, prepared and tested in accordance with "Test Method for Compressive Strength of Hydraulic Cement Mortars (Using 2-in. or 50-mm Cube Specimens)" (ASTM C 109).

3.5 — Steel reinforcement

3.5.1 — Reinforcement shall be deformed reinforcement, except that plain reinforcement shall be permitted for spirals or tendons; and reinforcement consisting of structural steel, steel pipe, or steel tubing shall be permitted as specified in this code.

3.5.2 — Welding of reinforcing bars shall conform to "Structural Welding Code — Reinforcing Steel," ANSI/AWS D1.4 of the American Welding Society. Type and location of welded splices and other required welding of reinforcing bars shall be indicated on the design drawings or in the project specifications. ASTM reinforcing bar specifications, except for ASTM A 706, shall be supplemented to require a report of material properties necessary to conform to the requirements in ANSI/AWS D1.4.

AWS D1.4-92 new code

COMMENTARY

R3.5 — Steel reinforcement

R3.5.1 — Materials permitted for use as reinforcement are specified. Other metal elements, such as inserts, anchor bolts, or plain bars for dowels at isolation or contraction joints, are not normally considered to be reinforcement under the provisions of this code.

R3.5.2 — When welding of reinforcing bars is required, the weldability of the steel and compatible welding procedures need to be considered. The provisions in ANSI/AWS D1.4 Welding Code cover aspects of welding reinforcing bars, including criteria to qualify welding procedures.

Weldability of the steel is based on its chemical composition or carbon equivalent (CE). The Welding Code establishes preheat and interpass temperatures for a range of carbon equivalents and reinforcing bar sizes. Carbon equivalent is calculated from the chemical composition of the reinforcing bars. The Welding Code has two expressions for calculating carbon equivalent. A relatively short expression, considering only the elements carbon and manganese, is to be used for bars other than ASTM A 706 material. A more comprehensive expression is given for ASTM A 706 bars. The CE formula in the Welding Code for A 706 bars is identical to the CE formula in the ASTM A 706 specification.

The engineer should realize that the chemical analysis, for bars other than A 706, required to calculate the carbon equivalent is not routinely provided by the producer of the

CODE

3.5.3 — Deformed reinforcement

3.5.3.1 — Deformed reinforcing bars shall conform to one of the following specifications:

COMMENTARY

reinforcing bars. Hence, for welding reinforcing bars other than A 706 bars, the design drawings or project specifications should specifically require results of the chemical analysis to be furnished.

The ASTM A 706 specification covers low-alloy steel reinforcing bars intended for applications requiring controlled tensile properties or welding. Weldability is accomplished in the A 706 specification by limits or controls on chemical composition and on carbon equivalent.[3.2] The producer is required by the A 706 specification to report the chemical composition and carbon equivalent.

The ANSI/AWS D1.4 Welding Code requires the contractor to prepare written welding procedure specifications conforming to the requirements of the Welding Code. Appendix A of the Welding Code contains a suggested form which shows the information required for such a specification for each joint welding procedure.

Often it is necessary to weld to existing reinforcing bars in a structure when no mill test report of the existing reinforcement is available. This condition is particularly common in alterations or building expansions. ANSI/AWS D1.4 states for such bars that a chemical analysis may be performed on representative bars. If the chemical composition is not known or obtained, the Welding Code requires a minimum preheat. For bars other than A 706 material, the minimum preheat required is 300 F for bars No. 6 or smaller, and 400 F for No. 7 bars or larger. The required preheat for all sizes of A 706 is to be the temperature given in the Welding Code's table for minimum preheat corresponding to the range of CE "over 45 percent to 55 percent." Welding of the particular bars must then be performed in accordance with ANSI/AWS D 1.4. It should also be determined if additional precautions are in order, based on other considerations such as stress level in the bars, consequences of failure, and heat damage to existing concrete due to welding operations.

Welding of wire to wire, and of wire or welded wire fabric to reinforcing bars or structural steel elements is not covered by ANSI/AWS D1.4. If welding of this type is required on a project, the engineer should specify requirements or performance criteria for this welding. If cold drawn wires are to be welded, the welding procedures should address the potential loss of yield strength and ductility, achieved by the cold working process (during manufacture), when such wires are heated by welding. Machine and resistance welding as used in the manufacture of welded wire fabrics is covered by ASTM A 185 and A 497 and is not part of this concern.

R3.5.3 — Deformed reinforcement

R3.5.3.1 — ASTM A 615 covers specifications for deformed billet-steel reinforcing bars which are normally

CODE

(a) "Specification for Deformed and Plain Billet-Steel Bars for Concrete Reinforcement" (ASTM A 615).

(b) "Specification for Rail-Steel Deformed and Plain Bars for Concrete Reinforcement" including Supplementary Requirement S1 (ASTM A 616 including S1).

(c) "Specification for Axle-Steel Deformed and Plain Bars for Concrete Reinforcement" (ASTM A 617).

(d) "Specification for Low-Alloy Steel Deformed Bars for Concrete Reinforcement" (ASTM A 706).

3.5.3.2 — Deformed reinforcing bars with a specified yield strength f_y exceeding 60,000 psi shall be permitted, provided f_y shall be the stress corresponding to a strain of 0.35 percent and the bars otherwise conform to one of the ASTM specifications listed in 3.5.3.1. See 9.4.

3.5.3.3 — Bar mats for concrete reinforcement shall conform to "Specification for Fabricated Deformed Steel Bar Mats for Concrete Reinforcement" (ASTM A 184). Reinforcing bars used in bar mats shall conform to one of the specifications listed in 3.5.3.1.

3.5.3.4 — Deformed wire for concrete reinforcement shall conform to "Specification for Steel Wire, Deformed, for Concrete Reinforcement" (ASTM A 496), except that wire shall not be smaller than size D4 and for wire with a specified yield strength f_y exceeding 60,000 psi, f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 60,000 psi.

COMMENTARY

used in reinforced concrete construction in the United States. The specification also requires that all billet-steel reinforcing bars be marked with the letter S.

Rail-steel reinforcing bars used with this code must conform to ASTM A 616 including Supplementary Requirement S1, marked with the letter R, in addition to the rail symbol. S1 prescribes more restrictive requirements for bend tests.

ASTM A 706 covers low-alloy steel deformed bars intended for special applications where welding or bending, or both, are of importance. The specification requires that the bars be marked with the letter W for type of steel.

R3.5.3.2 — ASTM A 615 includes provisions for Grade 75 bars in sizes No. 6 through 18.

The 0.35 percent strain limit is necessary to ensure that the assumption of an elasto-plastic stress-strain curve in 10.2.4 will not lead to unconservative values of the member strength.

The 0.35 strain requirement is not applied to reinforcing bars having yield strengths of 60,000 psi or less. For steels, having strengths of 40,000 psi, as were once used extensively, the assumption of an elasto-plastic stress-strain curve is well justified by extensive test data. For higher strength steels, up to 60,000 psi, the stress-strain curve may or may not be elasto-plastic as assumed in 10.2.4, depending on the properties of the steel and the manufacturing process. However, when the stress-strain curve is not elasto-plastic, there is limited experimental evidence to suggest that the actual steel stress at ultimate strength may not be enough less than the specified yield strength to warrant the additional effort of testing to the more restrictive criterion applicable to steels having f_y greater than 60,000 psi. In such cases, the ϕ-factor can be expected to account for the strength deficiency.

CODE

3.5.3.5 — Welded plain wire fabric for concrete reinforcement shall conform to "Specification for Steel Welded Wire Fabric, Plain, for Concrete Reinforcement" (ASTM A 185), except that for wire with a specified yield strength f_y exceeding 60,000 psi, f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 60,000 psi. Welded intersections shall not be spaced farther apart than 12 in. in direction of calculated stress, except for wire fabric used as stirrups in accordance with 12.13.2.

3.5.3.6 — Welded deformed wire fabric for concrete reinforcement shall conform to "Specification for Steel Welded Wire Fabric, Deformed, for Concrete Reinforcement" (ASTM A 497), except that for wire with a specified yield strength f_y exceeding 60,000 psi, f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 60,000 psi. Welded intersections shall not be spaced farther apart than 16 in. in direction of calculated stress, except for wire fabric used as stirrups in accordance with 12.13.2.

3.5.3.7 — Galvanized reinforcing bars shall comply with "Specification for Zinc-Coated (Galvanized) Steel Bars for Concrete Reinforcement" (ASTM A 767). Epoxy-coated reinforcing bars shall comply with "Specification for Epoxy-Coated Reinforcing Steel Bars" (ASTM A 775) or with "Specification for Epoxy-Coated Prefabricated Steel Reinforcing Bars" (ASTM A 934). Galvanized or epoxy-coated reinforcement shall conform to one of the specifications listed in 3.5.3.1.

3.5.3.8 — Epoxy-coated wires and welded wire fabric shall comply with "Specification for Epoxy-Coated Steel Wire and Welded Wire Fabric for Reinforcement" (ASTM A 884). Epoxy-coated wires shall conform to 3.5.3.4 and epoxy-coated welded wire fabric shall conform to 3.5.3.5 or 3.5.3.6.

3.5.4 — Plain reinforcement

3.5.4.1 — Plain bars for spiral reinforcement shall conform to the specification listed in 3.5.3.1(a), (b), or (c).

3.5.4.2 — Plain wire for spiral reinforcement shall conform to "Specification for Steel Wire, Plain, for Concrete Reinforcement" (ASTM A 82), except that for wire with a specified yield strength f_y exceeding 60,000 psi, f_y shall be the stress corresponding to a strain of 0.35 percent if the yield strength specified in the design exceeds 60,000 psi.

COMMENTARY

R3.5.3.5 — Welded plain wire fabric must be made of wire conforming to "Specification for Steel Wire, Plain, for Concrete Reinforcement" (ASTM A 82). ASTM A 82 has a minimum yield strength of 70,000 psi. The code has assigned a yield strength value of 60,000 psi, but makes provision for the use of higher yield strengths provided the stress corresponds to a strain of 0.35 percent.

R3.5.3.6 — Welded deformed wire fabric must be made of wire conforming to "Specification for Steel Wire, Deformed, for Concrete Reinforcement" (ASTM A 496). ASTM A 496 has a minimum yield strength of 70,000 psi. The code has assigned a yield strength value of 60,000 psi, but makes provision for the use of higher yield strengths provided the stress corresponds to a strain of 0.35 percent.

R3.5.3.7 — Galvanized reinforcing bars (A 767) and epoxy-coated reinforcing bars (A 775) were added to the 1983 code, and epoxy-coated prefabricated reinforcing bars (A 934) were added to the 1995 code recognizing their usage, especially for conditions where corrosion resistance of reinforcement is of particular concern. They have typically been used in parking decks, bridge decks, and other highly corrosive environments.

R3.5.4 — Plain reinforcement

Plain bars and plain wire are permitted only for spiral reinforcement (either as lateral reinforcement for compression members, for torsion members, or for confining reinforcement for splices).

CODE

3.5.5 — Prestressing tendons

3.5.5.1 — Tendons for prestressed reinforcement shall conform to one of the following specifications:

(a) Wire conforming to "Specification for Uncoated Stress-Relieved Steel Wire for Prestressed Concrete" (ASTM A 421).

(b) Low-relaxation wire conforming to "Specification for Uncoated Stress-Relieved Steel Wire for Prestressed Concrete" including Supplement "Low-Relaxation Wire" (ASTM A 421).

(c) Strand conforming to "Specification for Steel Strand, Uncoated Seven-Wire for Prestressed Concrete" (ASTM A 416).

(d) Bar conforming to "Specification for Uncoated High-Strength Steel Bar for Prestressed Concrete" (ASTM A 722).

3.5.5.2 — Wire, strands, and bars not specifically listed in ASTM A 421, A 416, or A 722 are allowed provided they conform to minimum requirements of these specifications and do not have properties that make them less satisfactory than those listed in ASTM A 421, A 416, or A 722.

3.5.6 — Structural steel, steel pipe, or tubing

3.5.6.1 — Structural steel used with reinforcing bars in composite compression members meeting requirements of 10.16.7 or 10.16.8 shall conform to one of the following specifications:

(a) "Specification for Structural Steel" (ASTM A 36).

(b) "Specification for High-Strength Low-Alloy Structural Steel" (ASTM A 242).

(c) "Specification for High-Strength Low-Alloy Columbium-Vanadium Steels of Structural Quality" (ASTM A 572).

(d) "Specification for High-Strength Low-Alloy Structural Steel with 50 ksi (345 MPa) Minimum Yield Point to 4 in. (100 mm) Thick" (ASTM A 588).

3.5.6.2 — Steel pipe or tubing for composite compression members composed of a steel encased concrete core meeting requirements of 10.16.6 shall conform to one of the following specifications:

(a) Grade B of "Specification for Pipe, Steel, Black and Hot-Dipped, Zinc-Coated Welded and Seamless" (ASTM A 53).

COMMENTARY

R3.5.5 — Prestressing tendons

R3.5.5.1 — Since low-relaxation tendons are addressed in a supplement to ASTM A 421 which applies only when low-relaxation material is specified, the appropriate ASTM reference is listed as a separate entity.

CODE

(b) "Specification for Cold-Formed Welded and Seamless Carbon Steel Structural Tubing in Rounds and Shapes" (ASTM A 500).

(c) "Specification for Hot-Formed Welded and Seamless Carbon Steel Structural Tubing" (ASTM A 501).

3.6 — Admixtures

3.6.1 — Admixtures to be used in concrete shall be subject to prior approval by the engineer.

3.6.2 — An admixture shall be shown capable of maintaining essentially the same composition and performance throughout the work as the product used in establishing concrete proportions in accordance with 5.2.

3.6.3 — Calcium chloride or admixtures containing chloride from other than impurities from admixture ingredients shall not be used in prestressed concrete, in concrete containing embedded aluminum, or in concrete cast against stay-in-place galvanized steel forms. See 4.3.2 and 4.4.1.

3.6.4 — Air-entraining admixtures shall conform to "Specification for Air-Entraining Admixtures for Concrete" (ASTM C 260).

3.6.5 — Water-reducing admixtures, retarding admixtures, accelerating admixtures, water-reducing and retarding admixtures, and water-reducing and accelerating admixtures shall conform to "Specification for Chemical Admixtures for Concrete" (ASTM C 494) or "Specification for Chemical Admixtures for Use in Producing Flowing Concrete" (ASTM C 1017).

3.6.6 — Fly ash or other pozzolans used as admixtures shall conform to "Specification for Fly Ash and Raw or Calcined Natural Pozzolan for Use as a Mineral Admixture in Portland Cement Concrete" (ASTM C 618).

3.6.7 — Ground granulated blast-furnace slag used as an admixture shall conform to "Specification for Ground Granulated Blast-Furnace Slag for Use in Concrete and Mortars" (ASTM C 989).

COMMENTARY

R3.6 — Admixtures

R3.6.3 — Admixtures containing any chloride, other than impurities from admixture ingredients, must not be used in prestressed concrete or in concrete with aluminum embedments. Concentrations of chloride ion may produce corrosion of embedded aluminum (e.g., conduit), especially if the aluminum is in contact with embedded steel and the concrete is in a humid environment. Serious corrosion of galvanized steel sheet and galvanized steel stay-in-place forms occurs, especially in humid environments or where drying is inhibited by the thickness of the concrete or coatings or impermeable coverings. See 4.4.1 for specific limits on chloride ion concentration in concrete.

R3.6.7 — Ground granulated blast-furnace slag conforming to ASTM C 989 is used as an admixture in concrete in much the same way as fly ash. Generally, it should be used with portland cements conforming to ASTM C 150 and only

CODE

3.6.8 — Admixtures used in concrete containing C 845 expansive cements shall be compatible with the cement and produce no deleterious effects.

3.6.9 — Silica fume used as an admixture shall conform to "Specification for Silica Fume for Use in Hydraulic-Cement Concrete and Mortar" (ASTM C 1240).

3.7 — Storage of materials

3.7.1 — Cementitious materials and aggregates shall be stored in such manner as to prevent deterioration or intrusion of foreign matter.

3.7.2 — Any material that has deteriorated or has been contaminated shall not be used for concrete.

3.8 — Standards cited in this code

3.8.1 — Standards of the American Society for Testing and Materials referred to in this code are listed below with their serial designations, including year of adoption or revision, and are declared to be part of this code as if fully set forth herein:

A 36-94	Standard Specification for Structural Steel
A 53-93a	Standard Specification for Pipe, Steel, Black and Hot-Dipped, Zinc-Coated Welded and Seamless
A 82-94	Standard Specification for Steel Wire, Plain, for Concrete Reinforcement
A 184-90	Standard Specification for Fabricated Deformed Steel Bar Mats for Concrete Reinforcement
A 185-94	Standard Specification for Steel Welded Wire Fabric, Plain, for Concrete Reinforcement

COMMENTARY

rarely would it be appropriate to use ASTM C 989 slag with an ASTM C 595 blended cement which already contains a pozzolan or slag. Such use with ASTM C 595 cements might be considered for massive concrete placements where slow strength gain can be tolerated and where low heat of hydration is of particular importance. ASTM C 989 includes appendices which discuss effects of ground granulated blast-furnace slag on concrete strength, sulfate resistance, and alkali-aggregate reaction.

R3.6.8 — The use of admixtures in concrete containing C 845 expansive cements has reduced levels of expansion or increased shrinkage values. See ACI 223.[3.3]

R3.8 — Standards cited in this code

The ASTM standard specifications listed are the latest editions at the time these code provisions were adopted. Since these specifications are revised frequently, generally in minor details only, the user of the code should check directly with the sponsoring organization if it is desired to reference the latest edition. However, such a procedure obligates the user of the specification to evaluate if any changes in the later edition are significant in the use of the specification.

Standard specifications or other material to be legally adopted by reference into a building code must refer to a specific document. This can be done by simply using the complete serial designation since the first part indicates the subject and the second part the year of adoption. All standard documents referenced in this code are listed in 3.8, with the title and complete serial designation. In other sections of the code, the designations do not include the date so that all may be kept up-to-date by simply revising 3.8.

CODE

A 242-93a	Standard Specification for High-Strength Low-Alloy Structural Steel
A 416-94	Standard Specification for Steel Strand, Uncoated Seven-Wire for Prestressed Concrete
A 421-91	Standard Specification for Uncoated Stress-Relieved Steel Wire for Prestressed Concrete
A 496-94	Standard Specification for Steel Wire, Deformed, for Concrete Reinforcement
A 497-94a	Standard Specification for Steel Welded Wire Fabric, Deformed, for Concrete Reinforcement
A 500-93	Standard Specification for Cold-Formed Welded and Seamless Carbon Steel Structural Tubing in Rounds and Shapes
A 501-93	Standard Specification for Hot-Formed Welded and Seamless Carbon Steel Structural Tubing
A 572-94b	Standard Specification for High-Strength Low-Alloy Columbium-Vanadium Steels of Structural Quality
A 588-94	Standard Specification for High-Strength Low-Alloy Structural Steel with 50 ksi (345 MPa) Minimum Yield Point to 4 in. (100 mm) Thick
A 615-94	Standard Specification for Deformed and Plain Billet-Steel Bars for Concrete Reinforcement
A 616-93[*]	Standard Specification for Rail-Steel Deformed and Plain Bars for Reinforcement, including Supplementary Requirement S1
A 617-93	Standard Specification for Axle-Steel Deformed and Plain Bars for Concrete Reinforcement
A 706-92b	Standard Specification for Low-Alloy Steel Deformed Bars for Concrete Reinforcement
A 722-90	Standard Specification for Uncoated

COMMENTARY

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[*] Supplementary Requirement (S1) of ASTM A 616 shall be considered a mandatory requirement whenever ASTM A 616 is referenced in this code.

	High-Strength Steel Bar for Prestressing Concrete
A 767-90	Standard Specification for Zinc-Coated (Galvanized) Steel Bars for Concrete Reinforcement
A 775-94d	Standard Specification for Epoxy-Coated Reinforcing Steel Bars
A 884-94a	Standard Specification for Epoxy-Coated Steel Wire and Welded Wire Fabric for Reinforcement
A 934-95	Standard Specification for Epoxy-Coated Prefabricated Steel Reinforcing Bars
C 31-91	Standard Practice for Making and Curing Concrete Test Specimens in the Field
C 33-93	Standard Specification for Concrete Aggregates
C 39-93a	Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens
C 42-90	Standard Method of Obtaining and Testing Drilled Cores and Sawed Beams of Concrete
C 94-94	Standard Specification for Ready-Mixed Concrete
C 109-93	Standard Test Method for Compressive Strength of Hydraulic Cement Mortars (Using 2-in. or 50-mm Cube Specimens)
C 144-93	Standard Specification for Aggregate for Masonry Mortar
C 150-94	Standard Specification for Portland Cement
C 172-90	Standard Method of Sampling Freshly Mixed Concrete
C 192-90a	Standard Method of Making and Curing Concrete Test Specimens in the Laboratory
C 260-94	Standard Specification for Air-Entraining Admixtures for Concrete
C 330-89	Standard Specification for Lightweight Aggregates for Structural Concrete

CODE

C 494-92	Standard Specification for Chemical Admixtures for Concrete
C 496-90	Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens
C 567-91	Standard Test Method for Unit Weight of Structural Lightweight Concrete
C 595-94a	Standard Specification for Blended Hydraulic Cements
C 618-94a	Standard Specification for Fly Ash and Raw or Calcined Natural Pozzolan for Use as a Mineral Admixture in Portland Cement Concrete
C 685-94	Standard Specification for Concrete Made by Volumetric Batching and Continuous Mixing
C 845-90	Standard Specification for Expansive Hydraulic Cement
C 989-93	Standard Specification for Ground Granulated Blast-Furnace Slag for Use in Concrete and Mortars
C 1017-92	Standard Specification for Chemical Admixtures for Use in Producing Flowing Concrete
C 1218-92$^{\varepsilon 1}$	Standard Test Method for Water-Soluble Chloride in Mortar and Concrete
C 1240-93	Standard Specification for Silica Fume for Use in Hydraulic-Cement Concrete and Mortar

3.8.2 — "Structural Welding Code—Reinforcing Steel" (ANSI/AWS D1.4-92) of the American Welding Society is declared to be part of this code as if fully set forth herein.

3.8.3 — Section 2.4 Combining Loads Using Strength of Design of "Minimum Design Loads for Buildings and Other Structures" (ASCE 7-88) is declared to be part of this code as if fully set forth herein, for the purpose cited in 9.3.1.1 and Appendix C.

3.8.4 — "Specification for Unbonded Single Strand Tendons," July 1993, of the Post-Tensioning Institute is declared to be part of this code as if fully set forth herein.

COMMENTARY

R3.8.3 — ASCE 7 is available from ASCE, 345 East 47th Street, New York, NY, 10017-2398.

R3.8.4 — The 1993 specification is available from: Post Tensioning Institute, 1717 W. Northern Ave., Suite 114, Phoenix, AZ, 85021.

PART 3 — CONSTRUCTION REQUIREMENTS

CHAPTER 4 — DURABILITY REQUIREMENTS

CODE

4.0 — Notation

f_c' = specified compressive strength of concrete, psi

4.1 — Water-cementitious materials ratio

4.1.1 — The water-cementitious materials ratios specified in Tables 4.2.2 and 4.3.1 shall be calculated using the weight of cement meeting ASTM C 150, C 595, or C 845 plus the weight of fly ash and other pozzolans meeting ASTM C 618, slag meeting ASTM C 989, and silica fume meeting ASTM C 1240, if any, except that when concrete is exposed to deicing chemicals, 4.2.3 further limits the amount of fly ash, pozzolans, silica fume, slag or the combination of these materials.

COMMENTARY

Chapters 4 and 5 of earlier editions of the code were reformatted in 1989 to emphasize the importance of considering durability requirements before the designer selects f_c' and cover over the reinforcing steel.

Maximum water-cementitious materials ratios of 0.40 to 0.50 that may be required for concretes exposed to freezing and thawing, sulfate soils or waters, or for preventing corrosion of reinforcement will typically be equivalent to requiring an f_c' of 5000 to 4000 psi, respectively. Generally, the required average concrete strengths, f_{cr}', will be 500 to 700 psi higher than the specified compressive strength, f_c'. Since it is difficult to accurately determine the water-cementitious materials ratio of concrete during production, the f_c' specified should be reasonably consistent with the water-cementitious materials ratio required for durability. Selection of an f_c' which is consistent with the water-cementitious materials ratio selected for durability will help ensure that the required water-cementitious materials ratio is actually obtained in the field. Because the usual emphasis on inspection is for strength, test results substantially higher than the specified strength may lead to a lack of concern for quality and production of concrete which exceeds the maximum water-cementitious materials ratio. Thus an f_c' of 3000 psi and a maximum water-cementitious materials ratio of 0.45 should not be specified for a parking structure, if the structure will be exposed to deicing salts.

The code does not include provisions for especially severe exposures, such as acids or high temperatures, and is not concerned with aesthetic considerations such as surface finishes. These items are beyond the scope of the code and must be covered specifically in the project specifications. Concrete ingredients and proportions must be selected to meet the minimum requirements stated in the code and the additional requirements of the contract documents.

R4.1 — Water-cementitious materials ratio

R4.1.1 — For concrete exposed to deicing chemicals the quantity of fly ash, other pozzolans, silica fume, slag, or blended cements used in the concrete is subject to the percentage limits in 4.2.3. Further, in 4.3 for sulfate exposures, the pozzolan should be Class F by ASTM C 618,[4.1] or have been tested by ASTM C 1012[4.2] or determined by service record to improve sulfate resistance.

CODE

4.2 — Freezing and thawing exposures

4.2.1 — Normal weight and lightweight concrete exposed to freezing and thawing or deicing chemicals shall be air-entrained with air content indicated in Table 4.2.1. Tolerance on air content as delivered shall be ± 1.5 percent. For specified compressive strength f_c' greater than 5000 psi, reduction of air content indicated in Table 4.2.1 by 1.0 percent shall be permitted.

TABLE 4.2.1—TOTAL AIR CONTENT FOR FROST-RESISTANT CONCRETE

Nominal maximum aggregate size, in.*	Air content, percent	
	Severe exposure	Moderate exposure
3/8	7 1/2	6
1/2	7	5 1/2
3/4	6	5
1	6	4 1/2
1 1/2	5 1/2	4 1/2
2†	5	4
3†	4 1/2	3 1/2

* See ASTM C 33 for tolerance on oversize for various nominal maximum size designations.

† These air contents apply to total mix, as for the preceding aggregate sizes. When testing these concretes, however, aggregate larger than 1 1/2 in. is removed by handpicking or sieving and air content is determined on the minus 1 1/2 in. fraction of mix (tolerance on air content as delivered applies to this value.). Air content of total mix is computed from value determined on the minus 1 1/2 in. fraction.

4.2.2 — Concrete that will be subject to the exposures given in Table 4.2.2 shall conform to the corresponding maximum water-cementitious materials ratios and minimum specified concrete compressive strength requirements of that table. In addition, concrete that will be exposed to deicing chemicals shall conform to the limitations of 4.2.3.

Table 4.2.2—REQUIREMENTS FOR SPECIAL EXPOSURE CONDITIONS

Exposure condition	Maximum water-cementitious materials ratio, by weight, normal weight aggregate concrete	Minimum f_c', normal weight and lightweight aggregate concrete, psi
Concrete intended to have low permeability when exposed to water	0.50	4000
Concrete exposed to freezing and thawing in a moist condition or to deicing chemicals	0.45	4500
For corrosion protection of reinforcement in concrete exposed to chlorides from deicing chemicals, salt, salt water, brackish water, seawater, or spray from these sources.	0.40	5000

COMMENTARY

R4.2 — Freezing and thawing exposures

R4.2.1—A table of required air contents for frost-resistant concrete is included in the code, based on **"Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete"** (ACI 211.1).[4.3] Values are provided for both severe and moderate exposures depending on the exposure to moisture or deicing salts. Entrained air will not protect concrete containing coarse aggregates that undergo disruptive volume changes when frozen in a saturated condition. In Table 4.2.1, a severe exposure is where the concrete in a cold climate may be in almost continuous contact with moisture prior to freezing, or where deicing salts are used. Examples are pavements, bridge decks, sidewalks, parking garages, and water tanks. A moderate exposure is where the concrete in a cold climate will be only occasionally exposed to moisture prior to freezinrg, and where no deicing salts are used. Examples are certain exterior walls, beams, girders, and slabs not in direct contact with soil. Section 4.2.1 permits 1 percent lower air content for concrete with f_c' greater than 5000 psi. Such high-strength concretes will have lower water-cementitious materials ratios and porosity and, therefore, improved frost resistance.

R4.2.2 — Maximum water-cementitious materials ratios are not specified for lightweight aggregate concrete because determination of the absorption of these aggregates is uncertain, making calculation of water-cementitious materials ratio uncertain. The use of a minimum specified strength will ensure the use of a high-quality cement paste. For normal weight aggregate concrete use of both minimum strength and maximum water-cementitious materials ratio provide additional assurance that this objective is met.

CODE

4.2.3 — For concrete exposed to deicing chemicals, the maximum weight of fly ash, other pozzolans, silica fume, or slag that is included in the concrete shall not exceed the percentages of the total weight of cementitious materials given in Table 4.2.3.

TABLE 4.2.3—REQUIREMENTS FOR CONCRETE EXPOSED TO DEICING CHEMICALS

Cementitious materials	Maximum percent of total cementitious materials by weight[*]
Fly ash or other pozzolans conforming to ASTM C 618	25
Slag conforming to ASTM C 989	50
Silica fume conforming to ASTM C 1240	10
Total of fly ash or other pozzolans, slag, and silica fume	50[†]
Total of fly ash or other pozzolans and silica fume	35[†]

[*] The total cementitious material also includes ASTM C 150, C 595, and C 845 cement.
The maximum percentages above shall include:
(a) Fly ash or other pozzolans present in Type IP or I(PM) blended cement, ASTM C 595
(b) Slag used in the manufacture of a IS or I(SM) blended cement, ASTM C 595
(c) Silica fume, ASTM C 1240, present in a blended cement

[†] Fly ash or other pozzolans and silica fume shall constitute no more than 25 and 10 percent, respectively, of the total weight of the cementitious materials.

COMMENTARY

R4.2.3 — Section 4.2.3 and Table 4.2.3 establish limitations on the amount of fly ash, other pozzolans, silica fume, and slag that can be included in concrete exposed to deicing chemicals.[4.4-4.6] Recent research has demonstrated that the use of fly ash, slag, and silica fume produce concrete with a finer pore structure and, therefore, lower permeability.[4.7-4.9]

4.3 — Sulfate exposures

4.3.1 — Concrete to be exposed to sulfate-containing solutions or soils shall conform to requirements of Table 4.3.1 or shall be concrete made with a cement that provides sulfate resistance and that has a maximum water-cementitious materials ratio and minimum compressive strength from Table 4.3.1.

R4.3 — Sulfate exposures

R4.3.1 — Concrete exposed to injurious concentrations of sulfates from soil and water should be made with a sulfate-resisting cement. Table 4.3.1 lists the appropriate types of cement and the maximum water-cementitious materials ratios and minimum strengths for various exposure conditions. In selecting a cement for sulfate resistance, the principal consideration is its C_3A content. For moderate exposures, Type II cement is limited to a maximum C_3A content of 8.0 percent under ASTM C 150. The blended cements under ASTM C 595 made with portland cement clinker with less than 8 percent C_3A qualify for the MS designation, and therefore, are appropriate for use in moderate sulfate exposures. The appropriate types under ASTM C 595 are IP(MS), IS(MS), I(PM)(MS), and I(SM)(MS). For severe exposures, Type V cement with a maximum C_3A

TABLE 4.3.1—REQUIREMENTS FOR CONCRETE EXPOSED TO SULFATE-CONTAINING SOLUTIONS

Sulfate exposure	Water soluble sulfate (SO_4) in soil, percent by weight	Sulfate (SO_4) in water, ppm	Cement type	Maximum water-cementitious materials ratio, by weight, normal weight aggregate concrete[*]	Minimum f_c', normal weight and lightweight aggregate concrete, psi[*]
Negligible	0.00-0.10	0-150	—	—	—
Moderate[†]	0.10-0.20	150-1500	II, IP(MS), IS(MS), P(MS), I(PM)(MS), I(SM)(MS)	0.50	4000
Severe	0.20-2.00	1500-10,000	V	0.45	4500
Very severe	Over 2.00	Over 10,000	V plus pozzolan[‡]	0.45	4500

[*] A lower water-cementitious materials ratio or higher strength may be required for low permeability or for protection against corrosion of embedded items or freezing and thawing (Table 4.2.2).

[†] Seawater.

[‡] Pozzolan that has been determined by test or service record to improve sulfate resistance when used in concrete containing Type V cement.

CODE

COMMENTARY

content of 5 percent is specified. In certain areas, the C_3A content of other available types such as Type III or Type I may be less than 8 or 5 percent and are usable in moderate or severe sulfate exposures. Note that sulfate-resisting cement will not increase resistance to some chemically aggressive solutions, for example ammonium nitrate. The project specifications should cover all special cases.

The judicious employment of a good quality fly ash (ASTM C 618, Class F) also has been shown to improve the sulfate resistance of concrete.[4.9] Certain Type IP cements made by blending Class F pozzolan with portland cement having a tricalcium aluminate (C_3A) content greater than 8 percent can provide sulfate resistance for moderate exposures.

A note to Table 4.3.1 lists seawater as "moderate exposure," even though it generally contains more than 1500 ppm SO_4. In seawater exposures, other types of cement with C_3A up to 10 percent may be used if the maximum water-cementitious materials ratio is reduced to 0.40.

ASTM test method C 1012[4.2] can be used to evaluate the sulfate resistance of mixtures using combinations of cementitious materials.

In addition to the proper selection of cement, other requirements for durable concrete exposed to concentrations of sulfate such as: low water-cementitious materials ratio, strength, adequate air entrainment, low slump, adequate consolidation, uniformity, adequate cover of reinforcement, and sufficient moist curing to develop the potential properties of the concrete, are essential.

4.3.2 — Calcium chloride as an admixture shall not be used in concrete to be exposed to severe or very severe sulfate-containing solutions, as defined in Table 4.3.1.

4.4 — Corrosion protection of reinforcement

4.4.1 — For corrosion protection of reinforcement in concrete, maximum water soluble chloride ion concentrations in hardened concrete at ages from 28 to 42 days contributed from the ingredients including water, aggregates, cementitious materials, and admixtures shall not exceed the limits of Table 4.4.1. When testing is performed to determine water soluble chloride ion content, test procedures shall conform to ASTM C 1218.

R4.4 — Corrosion protection of reinforcement

R4.4.1 — Additional information on the effects of chlorides on the corrosion of reinforcing steel is given in **"Guide to Durable Concrete"** reported by ACI Committee 201[4.10] and **"Corrosion of Metals in Concrete"** reported by ACI Committee 222.[4.11] Test procedures must conform to those given in ASTM C 1218. An initial evaluation may be obtained by testing individual concrete ingredients for total chloride ion content. If total chloride ion content, calculated on the basis of concrete proportions, exceeds those permitted in Table 4.4.1, it may be necessary to test samples of the hardened concrete for water soluble chloride ion content described in the guide. Some of the total chloride ions present in the ingredients will either be insoluble or will react with the cement during hydration and become insoluble under the test procedures described.

CODE

TABLE 4.4.1—MAXIMUM CHLORIDE ION CONTENT FOR CORROSION PROTECTION OF REINFORCEMENT

Type of member	Maximum water soluble chloride ion (Cl^-) in concrete, percent by weight of cement
Prestressed concrete	0.06
Reinforced concrete exposed to chloride in service	0.15
Reinforced concrete that will be dry or protected from moisture in service	1.00
Other reinforced concrete construction	0.30

4.4.2 — If concrete with reinforcement will be exposed to chlorides from deicing chemicals, salt, salt water, brackish water, seawater, or spray from these sources, requirements of Table 4.2.2 for water-cementitious materials ratio and concrete strength, and the minimum concrete cover requirements of 7.7 shall be satisfied. See 18.14 for unbonded prestressing tendons.

COMMENTARY

When concretes are tested for soluble chloride ion content the tests should be made at an age of 28 to 42 days. The limits in Table 4.4.1 are to be applied to chlorides contributed from the concrete ingredients, not those from the environment surrounding the concrete.

The chloride ion limits in Table 4.4.1 differ from those recommended in ACI 201.2R and ACI 222R. For reinforced concrete that will be dry in service, a limit of one percent has been included to control total soluble chlorides. Table 4.4.1 includes limits of 0.15 and 0.30 percent for reinforced concrete that will be exposed to chlorides or will be damp in service, respectively. These limits compare to 0.10 and 0.15 recommended in ACI 201.2R. ACI 222R recommends limits of 0.08 and 0.20 percent by weight of cement for chlorides in prestressed and reinforced concrete, respectively, based on tests for acid soluble chlorides, not the test for water soluble chlorides required here.

When epoxy- or zinc-coated bars are used, the limits in Table 4.4.1 may be more restrictive than necessary.

R4.4.2 — When concretes are exposed to external sources of chlorides the water-cementitious materials ratio and specified compressive strength f_c' of 4.2.2 are the minimum requirements that must be considered. The designer should evaluate conditions in structures where chlorides may be applied, in parking structures where chlorides may be tracked in by vehicles or in structures near seawater. Epoxy- or zinc-coated bars or cover greater than the minimum required in 7.7 may be desirable. Use of slag meeting ASTM C 989 or fly ash meeting ASTM C 618 and increased levels of specified strength provide increased protection. Use of silica fume meeting ASTM C 1240 with an appropriate high-range water reducer, ASTM C 494, Types F and G, or ASTM C 1017 can also provide additional protection.[4.12] Performance tests for chloride permeability by AASHTO T 277[4.13] of concrete mixtures prior to use will also provide additional assurance.

Notes

CHAPTER 5 — CONCRETE QUALITY, MIXING, AND PLACING

CODE

5.0 — Notation

f_c' = specified compressive strength of concrete, psi
f_{cr}' = required average compressive strength of concrete used as the basis for selection of concrete proportions, psi
f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi
s = standard deviation, psi

5.1 — General

5.1.1 — Concrete shall be proportioned to provide an average compressive strength as prescribed in 5.3.2 as well as satisfy the durability criteria of Chapter 4. Concrete shall be produced to minimize frequency of strengths below f_c' as prescribed in 5.6.2.3.

5.1.2 — Requirements for f_c' shall be based on tests of cylinders made and tested as prescribed in 5.6.2.

5.1.3 — Unless otherwise specified, f_c' shall be based on 28-day tests. If other than 28 days, test age for f_c' shall be as indicated in design drawings or specifications.

5.1.4 — Where design criteria in 9.5.2.3, 11.2, and 12.2.4 provide for use of a splitting tensile strength value of concrete, laboratory tests shall be made in accordance with "Specification for Lightweight Aggregates for Structural Concrete" (ASTM C 330) to establish value of f_{ct} corresponding to specified value of f_c'.

COMMENTARY

The requirements for proportioning of concrete mixtures are based on the philosophy that concrete should provide both adequate durability (Chapter 4) and strength. The criteria for acceptance of concrete are based on the philosophy that the code is intended primarily to protect the safety of the public. Chapter 5 describes procedures by which concrete of adequate strength can be obtained, and provides procedures for checking the quality of the concrete during and after its placement in the work.

Chapter 5 also prescribes minimum criteria for mixing and placing concrete.

The purpose of 5.3, together with Chapter 4, is to establish the required mixture proportions, and not to constitute a basis for confirming the adequacy of concrete strength, which is covered in 5.6 (evaluation and acceptance of concrete).

R5.1 — General

R5.1.1 — The basic premises governing the designation and evaluation of concrete strength are presented. It is emphasized that the average strength of concrete produced must always exceed the specified value of f_c' used in the structural design calculations. This is based on probabilistic concepts, and is intended to ensure that adequate concrete strength will be developed in the structure. The durability requirements prescribed in Chapter 4 must be satisfied in addition to attaining the average concrete strength in accordance with 5.3.2.

R5.1.4 — Code Sections 9.5.2.3 (modulus of rupture), 11.2 (concrete shear strength) and 12.2.4 (development of reinforcement) require modification in the design criteria for the use of lightweight aggregate concrete. Two alternate modification procedures are provided. One alternate is based on laboratory tests to determine the relationship between splitting tensile strength f_{ct} and specified compressive strength f_c' for the lightweight concrete. For a lightweight aggregate from a given source, it is intended that appropriate values of f_{ct} be obtained in advance of design.

CODE

5.1.5 — Splitting tensile strength tests shall not be used as a basis for field acceptance of concrete.

5.2 — Selection of concrete proportions

5.2.1 — Proportions of materials for concrete shall be established to provide:

(a) Workability and consistency to permit concrete to be worked readily into forms and around reinforcement under conditions of placement to be employed, without segregation or excessive bleeding.

(b) Resistance to special exposures as required by Chapter 4.

(c) Conformance with strength test requirements of 5.6.

5.2.2 — Where different materials are to be used for different portions of proposed work, each combination shall be evaluated.

5.2.3 — Concrete proportions, including water-cementitious materials ratio, shall be established on the basis of field experience and/or trial mixtures with materials to be employed (see 5.3), except as permitted in 5.4 or required by Chapter 4.

5.3 — Proportioning on the basis of field experience and/or trial mixtures

COMMENTARY

R5.1.5 — Tests for splitting tensile strength of concrete (as required by 5.1.4) are not intended for control of, or acceptance of, the strength of concrete in the field. Indirect control will be maintained through the normal compressive strength test requirements provided by 5.6 (evaluation and acceptance of concrete).

R5.2 — Selection of concrete proportions

Recommendations for selecting proportions for concrete are given in detail in **"Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete"** (ACI 211.1).[5.1] (Provides two methods for selecting and adjusting proportions for normal weight concrete: the estimated weight and absolute volume methods. Example calculations are shown for both methods. Proportioning of heavyweight concrete by the absolute volume method is presented in an appendix.)

Recommendations for lightweight concrete are given in **"Standard Practice for Selecting Proportions for Structural Lightweight Concrete"** (ACI 211.2).[5.2] (Provides a method of proportioning and adjusting structural grade concrete containing lightweight aggregates.)

R5.2.1 — The selected water-cementitious materials ratio must be low enough, or the compressive strength high enough (for lightweight concrete) to satisfy both the strength criteria (see 5.3 or 5.4) and the special exposure requirements (Chapter 4). The code does not include provisions for especially severe exposures, such as acids or high temperatures, and is not concerned with aesthetic considerations such as surface finishes. These items are beyond the scope of the code and must be covered specifically in the project specifications. Concrete ingredients and proportions must be selected to meet the minimum requirements stated in the code and the additional requirements of the contract documents.

R5.2.3 — The code emphasizes the use of field experience or laboratory trial mixtures (see 5.3) as the preferred method for selecting concrete mixture proportions. When no prior experience or trial mixture data is available, estimation of the water-cementitious materials ratio as prescribed in 5.4 is permitted, but only when special permission is given.

R5.3 — Proportioning on the basis of field experience and/or trial mixtures

In selecting a suitable concrete mixture there are three basic steps. The first is the determination of the standard deviation, and the second, the determination of the required aver-

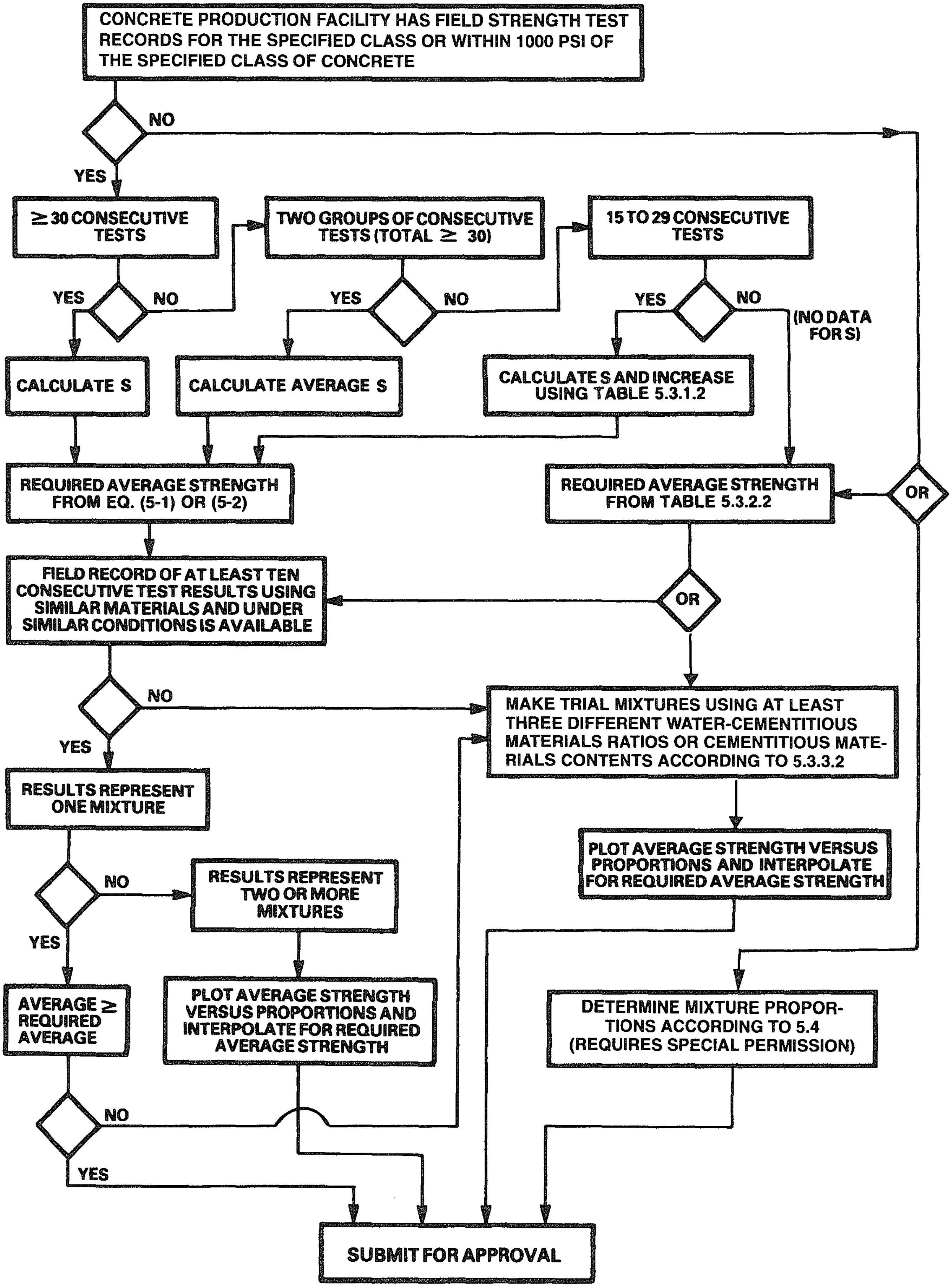
CONCRETE PRODUCTION FACILITY HAS FIELD STRENGTH TEST RECORDS FOR THE SPECIFIED CLASS OR WITHIN 1000 PSI OF THE SPECIFIED CLASS OF CONCRETE
NO
YES
≥ 30 CONSECUTIVE TESTS
TWO GROUPS OF CONSECUTIVE TESTS (TOTAL ≥ 30)
15 TO 29 CONSECUTIVE TESTS
YES
NO
YES
NO
YES
NO
(NO DATA FOR S)
CALCULATE S
CALCULATE AVERAGE S
CALCULATE S AND INCREASE USING TABLE 5.3.1.2
REQUIRED AVERAGE STRENGTH FROM EQ. (5-1) OR (5-2)
REQUIRED AVERAGE STRENGTH FROM TABLE 5.3.2.2
OR
FIELD RECORD OF AT LEAST TEN CONSECUTIVE TEST RESULTS USING SIMILAR MATERIALS AND UNDER SIMILAR CONDITIONS IS AVAILABLE
OR
NO
YES
MAKE TRIAL MIXTURES USING AT LEAST THREE DIFFERENT WATER-CEMENTITIOUS MATERIALS RATIOS OR CEMENTITIOUS MATERIALS CONTENTS ACCORDING TO 5.3.3.2
RESULTS REPRESENT ONE MIXTURE
NO
RESULTS REPRESENT TWO OR MORE MIXTURES
PLOT AVERAGE STRENGTH VERSUS PROPORTIONS AND INTERPOLATE FOR REQUIRED AVERAGE STRENGTH
YES
AVERAGE ≥ REQUIRED AVERAGE
PLOT AVERAGE STRENGTH VERSUS PROPORTIONS AND INTERPOLATE FOR REQUIRED AVERAGE STRENGTH
DETERMINE MIXTURE PROPORTIONS ACCORDING TO 5.4 (REQUIRES SPECIAL PERMISSION)
NO
YES
SUBMIT FOR APPROVAL

Fig. R5.3—Flow chart for selection and documentation of concrete proportions

5.3.1 — Standard deviation

5.3.1.1 — Where a concrete production facility has test records, a standard deviation shall be established. Test records from which a standard deviation is calculated:

(a) Shall represent materials, quality control procedures, and conditions similar to those expected and changes in materials and proportions within the test records shall not have been more restricted than those for proposed work.

(b) Shall represent concrete produced to meet a specified strength or strengths f_c' within 1000 psi of that specified for proposed work.

(c) Shall consist of at least 30 consecutive tests or two groups of consecutive tests totaling at least 30 tests as defined in 5.6.1.4, except as provided in 5.3.1.2.

5.3.1.2 — Where a concrete production facility does not have test records meeting requirements of 5.3.1.1, but does have a record based on 15 to 29 consecutive tests, a standard deviation shall be established as the product of the calculated standard deviation and modification factor of Table 5.3.1.2. To be acceptable, test record shall meet requirements (a) and (b) of 5.3.1.1, and represent only a single record of consecutive tests that span a period of not less than 45 calendar days.

TABLE 5.3.1.2—MODIFICATION FACTOR FOR STANDARD DEVIATION WHEN LESS THAN 30 TESTS ARE AVAILABLE

No. of tests*	Modification factor for standard deviation†
Less than 15	Use table 5.3.2.2
15	1.16
20	1.08
25	1.03
30 or more	1.00

* Interpolate for intermediate numbers of tests.

† Modified standard deviation to be used to determine required average strength f_{cr}' from 5.3.2.1.

age strength. The third step is the selection of mixture proportions required to produce that average strength, either by conventional trial mixture procedures or by a suitable experience record. Fig. R5.3 is a flow chart outlining the mix selection and documentation procedure.

The mixture selected must yield an average strength appreciably higher than the specified strength f_c'. The degree of mixture overdesign depends on the variability of the test results.

R5.3.1 — Standard deviation

When a concrete production facility has a suitable record of 30 consecutive tests of similar materials and conditions expected, the standard deviation is calculated from those results in accordance with the following formula:

$$s = \left[\frac{\Sigma (X_i - \bar{X})^2}{(n-1)}\right]^{1/2}$$

where:

s = standard deviation, psi
X_i = individual strength tests as defined in 5.6.1.4
$\bar{X}$ = average of n strength test results
n = number of consecutive strength tests

The standard deviation is used to determine the average strength required in 5.3.2.1.

If two test records are used to obtain at least 30 tests, the standard deviation used shall be the statistical average of the values calculated from each test record in accordance with the following formula:

$$\bar{s} = \left[\frac{(n_1 - 1)(s_1)^2 + (n_2 - 1)(s_2)^2}{(n_1 + n_2 - 2)}\right]^{1/2}$$

where

$\bar{s}$ = statistical average standard deviation where two test records are used to estimate the standard deviation
s_1, s_2 = standard deviations calculated from two test records, 1 and 2, respectively
n_1, n_2 = number of tests in each test record, respectively

If less than 30, but at least 15 tests are available, the calculated standard deviation is increased by the factor given in Table 5.3.1.2. This procedure results in a more conservative (increased) required average strength. The factors in Table 5.3.1.2 are based on the sampling distribution of the standard deviation and provide protection (equivalent to that

5.3.2 — Required average strength

5.3.2.1 — Required average compressive strength f_{cr}' used as the basis for selection of concrete proportions shall be the larger of Eq. (5-1) or (5-2) using a standard deviation calculated in accordance with 5.3.1.1 or 5.3.1.2.

$$f_{cr}' = f_c' + 1.34s \qquad (5\text{-}1)$$

or

from a record of 30 tests) against the possibility that the smaller sample underestimates the true or universe population standard deviation.

The standard deviation used in the calculation of required average strength must be developed under conditions "similar to those expected" [see 5.3.1.1(a)]. This requirement is important to ensure acceptable concrete.

Concrete for background tests to determine standard deviation is considered to be "similar" to that required if made with the same general types of ingredients under no more restrictive conditions of control over material quality and production methods than on the proposed work, and if its specified strength does not deviate more than 1000 psi from the f_c' required [see 5.3.1.1(b)]. A change in the type of concrete or a major increase in the strength level may increase the standard deviation. Such a situation might occur with a change in type of aggregate (i.e., from natural aggregate to lightweight aggregate or vice versa) or a change from non-air-entrained concrete to air-entrained concrete. Also, there may be an increase in standard deviation when the average strength level is raised by a significant amount, although the increment of increase in standard deviation should be somewhat less than directly proportional to the strength increase. When there is reasonable doubt, any estimated standard deviation used to calculate the required average strength should always be on the conservative (high) side.

Note that the code uses the standard deviation in pounds per square inch instead of the coefficient of variation in percent. The latter is equal to the former expressed as a percent of the average strength.

When a suitable record of test results is not available, the average strength must exceed the design strength by an amount that ranges from 1000 to 1400 psi, depending on the design strength. See Table 5.3.2.2.

Even when the average strength and standard deviation are of the levels assumed, there will be occasional tests that fail to meet the acceptance criteria prescribed in 5.6.2.3 (perhaps 1 test in 100).

R5.3.2 — Required average strength

R5.3.2.1 — Once the standard deviation has been determined, the required average strength is obtained from the larger of Eq. (5-1) or (5-2). Eq. (5-1) provides a probability of 1-in-100 that averages of three consecutive tests will be below the specified strength f_c'. Eq. (5-2) provides a similar probability of individual tests more than 500 psi below the specified strength f_c'. These equations assume that the standard deviation used is equal to the population value appropriate for an infinite or very large number of tests. For this

CODE

$$f'_{cr} = f'_c + 2.33s - 500 \qquad (5\text{-}2)$$

5.3.2.2 — When a concrete production facility does not have field strength test records for calculation of standard deviation meeting requirements of 5.3.1.1 or 5.3.1.2, required average strength f'_{cr} shall be determined from Table 5.3.2.2 and documentation of average strength shall be in accordance with requirements of 5.3.3.

TABLE 5.3.2.2—REQUIRED AVERAGE COMPRESSIVE STRENGTH WHEN DATA ARE NOT AVAILABLE TO ESTABLISH A STANDARD DEVIATION

Specified compressive strength, f'_c, psi	Required average compressive strength, f'_{cr}, psi
Less than 3000 psi	$f'_c + 1000$
3000 to 5000	$f'_c + 1200$
Over 5000	$f'_c + 1400$

5.3.3 — Documentation of average strength

Documentation that proposed concrete proportions will produce an average compressive strength equal to or greater than required average compressive strength (see 5.3.2) shall consist of a field strength test record, several strength test records, or trial mixtures.

5.3.3.1 — When test records are used to demonstrate that proposed concrete proportions will produce the required average strength f'_{cr} (see 5.3.2), such records shall represent materials and conditions similar to those expected. Changes in materials, condi-

COMMENTARY

reason, use of standard deviations estimated from records of 100 or more tests is desirable. When 30 tests are available, the probability of failure will likely be somewhat greater than 1-in-100. The additional refinements required to achieve the 1-in-100 probability are not considered necessary, because of the uncertainty inherent in assuming that conditions operating when the test record was accumulated will be similar to conditions when the concrete will be produced.

Additionally, the change adopted in ACI 318-77 (requiring action to increase the average strength whenever either of the acceptance criteria of 5.6.2.3 is not met) is considered to provide significant additional protection against subsequent low tests.

R.5.3.3 — Documentation of average strength

Once the required average strength f'_{cr} is known, the next step is to select mixture proportions that will produce an average strength at least as great as the required average strength, and also meet special exposure requirements of Chapter 4. The documentation may consist of a strength test record, several strength test records, or suitable laboratory trial mixtures. Generally, if a test record is used, it will be the same one that was used for computation of the standard deviation. However, if this test record shows either lower or higher average strength than the required average strength, different proportions may be necessary or desirable. In such instances, the average from a record of as few as 10 tests may be used, or the proportions may be established by interpolation between the strengths and proportions of two such records of consecutive tests. All test records for establishing proportions necessary to produce the average strength must meet the requirements of 5.3.3.1 for "similar materials and conditions."

The 1971 code required trial mixtures to be mixed at the maximum permitted slump and air content. Since 1977, the code has provided tolerances at the maximum permissible slump and air content. The code text makes it clear that these tolerances on slump and air content apply only to the trial mixtures and not to records of field tests or to later production of the concrete in the field.

CODE

COMMENTARY

tions, and proportions within the test records shall not have been more restricted than those for proposed work. For the purpose of documenting average strength potential, test records consisting of less than 30 but not less than 10 consecutive tests are acceptable provided test records encompass a period of time not less than 45 days. Required concrete proportions shall be permitted to be established by interpolation between the strengths and proportions of two or more test records each of which meets other requirements of this section.

5.3.3.2 — When an acceptable record of field test results is not available, concrete proportions established from trial mixtures meeting the following restrictions shall be permitted:

(a) Combination of materials shall be those for proposed work.

(b) Trial mixtures having proportions and consistencies required for proposed work shall be made using at least three different water-cementitious materials ratios or cementitious materials contents that will produce a range of strengths encompassing the required average strength f'_{cr}.

(c) Trial mixtures shall be designed to produce a slump within ± 0.75 in. of maximum permitted, and for air-entrained concrete, within ± 0.5 percent of maximum allowable air content.

(d) For each water-cementitious materials ratio or cementitious materials content, at least three test cylinders for each test age shall be made and cured in accordance with "Method of Making and Curing Concrete Test Specimens in the Laboratory" (ASTM C 192). Cylinders shall be tested at 28 days or at test age designated for determination of f'_c.

(e) From results of cylinder tests a curve shall be plotted showing relationship between water-cementitious materials ratio or cementitious materials content and compressive strength at designated test age.

(f) Maximum water-cementitious materials ratio or minimum cementitious materials content for concrete to be used in proposed work shall be that shown by the curve to produce the average strength required by 5.3.2, unless a lower water-cementitious materials ratio or higher strength is required by Chapter 4.

CODE

5.4 — Proportioning without field experience or trial mixtures

5.4.1 — If data required by 5.3 are not available, concrete proportions shall be based upon other experience or information, if approved by the engineer/architect. The required average compressive strength f_{cr}' of concrete produced with materials similar to those proposed for use shall be at least 1200 psi greater than the specified compressive strength f_c'. This alternative shall not be used for specified compressive strength greater than 4000 psi.

5.4.2 — Concrete proportioned by this section shall conform to the durability requirements of Chapter 4 and to compressive strength test criteria of 5.6.

5.5 — Average strength reduction

As data become available during construction, it shall be permitted to reduce the amount by which f_{cr}' must exceed the specified value of f_c', provided:

(a) 30 or more test results are available and average of test results exceeds that required by 5.3.2.1, using a standard deviation calculated in accordance with 5.3.1.1, or

(b) 15 to 29 test results are available and average of test results exceeds that required by 5.3.2.1 using a standard deviation calculated in accordance with 5.3.1.2, and

(c) special exposure requirements of Chapter 4 are met.

5.6 — Evaluation and acceptance of concrete

5.6.1 — Frequency of testing

5.6.1.1 — Samples for strength tests of each class of concrete placed each day shall be taken not less than once a day, nor less than once for each 150 yd^3 of concrete, nor less than once for each 5000 ft^2 of surface area for slabs or walls.

COMMENTARY

R5.4 — Proportioning without field experience or trial mixtures

R5.4.1 — When no prior experience (5.3.3.1) or trial mixture data (5.3.3.2) meeting the requirements of these sections is available, other experience may be used only when special permission is given. Because combinations of different ingredients may vary considerably in strength level, this procedure is not permitted for f_c' greater than 4000 psi and the required average strength should exceed f_c' by 1200 psi. The purpose of this provision is to allow work to continue when there is an unexpected interruption in concrete supply and there is not sufficient time for tests and evaluation or in small structures where the cost of trial mixture data is not justified.

R5.6 — Evaluation and acceptance of concrete

Once the mixture proportions have been selected and the job started, the criteria for evaluation and acceptance of the concrete can be obtained from 5.6.

An effort has been made in the code to provide a clear-cut basis for judging the acceptability of the concrete, as well as to indicate a course of action to be followed when the results of strength tests are not satisfactory.

R5.6.1 — Frequency of testing

R5.6.1.1 — The following three criteria establish the required minimum sampling frequency for each class of concrete:

(a) Once each day a given class is placed, nor less than

CODE

5.6.1.2 — On a given project, if total volume of concrete is such that frequency of testing required by 5.6.1.1 would provide less than five strength tests for a given class of concrete, tests shall be made from at least five randomly selected batches or from each batch if fewer than five batches are used.

5.6.1.3 — When total quantity of a given class of concrete is less than 50 yd^3, strength tests are not required when evidence of satisfactory strength is submitted to and approved by the building official.

5.6.1.4 — A strength test shall be the average of the strengths of two cylinders made from the same sample of concrete and tested at 28 days or at test age designated for determination of f_c'.

5.6.2 — Laboratory-cured specimens

5.6.2.1 — Samples for strength tests shall be taken in accordance with "Method of Sampling Freshly Mixed Concrete" (ASTM C 172).

5.6.2.2 — Cylinders for strength tests shall be molded and laboratory-cured in accordance with "Practice for Making and Curing Concrete Test Specimens in the Field" (ASTM C 31) and tested in accordance with "Test Method for Compressive Strength of Cylindrical Concrete Specimens" (ASTM C 39).

5.6.2.3 — Strength level of an individual class of concrete shall be considered satisfactory if both of the following requirements are met:

(a) Every arithmetic average of any three consecutive strength tests equals or exceeds f_c'.

COMMENTARY

(b) Once for each 150 yd^3 of each class placed each day, nor less than

(c) Once for each 5000 ft^2 of slab or wall surface area placed each day.

In calculating surface area, only one side of the slab or wall should be considered. If the average wall or slab thickness is less than $9^3/_4$ in. Criteria (c) will require more frequent sampling than once for each 150 yd^3 placed.

R5.6.1.2 — Samples for strength tests must be taken on a strictly random basis if they are to measure properly the acceptability of the concrete. To be representative, the choice of times of sampling, or the batches of concrete to be sampled, must be made on the basis of chance alone, within the period of placement. If batches to be sampled are selected on the basis of appearance, convenience, or other possibly biased criteria, the statistical concepts lose their validity. Obviously, not more than one test (average of two cylinders made from a sample, 5.6.1.4) should be taken from a single batch, and water may not be added to the concrete after the sample is taken.

ASTM D 3665 describes procedures for random selection of the batches to be tested.

R5.6.2 — Laboratory-cured specimens

R5.6.2.3 — A single set of criteria is given for acceptability of strength and is applicable to all concrete used in structures designed in accordance with the code, regardless of design method used. The concrete strength is considered to be satisfactory as long as averages of any three consecutive strength tests remain above the specified f_c' and no individ-

CODE

(b) No individual strength test (average of two cylinders) falls below f_c' by more than 500 psi.

5.6.2.4 — If either of the requirements of 5.6.2.3 are not met, steps shall be taken to increase the average of subsequent strength test results. Requirements of 5.6.4 shall be observed if requirement of 5.6.2.3(b) is not met.

COMMENTARY

ual strength test falls below the specified f_c' by more than 500 psi. Evaluation and acceptance of the concrete can be judged immediately as test results are received during the course of the work. Strength tests failing to meet these criteria will occur occasionally (probably about once in 100 tests) even though concrete strength and uniformity are satisfactory. Allowance should be made for such statistically expected variations in deciding whether the strength level being produced is adequate. In terms of the probability of failure, the criterion of minimum individual strength test result of 500 psi less than f_c' adapts itself readily to small numbers of tests. For example, if only five strength tests are made on a small job, it is apparent that, if any of the strength test results (average of two cylinders) is more than 500 psi below f_c', the criterion is not met.

R5.6.2.4 — When concrete fails to meet either of the strength requirements of 5.6.2.3, steps must be taken to increase the average of the concrete test results. If sufficient concrete has been produced to accumulate at least 15 tests, these should be used to establish a new target average strength as described in 5.3.

If fewer than 15 tests have been made on the class of concrete in question, the new target level should be at least as great as the average level used in the initial selection of proportions. If the average of the available tests made on the project equals or exceeds the level used in the initial selection of proportions, a further increase in average level is required.

The steps taken to increase the average level of test results will depend on the particular circumstances, but could include one or more of the following:

(a) an increase in cementitious materials content,

(b) changes in mixture proportions,

(c) reductions in or better control of levels of slump supplied,

(d) a reduction in delivery time,

(e) closer control of air content, or

(f) an improvement in the quality of the testing, including strict compliance with standard test procedures.

Such changes in operating and testing procedures, or changes in cementitious materials content, or slump should not require a formal resubmission under the procedures of 5.3; however, important changes in sources of cement, aggregates, or admixtures, should be accompanied by evidence that the average strength level will be improved.

CODE

5.6.3 — Field-cured specimens

5.6.3.1 — If required by the building official, results of strength tests of cylinders cured under field conditions shall be provided.

5.6.3.2 — Field-cured cylinders shall be cured under field conditions in accordance with "Practice for Making and Curing Concrete Test Specimens in the Field" (ASTM C 31).

5.6.3.3 — Field-cured test cylinders shall be molded at the same time and from the same samples as laboratory-cured test cylinders.

5.6.3.4 — Procedures for protecting and curing concrete shall be improved when strength of field-cured cylinders at test age designated for determination of f_c' is less than 85 percent of that of companion laboratory-cured cylinders. The 85 percent limitation shall not apply if field-cured strength exceeds f_c' by more than 500 psi.

5.6.4 — Investigation of low-strength test results

5.6.4.1 — If any strength test (see 5.6.1.4) of laboratory-cured cylinders falls below specified value of f_c' by more than 500 psi [see 5.6.2.3(b)] or if tests of field-cured cylinders indicate deficiencies in protection and curing (see 5.6.3.4), steps shall be taken to assure that load-carrying capacity of the structure is not jeopardized.

5.6.4.2 — If the likelihood of low-strength concrete is confirmed and calculations indicate that load-carrying capacity is significantly reduced, tests of cores drilled from the area in question in accordance with "Method of Obtaining and Testing Drilled Cores and Sawed

COMMENTARY

Laboratories testing cylinders or cores to determine compliance with these requirements should be accredited or inspected for conformance to the requirement of ASTM C 1077[5.3] by a recognized agency such as the American Association for Laboratory Accreditation (A2LA), AASHTO Materials Reference Laboratory (AMRL), National Voluntary Laboratory Accreditation Program (NVLAP), Cement and Concrete Reference Laboratory (CCRL), or their equivalent.

R5.6.3 — Field-cured specimens

R5.6.3.1 — Strength tests of cylinders cured under field conditions may be required to check the adequacy of curing and protection of concrete in the structure.

R5.6.3.4 — Positive guidance is provided in the code concerning the interpretation of tests of field-cured cylinders. Research has shown that cylinders protected and cured to simulate good field practice should test not less than about 85 percent of standard laboratory moist-cured cylinders. This percentage has been set merely as a rational basis for judging the adequacy of field curing. The comparison is made between the actual measured strengths of companion job-cured and laboratory-cured cylinders, not between job-cured cylinders and the specified value of f_c'. However, results for the job-cured cylinders are considered satisfactory if the job-cured cylinders exceed the specified f_c' by more than 500 psi, even though they fail to reach 85 percent of the strength of companion laboratory-cured cylinders.

R5.6.4 — Investigation of low-strength test results

Instructions are provided concerning the procedure to be followed when strength tests have failed to meet the specified acceptance criteria. For obvious reasons, these instructions cannot be dogmatic. The building official must apply judgment as to the true significance of low test results and whether they indicate need for concern. If further investigation is deemed necessary, such investigation may include nondestructive tests, or in extreme cases, strength tests of cores taken from the structure.

Nondestructive tests of the concrete in place, such as by probe penetration, impact hammer, ultrasonic pulse velocity or pull out may be useful in determining whether or not a

CODE

Beams of Concrete" (ASTM C 42) shall be permitted. In such cases, three cores shall be taken for each strength test more than 500 psi below the specified value of f_c'.

5.6.4.3 — If concrete in the structure will be dry under service conditions, cores shall be air dried (temperature 60 to 80 F, relative humidity less than 60 percent) for 7 days before test and shall be tested dry. If concrete in the structure will be more than superficially wet under service conditions, cores shall be immersed in water for at least 40 hr and be tested wet.

5.6.4.4 — Concrete in an area represented by core tests shall be considered structurally adequate if the average of three cores is equal to at least 85 percent of f_c' and if no single core is less than 75 percent of f_c'. Additional testing of cores extracted from locations represented by erratic core strength results shall be permitted.

5.6.4.5 — If criteria of 5.6.4.4 are not met and if the structural adequacy remains in doubt, the responsible authority shall be permitted to order a strength evaluation in accordance with Chapter 20 for the questionable portion of the structure, or take other appropriate action.

5.7 — Preparation of equipment and place of deposit

5.7.1 — Preparation before concrete placement shall include the following:

(a) All equipment for mixing and transporting concrete shall be clean.

(b) All debris and ice shall be removed from spaces to be occupied by concrete.

(c) Forms shall be properly coated.

(d) Masonry filler units that will be in contact with concrete shall be well drenched.

(e) Reinforcement shall be thoroughly clean of ice or other deleterious coatings.

(f) Water shall be removed from place of deposit

COMMENTARY

portion of the structure actually contains low-strength concrete. Such tests are of value primarily for comparisons within the same job rather than as quantitative measures of strength. For cores, if required, conservatively safe acceptance criteria are provided which should assure structural adequacy for virtually any type of construction.[5.4-5.7] Lower strength may, of course, be tolerated under many circumstances, but this again becomes a matter of judgment on the part of the building official and design engineer. When the core tests fail to provide assurance of structural adequacy, it may be practical, particularly in the case of floor or roof systems, for the building official to require a load test (Chapter 20). Short of load tests, if time and conditions permit, an effort may be made to improve the strength of the concrete in place by supplemental wet curing. Effectiveness of such a treatment must be verified by further strength evaluation using procedures previously discussed.

It should be noted that core tests having an average of 85 percent of the specified strength are entirely realistic. To expect core tests to be equal to f_c' is not realistic, since differences in the size of specimens, conditions of obtaining samples, and procedures for curing, do not permit equal values to be obtained.

The code, as stated, concerns itself with assuring structural safety, and the instructions in 5.6 are aimed at that objective. It is not the function of the code to assign responsibility for strength deficiencies, whether or not they are such as to require corrective measures.

Under the requirements of this section, cores taken to confirm structural adequacy will usually be taken at ages later than those specified for determination of f_c'.

R5.7 — Preparation of equipment and place of deposit

Recommendations for mixing, handling and transporting, and placing concrete are given in detail in **"Guide for Measuring, Mixing, Transporting, and Placing Concrete"** reported by ACI Committee 304.[5.8] (Presents methods and procedures for control, handling and storage of materials, measurement, batching tolerances, mixing, methods of placing, transporting, and forms.)

Attention is directed to the need for using clean equipment and for cleaning forms and reinforcement thoroughly before beginning to deposit concrete. In particular, sawdust, nails, wood pieces, and other debris that may collect inside the forms must be removed. Reinforcement must be thoroughly cleaned of ice, dirt, loose rust, mill scale, or other coatings. Water should be removed from the forms.

CODE

before concrete is placed unless a tremie is to be used or unless otherwise permitted by the building official.

(g) All laitance and other unsound material shall be removed before additional concrete is placed against hardened concrete.

5.8 — Mixing

5.8.1 — All concrete shall be mixed until there is a uniform distribution of materials and shall be discharged completely before mixer is recharged.

5.8.2 — Ready-mixed concrete shall be mixed and delivered in accordance with requirements of "Specification for Ready-Mixed Concrete" (ASTM C 94) or "Specification for Concrete Made by Volumetric Batching and Continuous Mixing" (ASTM C 685).

5.8.3 — Job-mixed concrete shall be mixed in accordance with the following:

(a) Mixing shall be done in a batch mixer of approved type.

(b) Mixer shall be rotated at a speed recommended by the manufacturer.

(c) Mixing shall be continued for at least 1$^1/_2$ minutes after all materials are in the drum, unless a shorter time is shown to be satisfactory by the mixing uniformity tests of "Specification for Ready-Mixed Concrete" (ASTM C 94).

(d) Materials handling, batching, and mixing shall conform to applicable provisions of "Specification for Ready-Mixed Concrete" (ASTM C 94).

(e) A detailed record shall be kept to identify:

(1) number of batches produced;

(2) proportions of materials used;

(3) approximate location of final deposit in structure;

(4) time and date of mixing and placing.

5.9 — Conveying

5.9.1 — Concrete shall be conveyed from mixer to place of final deposit by methods that will prevent separation or loss of materials.

COMMENTARY

R5.8 — Mixing

Concrete of uniform and satisfactory quality requires the materials to be thoroughly mixed until uniform in appearance and all ingredients are distributed. Samples taken from different portions of a batch should have essentially the same unit weight, air content, slump, and coarse aggregate content. Test methods for uniformity of mixing are given in ASTM C 94. The necessary time of mixing will depend on many factors including batch size, stiffness of the batch, size and grading of the aggregate, and the efficiency of the mixer. Excessively long mixing times should be avoided to guard against grinding of the aggregates.

R5.9 — Conveying

Each step in the handling and transporting of concrete needs to be carefully controlled to maintain uniformity within a batch and from batch to batch. It is essential to avoid segre-

CODE

5.9.2 — Conveying equipment shall be capable of providing a supply of concrete at site of placement without separation of ingredients and without interruptions sufficient to permit loss of plasticity between successive increments.

5.10 — Depositing

5.10.1 — Concrete shall be deposited as nearly as practical in its final position to avoid segregation due to rehandling or flowing.

5.10.2 — Concreting shall be carried on at such a rate that concrete is at all times plastic and flows readily into spaces between reinforcement.

5.10.3 — Concrete that has partially hardened or been contaminated by foreign materials shall not be deposited in the structure.

5.10.4 — Retempered concrete or concrete that has been remixed after initial set shall not be used unless approved by the engineer.

5.10.5 — After concreting is started, it shall be carried on as a continuous operation until placing of a panel or section, as defined by its boundaries or predetermined joints, is completed except as permitted or prohibited by 6.4.

5.10.6 — Top surfaces of vertically formed lifts shall be generally level.

5.10.7 — When construction joints are required, joints shall be made in accordance with 6.4.

5.10.8 — All concrete shall be thoroughly consolidated by suitable means during placement and shall be thoroughly worked around reinforcement and embedded fixtures and into corners of forms.

COMMENTARY

gation of the coarse aggregate from the mortar or of water from the other ingredients.

The code requires the equipment for handling and transporting concrete to be capable of supplying concrete to the place of deposit continuously and reliably under all conditions and for all methods of placement. The provisions of 5.9 apply to all placement methods, including pumps, belt conveyors, pneumatic systems, wheelbarrows, buggies, crane buckets, and tremies.

Serious loss in strength can result when concrete is pumped through pipe made of aluminum or aluminum alloy.[5.9] Hydrogen gas generated by the reaction between the cement alkalies and the aluminum eroded from the interior of the pipe surface has been shown to cause strength reduction as much as 50 percent. Hence, equipment made of aluminum or aluminum alloys should not be used for pump lines, tremies, or chutes other than short chutes such as those used to convey concrete from a truck mixer.

R5.10 — Depositing

Rehandling concrete can cause segregation of the materials. Hence the code cautions against this practice. Retempering of partially set concrete with the addition of water should not be permitted, unless authorized. This does not preclude the practice (recognized in ASTM C 94) of adding water to mixed concrete to bring it up to the specified slump range so long as prescribed limits on the maximum mixing time and water-cementitious materials ratio are not violated.

Section 5.10.4 of the 1971 ACI Building Code contained a requirement that "where conditions make consolidation difficult or where reinforcement is congested, batches of mortar containing the same proportions of cement, sand, and water as used in the concrete, shall first be deposited in the forms to a depth of at least 1 in." That requirement was deleted from the 1977 code since the conditions for which it was applicable could not be defined precisely enough to justify its inclusion as a code requirement. The practice, however, has merit and should be incorporated in job specifications where appropriate, with the specific enforcement the responsibility of the job inspector rather than the building official. The use of mortar batches aids in preventing honeycomb and poor bonding of the concrete with the reinforcement. The mortar should be placed immediately before depositing the concrete and must be plastic (neither stiff nor fluid) when the concrete is placed.

Recommendations for consolidation of concrete are given in detail in **"Guide for Consolidation of Concrete"** reported by ACI Committee 309.[5.10] (Presents current information on the mechanism of consolidation and gives recommendations on equipment characteristics and procedures for various classes of concrete.)

CODE

5.11 — Curing

5.11.1 — Concrete (other than high-early-strength) shall be maintained above 50 F and in a moist condition for at least the first 7 days after placement, except when cured in accordance with 5.11.3.

5.11.2 — High-early-strength concrete shall be maintained above 50 F and in a moist condition for at least the first 3 days, except when cured in accordance with 5.11.3.

5.11.3 — Accelerated curing

5.11.3.1 — Curing by high pressure steam, steam at atmospheric pressure, heat and moisture, or other accepted processes, shall be permitted to accelerate strength gain and reduce time of curing.

5.11.3.2 — Accelerated curing shall provide a compressive strength of the concrete at the load stage considered at least equal to required design strength at that load stage.

5.11.3.3 — Curing process shall be such as to produce concrete with a durability at least equivalent to the curing method of 5.11.1 or 5.11.2.

5.11.4 — When required by the engineer or architect, supplementary strength tests in accordance with 5.6.3 shall be performed to assure that curing is satisfactory.

COMMENTARY

R5.11 — Curing

Recommendations for curing concrete are given in detail in **"Standard Practice for Curing Concrete"** reported by ACI Committee 308.[5.11] (Presents basic principles of proper curing and describes the various methods, procedures, and materials for curing of concrete.)

R5.11.3 — Accelerated curing

The provisions of this section apply whenever an accelerated curing method is used, whether for precast or cast-in-place elements. The compressive strength of steam-cured concrete is not as high as that of similar concrete continuously cured under moist conditions at moderate temperatures. Also the elastic modulus E_c of steam-cured specimens may vary from that of specimens moist-cured at normal temperatures. When steam curing is to be used, it is advisable to base the concrete mix proportions on steam-cured test cylinders.

Accelerated curing procedures require careful attention to obtain uniform and satisfactory results. It is essential that moisture loss during the curing process be prevented.

R5.11.4 — In addition to requiring a minimum curing temperature and time for normal- and high-early-strength concrete, the code provides a specific criterion in 5.6.3 for judging the adequacy of field curing. At the test age for which the strength is specified (usually 28 days), field-cured cylinders should produce strength not less than 85 percent of that of the standard, laboratory-cured cylinders. For a reasonably valid comparison to be made, field-cured cylinders and companion laboratory-cured cylinders must come from the same sample. Field-cured cylinders must be cured under conditions identical to those of the structure. If the structure is protected from the elements, the cylinder should be protected similarly.

That is, cylinders related to members not directly exposed to weather should be cured adjacent to those members and provided with the same degree of protection and method of curing.

Obviously, the field cylinders should not be treated more favorably than the elements they represent. (See code and commentary, 5.6.3 for additional information.)

If the field-cured cylinders do not provide satisfactory strength by this comparison, measures should be taken to improve the curing of the structure. If the tests indicate a possible serious deficiency in strength of concrete in the

CODE

5.12 — Cold weather requirements

5.12.1 — Adequate equipment shall be provided for heating concrete materials and protecting concrete during freezing or near-freezing weather.

5.12.2 — All concrete materials and all reinforcement, forms, fillers, and ground with which concrete is to come in contact shall be free from frost.

5.12.3 — Frozen materials or materials containing ice shall not be used.

5.13 — Hot weather requirements

During hot weather, proper attention shall be given to ingredients, production methods, handling, placing, protection, and curing to prevent excessive concrete temperatures or water evaporation that could impair required strength or serviceability of the member or structure.

COMMENTARY

structure, core tests may be required, with or without supplemental wet curing, to check the structural adequacy, as provided in 5.6.4.

R5.12 — Cold weather requirements

Recommendations for cold weather concreting are given in detail in **"Cold Weather Concreting"** reported by ACI Committee 306.[5.12] (Presents requirements and methods for producing satisfactory concrete during cold weather.)

R5.13 — Hot weather requirements

Recommendations for hot weather concreting are given in detail in **"Hot Weather Concreting"** reported by ACI Committee 305.[5.13] (Defines the hot weather factors that affect concrete properties and construction practices and recommends measures to eliminate or minimize the undesirable effects.)

CHAPTER 6 — FORMWORK, EMBEDDED PIPES, AND CONSTRUCTION JOINTS

CODE

6.1 — Design of formwork

6.1.1 — Forms shall result in a final structure that conforms to shapes, lines, and dimensions of the members as required by the design drawings and specifications.

6.1.2 — Forms shall be substantial and sufficiently tight to prevent leakage of mortar.

6.1.3 — Forms shall be properly braced or tied together to maintain position and shape.

6.1.4 — Forms and their supports shall be designed so as not to damage previously placed structure.

6.1.5 — Design of formwork shall include consideration of the following factors:

(a) Rate and method of placing concrete

(b) Construction loads, including vertical, horizontal, and impact loads

(c) Special form requirements for construction of shells, folded plates, domes, architectural concrete, or similar types of elements.

6.1.6 — Forms for prestressed concrete members shall be designed and constructed to permit movement of the member without damage during application of prestressing force.

6.2 — Removal of forms, shores, and reshoring

6.2.1 — Removal of forms

Forms shall be removed in such a manner as not to impair safety and serviceability of the structure. Concrete to be exposed by form removal shall have sufficient strength not to be damaged by removal operation.

6.2.2 — Removal of shores and reshoring

The provisions of 6.2.2.1 through 6.2.2.3 shall apply to slabs and beams except where cast on the ground.

6.2.2.1 — Before starting construction, the contractor shall develop a procedure and schedule for

COMMENTARY

R6.1 — Design of formwork

Only minimum performance requirements for formwork, necessary to provide for public health and safety, are prescribed in Chapter 6. Formwork for concrete, including proper design, construction, and removal, demands sound judgment and planning to achieve adequate forms that are both economical and safe. Detailed information on formwork for concrete is given in: **"Guide to Formwork for Concrete."**$^{6.1}$ (Provides recommendations for design, construction, and materials for formwork, forms for special structures, and formwork for special methods of construction. Directed primarily to contractors, the suggested criteria will aid engineers and architects in preparing job specifications for the contractors.)

Formwork for Concrete$^{6.2}$ reported by ACI Committee 347. (A how-to-do-it handbook for contractors, engineers, and architects following the guidelines established in ACI 347R-88. Planning, building, and using formwork are discussed, including tables, diagrams, and formulas for form design loads.)

R6.2 — Removal of forms, shores, and reshoring

In determining the time for removal of forms, consideration should be given to the construction loads and to the possibilities of deflections.$^{6.3}$ The construction loads are frequently at least as great as the specified live loads. At early ages, a structure may be adequate to support the applied loads but may deflect sufficiently to cause permanent damage.

Evaluation of concrete strength during construction may be demonstrated by field-cured test cylinders or other procedures approved by the building official such as:

(a) Tests of cast-in-place cylinders in accordance with "Standard Test Method for Compressive Strength of Con-

CODE

removal of shores and installation of reshores and for calculating the loads transferred to the structure during the process.

(a) The structural analysis and concrete strength data used in planning and implementing form removal and shoring shall be furnished by the contractor to the building official when so requested.

(b) No construction loads shall be supported on, nor any shoring removed from, any part of the structure under construction except when that portion of the structure in combination with remaining forming and shoring system has sufficient strength to support safely its weight and loads placed thereon.

(c) Sufficient strength shall be demonstrated by structural analysis considering proposed loads, strength of forming and shoring system, and concrete strength data. Concrete strength data shall be based on tests of field-cured cylinders or, when approved by the building official, on other procedures to evaluate concrete strength.

6.2.2.2 — No construction loads exceeding the combination of superimposed dead load plus specified live load shall be supported on any unshored portion of the structure under construction, unless analysis indicates adequate strength to support such additional loads.

6.2.2.3 — Form supports for prestressed concrete members shall not be removed until sufficient prestressing has been applied to enable prestressed members to carry their dead load and anticipated construction loads.

COMMENTARY

crete Cylinders Cast-in-Place in Cylindrical Molds" (ASTM C 873). (This method is limited to use in slabs where the depth of concrete is from 5 to 12 in.)

(b) Penetration resistance in accordance with "Standard Test Method for Penetration Resistance of Hardened Concrete" (ASTM C 803).

(c) Pullout strength in accordance with "Standard Test Method for Pullout Strength of Hardened Concrete" (ASTM C 900).

(d) Maturity factor measurements and correlation in accordance with ASTM C 1074.[6.4]

Procedures (b), (c), and (d) require sufficient data, using job materials, to demonstrate correlation of measurements on the structure with compressive strength of molded cylinders or drilled cores.

Where the structure is adequately supported on shores, the side forms of beams, girders, columns, walls, and similar vertical forms, may generally be removed after 12 hr of cumulative curing time, provided the side forms support no loads other than the lateral pressure of the plastic concrete. "Cumulative curing time" represents the sum of time intervals, not necessarily consecutive, during which the temperature of the air surrounding the concrete is above 50 F. The 12-hr cumulative curing time is based on regular cements and ordinary conditions; the use of special cements or unusual conditions may require adjustment of the given limits. For example, concrete made with Type II or V (ASTM C 150) or ASTM C 595 cements, concrete containing retarding admixtures, and concrete to which ice was added during mixing (to lower the temperature of fresh concrete) may not have sufficient strength in 12 hr and should be investigated before removal of formwork.

The removal of formwork for multistory construction should be a part of a planned procedure considering the temporary support of the whole structure as well as that of each individual member. Such a procedure should be worked out prior to construction and should be based on a structural analysis taking into account the following items, as a minimum:

(a) The structural system that exists at the various stages of construction and the construction loads corresponding to those stages;

(b) The strength of the concrete at the various ages during construction;

(c) The influence of deformations of the structure and shoring system on the distribution of dead loads and construction loads during the various stages of construction;

CODE

6.3 — Conduits and pipes embedded in concrete

6.3.1 — Conduits, pipes, and sleeves of any material not harmful to concrete and within limitations of 6.3 shall be permitted to be embedded in concrete with approval of the engineer, provided they are not considered to replace structurally the displaced concrete.

6.3.2 — Conduits and pipes of aluminum shall not be embedded in structural concrete unless effectively coated or covered to prevent aluminum-concrete reaction or electrolytic action between aluminum and steel.

6.3.3 — Conduits, pipes, and sleeves passing through a slab, wall, or beam shall not impair significantly the strength of the construction.

6.3.4 — Conduits and pipes, with their fittings, embedded within a column shall not displace more than 4 percent of the area of cross section on which strength is calculated or which is required for fire protection.

6.3.5 — Except when drawings for conduits and pipes are approved by the structural engineer, conduits and pipes embedded within a slab, wall, or beam (other than those merely passing through) shall satisfy the following:

6.3.5.1 — They shall not be larger in outside dimension than $^{1}/_{3}$ the overall thickness of slab, wall, or beam in which they are embedded.

COMMENTARY

(d) The strength and spacing of shores or shoring systems used, as well as the method of shoring, bracing, shore removal, and reshoring including the minimum time intervals between the various operations;

(e) Any other loading or condition that affects the safety or serviceability of the structure during construction.

For multistory construction, the strength of the concrete during the various stages of construction should be substantiated by field-cured test specimens or other approved methods.

R6.3 — Conduits and pipes embedded in concrete

R6.3.1 — Conduits, pipes, and sleeves not harmful to concrete can be embedded within the concrete, but the work must be done in such a manner that the structure will not be endangered. Empirical rules are given in 6.3 for safe installations under common conditions; for other than common conditions, special designs must be made. Many general building codes have adopted ANSI/ASME piping codes B 31.1 for power piping[6.5] and B 31.3 for chemical and petroleum piping.[6.6] The specifier should be sure that the appropriate piping codes are used in the design and testing of the system. The contractor should not be permitted to install conduits, pipes, ducts, or sleeves that are not shown on the plans or not approved by the engineer or architect.

For the integrity of the structure, it is important that all conduit and pipe fittings within the concrete be carefully assembled as shown on the plans or called for in the job specifications.

R6.3.2 — The code prohibits the use of aluminum in structural concrete unless it is effectively coated or covered. Aluminum reacts with concrete and, in the presence of chloride ions, may also react electrolytically with steel, causing cracking and/or spalling of the concrete. Aluminum electrical conduits present a special problem since stray electric current accelerates the adverse reaction.

CODE

6.3.5.2 — They shall not be spaced closer than 3 diameters or widths on center.

6.3.5.3 — They shall not impair significantly the strength of the construction.

6.3.6 — Conduits, pipes, and sleeves shall be permitted to be considered as replacing structurally in compression the displaced concrete provided:

6.3.6.1 — They are not exposed to rusting or other deterioration.

6.3.6.2 — They are of uncoated or galvanized iron or steel not thinner than standard Schedule 40 steel pipe.

6.3.6.3 — They have a nominal inside diameter not over 2 in. and are spaced not less than 3 diameters on centers.

6.3.7 — Pipes and fittings shall be designed to resist effects of the material, pressure, and temperature to which they will be subjected.

6.3.8 — No liquid, gas, or vapor, except water not exceeding 90 F nor 50 psi pressure, shall be placed in the pipes until the concrete has attained its design strength.

6.3.9 — In solid slabs, piping, unless it is for radiant heating or snow melting, shall be placed between top and bottom reinforcement.

6.3.10 — Concrete cover for pipes, conduits, and fittings shall not be less than $1^1/_2$ in. for concrete exposed to earth or weather, nor $^3/_4$ in. for concrete not exposed to weather or in contact with ground.

6.3.11 — Reinforcement with an area not less than 0.002 times area of concrete section shall be provided normal to piping.

6.3.12 — Piping and conduit shall be so fabricated and installed that cutting, bending, or displacement of reinforcement from its proper location will not be required.

6.4 — Construction joints

6.4.1 — Surface of concrete construction joints shall be cleaned and laitance removed.

COMMENTARY

R6.3.7 — The 1983 code limited the maximum pressure in embedded pipe to 200 psi, which was considered too restrictive. Nevertheless, the effects of such pressures and the expansion of embedded pipe should be considered in the design of the concrete member.

R6.4 — Construction joints

For the integrity of the structure, it is important that all construction joints be carefully defined in construction documents and constructed as required. Any deviations therefrom should be approved by the engineer or architect.

CODE

6.4.2 — Immediately before new concrete is placed, all construction joints shall be wetted and standing water removed.

6.4.3 — Construction joints shall be so made and located as not to impair the strength of the structure. Provision shall be made for transfer of shear and other forces through construction joints. See 11.7.9.

6.4.4 — Construction joints in floors shall be located within the middle third of spans of slabs, beams, and girders. Joints in girders shall be offset a minimum distance of two times the width of intersecting beams.

6.4.5 — Beams, girders, or slabs supported by columns or walls shall not be cast or erected until concrete in the vertical support members is no longer plastic.

6.4.6 — Beams, girders, haunches, drop panels, and capitals shall be placed monolithically as part of a slab system, unless otherwise shown in design drawings or specifications.

COMMENTARY

R6.4.2 — The requirements of the 1977 code for the use of neat cement on vertical joints have been removed, since it is rarely practical and can be detrimental where deep forms and steel congestion prevent proper access. Often wet blasting and other procedures are more appropriate. Since the code sets only minimum standards the engineer may have to specify special procedures if conditions warrant. The degree to which mortar batches are needed at the start of concrete placement depend on concrete proportions, congestion of steel, vibrator access, and other factors.

R6.4.3 — Construction joints should be located where they will cause the least weakness in the structure. When shear due to gravity load is not significant, as is usually the case in the middle of the span of flexural members, a simple vertical joint may be adequate. Lateral force design may require special design treatment of construction joints. Shear keys, intermittent shear keys, diagonal dowels, or the shear transfer method of 11.7 may be used whenever a force transfer is required.

R6.4.5 — Delay in placing concrete in members supported by columns and walls is necessary to prevent cracking at the interface of the slab and supporting member, caused by bleeding and settlement of plastic concrete in the supporting member.

R6.4.6 — Separate placement of slabs and beams, haunches, and similar elements is permitted when shown on the drawings and where provision has been made to transfer forces as required in 6.4.3.

Notes

CHAPTER 7 — DETAILS OF REINFORCEMENT

CODE

7.0 — Notation

d = distance from extreme compression fiber to centroid of tension reinforcement, in.
d_b = nominal diameter of bar, wire, or prestressing strand, in.
f_y = specified yield strength of nonprestressed reinforcement, psi
ℓ_d = development length, in. See Chapter 12

7.1 — Standard hooks

The term "standard hook" as used in this code shall mean one of the following:

7.1.1 — 180-deg bend plus $4d_b$ extension, but not less than 2½ in. at free end of bar.

7.1.2 — 90-deg bend plus $12d_b$ extension at free end of bar.

7.1.3 — For stirrup and tie hooks*

(a) No. 5 bar and smaller, 90-deg bend plus $6d_b$ extension at free end of bar, or

(b) No. 6, No. 7, and No. 8 bar, 90-deg bend plus $12d_b$ extension at free end of bar, or

(c) No. 8 bar and smaller, 135-deg bend plus $6d_b$ extension at free end of bar.

7.2 — Minimum bend diameters

7.2.1 — Diameter of bend measured on the inside of the bar, other than for stirrups and ties in sizes No. 3 through No. 5, shall not be less than the values in Table 7.2.

7.2.2 — Inside diameter of bend for stirrups and ties

COMMENTARY

Recommended methods and standards for preparing design drawings, typical details, and drawings for the fabrication and placing of reinforcing steel in reinforced concrete structures are given in ***ACI Detailing Manual — 1994***, reported by ACI Committee 315.[7.1]

All provisions in this code relating to bar, wire, or strand diameter (and area) are based on the nominal dimensions of the reinforcement as given in the appropriate ASTM specification. Nominal dimensions are equivalent to those of a circular area having the same weight per foot as the ASTM designated bar, wire, or strand sizes. Cross-sectional area of reinforcement is based on nominal dimensions.

R7.1 — Standard hooks

R7.1.3 — Standard stirrup and tie hooks are limited to No. 8 bars and smaller, and the 90-deg hook with $6d_b$ extension is further limited to No. 5 bars and smaller, in both cases as the result of research showing that larger bar sizes with 90-deg hooks and $6d_b$ extensions tend to "pop out" under high load.

R7.2 — Minimum bend diameters

Standard bends in reinforcing bars are described in terms of the inside diameter of bend since this is easier to measure than the radius of bend. The primary factors affecting the minimum bend diameter are feasibility of bending without breakage and avoidance of crushing the concrete inside the bend.

R7.2.2 — The minimum $4d_b$ bend for the bar sizes commonly used for stirrups and ties is based on accepted industry practice in the United States. Use of a stirrup bar size not greater than No. 5 for either the 90-deg or 135-deg standard

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*For closed ties and continuously wound ties defined as hoops in Chapter 21, a 135-deg bend plus an extension of at least $6d_b$, but not less than 3 in. (See definition of "hoop" in 21.1.)

CODE

shall not be less than $4d_b$ for No. 5 bar and smaller. For bars larger than No. 5, diameter of bend shall be in accordance with Table 7.2.

7.2.3 — Inside diameter of bend in welded wire fabric (plain or deformed) for stirrups and ties shall not be less than $4d_b$ for deformed wire larger than D6 and $2d_b$ for all other wires. Bends with inside diameter of less than $8d_b$ shall not be less than $4d_b$ from nearest welded intersection.

TABLE 7.2—MINIMUM DIAMETERS OF BEND

Bar size	Minimum diameter
No. 3 through No. 8	$6d_b$
No. 9, No. 10, and No. 11	$8d_b$
No. 14 and No. 18	$10d_b$

7.3 — Bending

7.3.1 — All reinforcement shall be bent cold, unless otherwise permitted by the engineer.

7.3.2 — Reinforcement partially embedded in concrete shall not be field bent, except as shown on the design drawings or permitted by the engineer.

7.4 — Surface conditions of reinforcement

7.4.1—At time concrete is placed, reinforcement shall be free from mud, oil, or other nonmetallic coatings that decrease bond. Epoxy coatings of bars in accord

COMMENTARY

stirrup hook will permit multiple bending on standard stirrup bending equipment.

R7.2.3 — Welded wire fabric, of plain or deformed wire, can be used for stirrups and ties. The wire at welded intersections does not have the same uniform ductility and bendability as in areas which were not heated. These effects of the welding temperature are usually dissipated in a distance of approximately four wire diameters. Minimum bend diameters permitted are in most cases the same as those required in the ASTM bend tests for wire material.

R7.3 — Bending

R7.3.1—The engineer may be the design engineer or architect or the engineer or architect employed by the owner to perform inspection. For unusual bends with inside diameters less than ASTM bend test requirements, special fabrication may be required.

R7.3.2 — Construction conditions may make it necessary to bend bars that have been embedded in concrete. Such field bending should not be done without authorization of the engineer. The engineer must determine whether the bars should be bent cold or if heating should be used. Bends should be gradual and must be straightened as required.

Tests[7.2,7.3] have shown that A 615 Grade 40 and Grade 60 reinforcing bars can be cold bent and straightened up to 90 deg at or near the minimum diameter specified in 7.2. If cracking or breakage is encountered, heating to a maximum temperature of 1500 F should be beneficial for avoiding this condition for the remainder of the bars. Bars that fracture during bending or straightening can be spliced outside the bend region.

Heating must be performed in a manner that will avoid damage to the concrete. If the bend area is within approximately 6 in. of the concrete, some protective insulation may need to be applied. Heating of the bar should be controlled by temperature-indicating crayons or other suitable means. The heated bars should not be artificially cooled (with water or forced air) until after cooling to at least 600 F.

R7.4 — Surface conditions of reinforcement

Specific limits on rust are based on tests,[7.4] plus a review of earlier tests and recommendations. Reference 7.4 provides guidance with regard to the effects of rust and mill scale on

CODE

with standards in this code shall be permitted.

7.4.2 — Reinforcement, except prestressing tendons, with rust, mill scale, or a combination of both shall be considered satisfactory, provided the minimum dimensions (including height of deformations) and weight of a hand-wire-brushed test specimen are not less than applicable ASTM specification requirements.

7.4.3 — Prestressing tendons shall be clean and free of oil, dirt, scale, pitting and excessive rust. A light oxide shall be permitted.

7.5 — Placing reinforcement

7.5.1 — Reinforcement, prestressing tendons, and ducts shall be accurately placed and adequately supported before concrete is placed, and shall be secured against displacement within tolerances permitted in 7.5.2.

7.5.2 — Unless otherwise specified by the engineer, reinforcement, prestressing tendons, and prestressing ducts shall be placed within the following tolerances:

7.5.2.1 — Tolerance for depth ***d***, and minimum concrete cover in flexural members, walls and compression members shall be as follows:

COMMENTARY

bond characteristics of deformed reinforcing bars. Research has shown that a normal amount of rust increases bond. Normal rough handling generally removes rust which is loose enough to injure the bond between the concrete and reinforcement.

R7.5 — Placing reinforcement

R7.5.1 — Reinforcement including prestressing tendons must be adequately supported in the forms to prevent displacement by concrete placement or workers. Beam stirrups should be supported on the bottom form of the beam by positive supports such as continuous longitudinal beam bolsters. If only the longitudinal beam bottom reinforcement is supported, construction traffic can dislodge the stirrups as well as any prestressing tendons tied to the stirrups.

R7.5.2 — Generally accepted practice, as reflected in **"Standard Tolerances for Concrete Construction and Materials"** (ACI 117)[7.5] has established tolerances on total depth (formwork or finish) and fabrication of truss bent reinforcing bars and closed ties, stirrups, and spirals. The engineer should specify more restrictive tolerances than those permitted by the code when it is necessary to minimize the accumulation of tolerances resulting in excessive reduction in effective depth or cover.

More restrictive tolerances have been placed on minimum clear distance to formed soffits because of its importance for durability and fire protection, and because bars are usually supported in such a manner that the specified tolerance is practical.

More restrictive tolerances than those required by the code may be desirable for prestressed concrete to achieve camber control within limits acceptable to the designer or owner. In such cases, the engineer should specify the necessary tolerances. Recommendations are given in Reference 7.6.

R7.5.2.1 — The code specifies a tolerance on depth ***d***, an essential component of strength of the member. Because reinforcing steel is placed with respect to edges of members and formwork surfaces, the depth ***d*** is not always conveniently measured in the field. Engineers should spec-

CODE

	Tolerance on **d**	Tolerance on minimum concrete cover
$d \leq 8$ in.	$\pm^3/_8$ in.	$-^3/_8$ in.
$d > 8$ in.	$\pm^1/_2$ in.	$-^1/_2$ in.

except that tolerance for the clear distance to formed soffits shall be minus $^1/_4$ in. and tolerance for cover shall not exceed minus $^1/_3$ the minimum concrete cover required in the design drawings or specifications.

7.5.2.2 — Tolerance for longitudinal location of bends and ends of reinforcement shall be ± 2 in. except at discontinuous ends of members where tolerance shall be ± $^1/_2$ in.

7.5.3 — Welded wire fabric (with wire size not greater than W5 or D5) used in slabs not exceeding 10 ft in span shall be permitted to be curved from a point near the top of slab over the support to a point near the bottom of slab at midspan, provided such reinforcement is either continuous over, or securely anchored at support.

7.5.4 — Welding of crossing bars shall not be permitted for assembly of reinforcement unless authorized by the engineer.

7.6 — Spacing limits for reinforcement

7.6.1 — The minimum clear spacing between parallel bars in a layer shall be d_b, but not less than 1 in. See also 3.3.2.

7.6.2 — Where parallel reinforcement is placed in two or more layers, bars in the upper layers shall be placed directly above bars in the bottom layer with clear distance between layers not less than 1 in.

7.6.3 — In spirally reinforced or tied reinforced compression members, clear distance between longitudinal bars shall be not less than $1.5d_b$ nor $1^1/_2$ in. See also 3.3.2.

7.6.4 — Clear distance limitation between bars shall apply also to the clear distance between a contact lap splice and adjacent splices or bars.

7.6.5 — In walls and slabs other than concrete joist construction, primary flexural reinforcement shall be spaced not farther apart than three times the wall or slab thickness, nor 18 in.

COMMENTARY

ify tolerances for bar placement, cover, and member size. See ACI 117.[7.5]

R7.5.4 — "Tack" welding (welding crossing bars) can seriously weaken a bar at the point welded by creating a metallurgical notch effect. This operation can be performed safely only when the material welded and welding operations are under continuous competent control, as in the manufacture of welded wire fabric.

R7.6 — Spacing limits for reinforcement

Although the minimum bar spacings are unchanged in this code, the development lengths given in Chapter 12 became a function of the bar spacings since the 1989 code. As a result, it may be desirable to use larger than minimum bar spacings in some cases. The minimum limits were originally established to permit concrete to flow readily into spaces between bars and between bars and forms without honeycomb, and to ensure against concentration of bars on a line that may cause shear or shrinkage cracking. Use of "nominal" bar diameter to define minimum spacing permits a uniform criteria for all bar sizes.

CODE

7.6.6 — Bundled bars

7.6.6.1 — Groups of parallel reinforcing bars bundled in contact to act as a unit shall be limited to four in any one bundle.

7.6.6.2 — Bundled bars shall be enclosed within stirrups or ties.

7.6.6.3 — Bars larger than No. 11 shall not be bundled in beams.

7.6.6.4 — Individual bars within a bundle terminated within the span of flexural members shall terminate at different points with at least $40d_b$ stagger.

7.6.6.5 — Where spacing limitations and minimum concrete cover are based on bar diameter d_b, a unit of bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area.

7.6.7 — Prestressing tendons and ducts

7.6.7.1 — Clear distance between pretensioning tendons at each end of a member shall be not less than $4d_b$ for wire, nor $3d_b$ for strands. See also 3.3.2. Closer vertical spacing and bundling of tendons shall be permitted in the middle portion of a span.

7.6.7.2 — Bundling of post-tensioning ducts shall be permitted if shown that concrete can be satisfactorily placed and if provision is made to prevent the tendons, when tensioned, from breaking through the duct.

7.7 — Concrete protection for reinforcement

7.7.1 — Cast-in-place concrete (nonprestressed)

The following minimum concrete cover shall be provided for reinforcement:

	Minimum cover, in.
(a) Concrete cast against and permanently exposed to earth	3
(b) Concrete exposed to earth or weather:	

COMMENTARY

R7.6.6 — Bundled bars

Bond research[7.7] showed that bar cutoffs within bundles should be staggered. Bundled bars should be tied, wired, or otherwise fastened together to ensure remaining in position whether vertical or horizontal.

A limitation that bars larger than No. 11 not be bundled in beams or girders is a practical limit for application to building size members. (The "**Standard Specifications for Highway Bridges**"[7.8] permits two-bar bundles for No. 14 and No. 18 bars in bridge girders.) Conformance to the crack control requirements of 10.6 will effectively preclude bundling of bars larger than No. 11 as tensile reinforcement. The code phrasing "bundled in contact to act as a unit," is intended to preclude bundling more than two bars in the same plane. Typical bundle shapes are triangular, square, or L-shaped patterns for three- or four-bar bundles. As a practical caution, bundles more than one bar deep in the plane of bending may not be hooked or bent as a unit. Where end hooks are required, it is preferable to stagger the individual bar hooks within a bundle.

R7.6.7 — Prestressing tendons and ducts

R7.6.7.2 — When ducts for post-tensioning tendons in a beam are arranged closely together vertically, provision must be made to prevent the tendons, when tensioned, from breaking through the duct. Horizontal disposition of ducts must allow proper placement of concrete. Generally a clear spacing of one and one-third times the size of the coarse aggregate, but not less than 1 in., has proven satisfactory. Where concentration of tendons or ducts tends to create a weakened plane in the concrete cover, reinforcement should be provided to control cracking.

R7.7 — Concrete protection for reinforcement

Concrete cover as protection of reinforcement against weather and other effects is measured from the concrete surface to the outermost surface of the steel to which the cover requirement applies. Where minimum cover is prescribed for a class of structural member, it is measured to the outer edge of stirrups, ties, or spirals if transverse reinforcement encloses main bars; to the outermost layer of bars if more than one layer is used without stirrups or ties; to the metal end fitting or duct on post-tensioned prestressing steel.

The condition "concrete surfaces exposed to the weather" refers to direct exposure to moisture changes and not just to

CODE

	Minimum cover, in.
No. 6 through No. 18 bars	2
No. 5 bar, W31 or D31 wire, and smaller	$1\frac{1}{2}$

(c) Concrete not exposed to weather or in contact with ground:

Slabs, walls, joists:	
No. 14 and No. 18 bars	$1\frac{1}{2}$
No. 11 bar and smaller	$\frac{3}{4}$
Beams, columns:	
Primary reinforcement, ties, stirrups, spirals	$1\frac{1}{2}$
Shells, folded plate members:	
No. 6 bar and larger	$\frac{3}{4}$
No. 5 bar, W31 or D31 wire, and smaller	$\frac{1}{2}$

7.7.2 — Precast concrete (manufactured under plant control conditions)

The following minimum concrete cover shall be provided for reinforcement:

(a) Concrete exposed to earth or weather:

Wall panels:	
No. 14 and No. 18 bars	$1\frac{1}{2}$
No. 11 bar and smaller	$\frac{3}{4}$
Other members:	
No. 14 and No. 18 bars	2
No. 6 through No. 11 bars	$1\frac{1}{2}$
No. 5 bar, W31 or D31 wire, and smaller	$1\frac{1}{4}$

(b) Concrete not exposed to weather or in contact with ground:

Slabs, walls, joists:	
No. 14 and No. 18 bars	$1\frac{1}{4}$
No. 11 bar and smaller	$\frac{5}{8}$
Beams, columns:	
Primary reinforcement	d_b but not less than $\frac{5}{8}$ and need not exceed $1\frac{1}{2}$
Ties, stirrups, spirals	$\frac{3}{8}$
Shells, folded plate members:	
No. 6 bar and larger	$\frac{5}{8}$
No. 5 bar, W31 or D31 wire, and smaller	$\frac{3}{8}$

7.7.3 — Prestressed concrete

7.7.3.1 — The following minimum concrete cover shall be provided for prestressed and nonprestressed reinforcement, ducts, and end fittings, except as provided in 7.7.3.2 and 7.7.3.3:

COMMENTARY

temperature changes. Slab or thin shell soffits are not usually considered directly "exposed" unless subject to alternate wetting and drying, including that due to condensation conditions or direct leakage from exposed top surface, run off, or similar effects.

Alternative methods of protecting the reinforcement from weather may be provided if they are equivalent to the additional concrete cover required by the code. When approved by the building official under the provisions of 1.4, reinforcement with alternative protection from the weather may have concrete cover not less than the cover required for reinforcement not exposed to weather.

The development lengths given in Chapter 12 are now a function of the bar cover. As a result, it may be desirable to use larger than minimum cover in some cases.

R7.7.2 — Precast concrete (manufactured under plant control conditions)

The lesser thicknesses for precast construction reflect the greater convenience of control for proportioning, placing, and curing inherent in precasting. The term "manufactured under plant controlled conditions" does not specifically imply that precast members must be manufactured in a plant. Structural elements precast at the job site will also qualify under this section if the control of form dimensions, placing of reinforcement, quality control of concrete, and curing procedure are equal to that normally expected in a plant.

	Minimum cover, in.
(a) Concrete cast against and permanently exposed to earth	3
(b) Concrete exposed to earth or weather:	
Wall panels, slabs, joists	1
Other members	$1\frac{1}{2}$
(c) Concrete not exposed to weather or in contact with ground:	
Slabs, walls, joists	$\frac{3}{4}$
Beams, columns:	
Primary reinforcement	$1\frac{1}{2}$
Ties, stirrups, spirals	1
Shells, folded plate members:	
No. 5 bar, W31 or D31 wire, and smaller	$\frac{3}{8}$
Other reinforcement	d_b but not less than $\frac{3}{4}$

7.7.3.2 — For prestressed concrete members exposed to earth, weather, or corrosive environments, and in which permissible tensile stress of 18.4.2(c) is exceeded, minimum cover shall be increased 50 percent.

7.7.3.3 — For prestressed concrete members manufactured under plant control conditions, minimum concrete cover for nonprestressed reinforcement shall be as required in 7.7.2.

7.7.4 — Bundled bars

For bundled bars, minimum concrete cover shall be equal to the equivalent diameter of the bundle, but need not be greater than 2 in.; except for concrete cast against and permanently exposed to earth, minimum cover shall be 3 in.

7.7.5 — Corrosive environments

In corrosive environments or other severe exposure conditions, amount of concrete protection shall be suitably increased, and denseness and nonporosity of protecting concrete shall be considered, or other protection shall be provided.

R7.7.5 — Corrosive environments

When concrete will be exposed to external sources of chlorides in service, such as deicing salts, brackish water, seawater, or spray from these sources, concrete must be proportioned to satisfy the special exposure requirements of Chapter 4. These include minimum air content, maximum water-cementitious materials ratio, minimum strength for normal weight and lightweight concrete, maximum chloride ion in concrete, and cement type. Additionally, for corrosion protection, a minimum concrete cover for reinforcement of 2 in. for walls and slabs and $2\frac{1}{2}$ in. for other members is recommended. For precast concrete manufactured under plant control conditions, a minimum cover of $1\frac{1}{2}$ and 2 in., respectively, is recommended.

CODE

7.7.6 — Future extensions

Exposed reinforcement, inserts, and plates intended for bonding with future extensions shall be protected from corrosion.

7.7.7 — Fire protection

When the general building code (of which this code forms a part) requires a thickness of cover for fire protection greater than the minimum concrete cover specified in 7.7, such greater thicknesses shall be used.

7.8 — Special reinforcement details for columns

7.8.1 — Offset bars

Offset bent longitudinal bars shall conform to the following:

7.8.1.1 — Slope of inclined portion of an offset bar with axis of column shall not exceed 1 in 6.

7.8.1.2 — Portions of bar above and below an offset shall be parallel to axis of column.

7.8.1.3 — Horizontal support at offset bends shall be provided by lateral ties, spirals, or parts of the floor construction. Horizontal support provided shall be designed to resist $1^1/_2$ times the horizontal component of the computed force in the inclined portion of an offset bar. Lateral ties or spirals, if used, shall be placed not more than 6 in. from points of bend.

7.8.1.4 — Offset bars shall be bent before placement in the forms. See 7.3.

7.8.1.5 — Where a column face is offset 3 in. or greater, longitudinal bars shall not be offset bent. Separate dowels, lap spliced with the longitudinal bars adjacent to the offset column faces, shall be provided. Lap splices shall conform to 12.17.

7.8.2 — Steel cores

Load transfer in structural steel cores of composite compression members shall be provided by the following:

7.8.2.1 — Ends of structural steel cores shall be accurately finished to bear at end bearing splices, with positive provision for alignment of one core above the other in concentric contact.

COMMENTARY

R7.8 — Special reinforcement details for columns

R7.8.2 — Steel cores

The 50 percent limit on transfer of compressive load by end bearing on ends of structural steel cores is intended to provide some tensile capacity at such splices (up to 50 percent), since the remainder of the total compressive stress in the steel core must be transmitted by dowels, splice plates, welds, etc. This provision should ensure that splices in composite compression members meet essentially the same tensile capacity as required for conventionally reinforced concrete compression members.

CODE

7.8.2.2 — At end bearing splices, bearing shall be considered effective to transfer not more than 50 percent of the total compressive stress in the steel core.

7.8.2.3 — Transfer of stress between column base and footing shall be designed in accordance with 15.8.

7.8.2.4 — Base of structural steel section shall be designed to transfer the total load from the entire composite member to the footing; or, the base shall be designed to transfer the load from the steel core only, provided ample concrete section is available for transfer of the portion of the total load carried by the reinforced concrete section to the footing by compression in the concrete and by reinforcement.

7.9 — Connections

7.9.1 — At connections of principal framing elements (such as beams and columns), enclosure shall be provided for splices of continuing reinforcement and for anchorage of reinforcement terminating in such connections.

7.9.2 — Enclosure at connections shall consist of external concrete or internal closed ties, spirals, or stirrups.

7.10 — Lateral reinforcement for compression members

7.10.1 — Lateral reinforcement for compression members shall conform to the provisions of 7.10.4 and 7.10.5 and, where shear or torsion reinforcement is required, shall also conform to provisions of Chapter 11.

7.10.2 — Lateral reinforcement requirements for composite compression members shall conform to 10.16. Lateral reinforcement requirements for prestressing tendons shall conform to 18.11.

7.10.3 — It shall be permitted to waive the lateral reinforcement requirements of 7.10, 10.16, and 18.11 where tests and structural analysis show adequate strength and feasibility of construction.

COMMENTARY

R7.9 — Connections

Confinement is essential at connections to assure that the flexural capacity of the members can be developed without deterioration of the joint under repeated loadings.[7.9,7.10]

R7.10 — Lateral reinforcement for compression members

R7.10.3 — Precast columns with cover less than $1^1/_2$ in., prestressed columns without longitudinal bars, columns smaller than minimum dimensions prescribed in earlier ACI Building Codes, columns of concrete with small size coarse aggregate, wall-like columns, and other special cases may require special designs for lateral reinforcement. Plain or deformed wire, W4, D4, or larger, may be used for ties or spirals. If such special columns are considered as spiral columns for load strength in design, the ratio of spiral reinforcement ρ_s must conform to 10.9.3.

CODE

7.10.4 — Spirals

Spiral reinforcement for compression members shall conform to 10.9.3 and to the following:

7.10.4.1 — Spirals shall consist of evenly spaced continuous bar or wire of such size and so assembled to permit handling and placing without distortion from designed dimensions.

7.10.4.2 — For cast-in-place construction, size of spirals shall not be less than $^3/_8$ in. diameter.

7.10.4.3 — Clear spacing between spirals shall not exceed 3 in., nor be less than 1 in. See also 3.3.2.

7.10.4.4 — Anchorage of spiral reinforcement shall be provided by $1^1/_2$ extra turns of spiral bar or wire at each end of a spiral unit.

7.10.4.5 — Splices in spiral reinforcement shall be lap splices of $48d_b$ but not less than 12 in., or welded.

7.10.4.6 — Spirals shall extend from top of footing or slab in any story to level of lowest horizontal reinforcement in members supported above.

7.10.4.7 — Where beams or brackets do not frame into all sides of a column, ties shall extend above termination of spiral to bottom of slab or drop panel.

7.10.4.8 — In columns with capitals, spirals shall extend to a level at which the diameter or width of capital is two times that of the column.

7.10.4.9 — Spirals shall be held firmly in place and true to line.

7.10.5 — Ties

Tie reinforcement for compression members shall conform to the following:

7.10.5.1 — All nonprestressed bars shall be enclosed by lateral ties, at least No. 3 in size for longitudinal bars No. 10 or smaller, and at least No. 4 in size for No. 11, No. 14, No. 18, and bundled longitudinal bars. Deformed wire or welded wire fabric of equivalent area shall be permitted.

7.10.5.2 — Vertical spacing of ties shall not exceed 16 longitudinal bar diameters, 48 tie bar or wire diameters, or least dimension of the compression member.

COMMENTARY

R7.10.4 — Spirals

For practical considerations in cast-in-place construction, the minimum diameter of spiral reinforcement is $^3/_8$ in. ($^3/_8$" ϕ, No. 3 bar, or W11 or D11 wire). This is the smallest size that can be used in a column with $1^1/_2$ in. or more cover and having concrete strengths of 3000 psi or more if the minimum clear spacing for placing concrete is to be maintained.

Standard spiral sizes are $^3/_8$ in., $^1/_2$ in., and $^5/_8$ in. diameter for hot rolled or cold drawn material, plain or deformed.

The code allows spirals to be terminated at the level of lowest horizontal reinforcement framing into the column. However, if one or more sides of the column are not enclosed by beams or brackets, ties are required from the termination of the spiral to the bottom of the slab or drop panel. If beams or brackets enclose all sides of the column but are of different depths, the ties should extend from the spiral to the level of the horizontal reinforcement of the shallowest beam or bracket framing into the column. These additional ties are to enclose the longitudinal column reinforcement and the portion of bars from beams bent into the column for anchorage. See also 7.9.

Spirals must be held firmly in place, at proper pitch and alignment, to prevent displacement during concrete placement. The code has traditionally required spacers to hold the fabricated spiral cage in place but was changed in 1989 to allow alternate methods of installation. When spacers are used, the following may be used for guidance: For spiral bar or wire smaller than $^5/_8$ in. diameter, a minimum of two spacers should be used for spirals less than 20 in. in diameter, three spacers for spirals 20 to 30 in. in diameter, and four spacers for spirals greater than 30 in. in diameter. For spiral bar or wire $^5/_8$ in. diameter or larger, a minimum of three spacers should be used for spirals 24 in. or less in diameter, and four spacers for spirals greater than 24 in. in diameter. The project specifications or subcontract agreements should be clearly written to cover the supply of spacers or field tying of the spiral reinforcement.

R7.10.5 — Ties

All longitudinal bars in compression must be enclosed within lateral ties. Where longitudinal bars are arranged in a circular pattern, only one circular tie per specified spacing is required. This requirement can be satisfied by a continuous circular tie (helix) at larger pitch than required for spirals under 10.9.3, the maximum pitch being equal to the required tie spacing.

The 1956 ACI Building Code required "lateral support equivalent to that provided by a 90-deg corner of a tie," for every vertical bar. Tie requirements were liberalized in 1963 by increasing the permissible included angle from 90 to 135 deg and exempting bars which are located within 6 in. clear

CODE

7.10.5.3 — Ties shall be arranged such that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie with an included angle of not more than 135 deg and no bar shall be farther than 6 in. clear on each side along the tie from such a laterally supported bar. Where longitudinal bars are located around the perimeter of a circle, a complete circular tie shall be permitted.

7.10.5.4 — Ties shall be located vertically not more than one-half a tie spacing above the top of footing or slab in any story, and shall be spaced as provided herein to not more than one-half a tie spacing below the lowest horizontal reinforcement in slab or drop panel above.

7.10.5.5 — Where beams or brackets frame from four directions into a column, termination of ties not more than 3 in. below lowest reinforcement in shallowest of such beams or brackets shall be permitted.

7.11 — Lateral reinforcement for flexural members

7.11.1 — Compression reinforcement in beams shall be enclosed by ties or stirrups satisfying the size and spacing limitations in 7.10.5 or by welded wire fabric of equivalent area. Such ties or stirrups shall be provided throughout the distance where compression reinforcement is required.

7.11.2 — Lateral reinforcement for flexural framing members subject to stress reversals or to torsion at

COMMENTARY

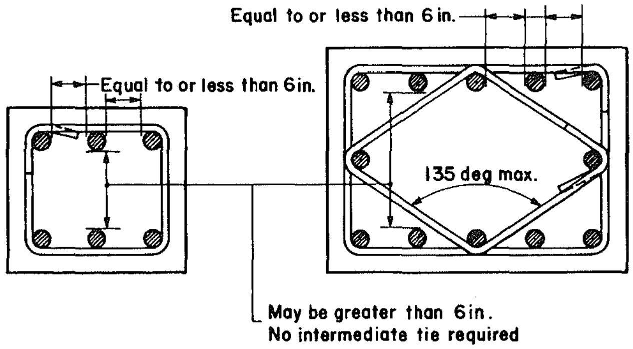

Equal to or less than 6 in.
Equal to or less than 6in.
135 deg max.
May be greater than 6in.
No intermediate tie required

Fig. R7.10.5—Sketch to clarify measurements between laterally supported column bars

on each side along the tie from adequately tied bars (see Fig. R7.10.5). Limited tests[7.11] on full-size, axially-loaded, tied columns containing full-length bars (without splices) showed no appreciable difference between ultimate strengths of columns with full tie requirements and no ties at all.

Since spliced bars and bundled bars were not included in the tests of Reference 7.11, it would be prudent to provide a set of ties at each end of lap spliced bars, above and below end-bearing splices, and at minimum spacings immediately below sloping regions of offset bent bars.

Standard tie hooks are intended for use with deformed bars only, and should be staggered where possible. See also 7.9.

Continuously wound bars or wires can be used as ties provided their pitch and area are at least equivalent to the area and spacing of separate ties. Anchorage at the end of a continuously wound bar or wire should be by a standard hook as for separate bars or by one additional turn of the tie pattern. A circular continuously wound bar or wire is considered a spiral if it conforms to 7.10.4, otherwise it is considered a tie.

R7.10.5.5 — With the 1983 code, the wording of this section was modified to clarify that ties may be terminated only when elements frame into all four sides of square and rectangular columns; and, for round or polygonal columns, such elements frame into the column from four directions.

R7.11 — Lateral reinforcement for flexural members

R7.11.1 — Compression reinforcement in beams and girders must be enclosed to prevent buckling; similar requirements for such enclosure have remained essentially unchanged through several editions of the code, except for minor clarification.

CODE

supports shall consist of closed ties, closed stirrups, or spirals extending around the flexural reinforcement.

7.11.3 — Closed ties or stirrups shall be formed in one piece by overlapping standard stirrup or tie end hooks around a longitudinal bar, or formed in one or two pieces lap spliced with a Class B splice (lap of $1.3\ell_d$) or anchored in accordance with 12.13.

7.12 — Shrinkage and temperature reinforcement

7.12.1 — Reinforcement for shrinkage and temperature stresses normal to flexural reinforcement shall be provided in structural slabs where the flexural reinforcement extends in one direction only.

7.12.1.1 — Shrinkage and temperature reinforcement shall be provided in accordance with either 7.12.2 or 7.12.3.

7.12.1.2 — Where shrinkage and temperature movements are significantly restrained, the requirements of 8.2.4 and 9.2.7 shall be considered.

7.12.2 — Deformed reinforcement conforming to 3.5.3 used for shrinkage and temperature reinforcement shall be provided in accordance with the following:

7.12.2.1 — Area of shrinkage and temperature reinforcement shall provide at least the following ratios of reinforcement area to gross concrete area, but not less than 0.0014:

(a) Slabs where Grade 40 or 50
deformed bars are used 0.0020

(b) Slabs where Grade 60
deformed bars or welded wire
fabric (plain or deformed) are used 0.0018

(c) Slabs where reinforcement
with yield stress exceeding 60,000
psi measured at a yield strain
of 0.35 percent is used .... $\frac{0.0018 \times 60{,}000}{f_y}$

COMMENTARY

R7.12 — Shrinkage and temperature reinforcement

R7.12.1 — Shrinkage and temperature reinforcement is required at right angles to the principal reinforcement to minimize cracking and to tie the structure together to assure its acting as assumed in the design. The provisions of this section are intended for structural slabs only; they are not intended for soil supported "slabs on grade."

R7.12.1.2 — The area of shrinkage and temperature reinforcement required by 7.12 has been satisfactory where shrinkage and temperature movements are permitted to occur. For cases where structural walls or large columns provide significant restraints to shrinkage and temperature movements, it may be necessary to increase the amount of reinforcement normal to the flexural reinforcement in 7.12.1.2 (see Reference 7.12). Top and bottom reinforcement are both effective in controlling cracks. Control strips during the construction period, which permit initial shrinkage to occur without causing an increase in stresses, are also effective in reducing cracks caused by restraints.

R7.12.2 — The amounts specified for deformed bars and welded wire fabric are empirical but have been used satisfactorily for many years. Splices and end anchorages of shrinkage and temperature reinforcement must be designed for the full specified yield strength in accordance with 12.1, 12.15, 12.18, and 12.19.

CODE

7.12.2.2 — Shrinkage and temperature reinforcement shall be spaced not farther apart than five times the slab thickness, nor 18 in.

7.12.2.3 — At all sections where required, reinforcement for shrinkage and temperature stresses shall develop the specified yield strength f_y in tension in accordance with Chapter 12.

7.12.3 — Prestressing tendons conforming to 3.5.5 used for shrinkage and temperature reinforcement shall be provided in accordance with the following:

7.12.3.1 — Tendons shall be proportioned to provide a minimum average compressive stress of 100 psi on gross concrete area using effective prestress, after losses, in accordance with 18.6.

7.12.3.2 — Spacing of tendons shall not exceed 6 ft.

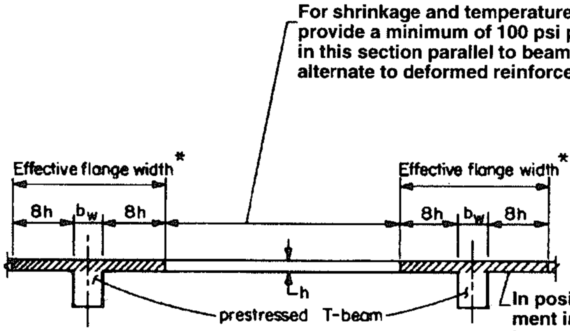
7.12.3.3 — When spacing of tendons exceeds 54 in., additional bonded shrinkage and temperature reinforcement conforming to 7.12.2 shall be provided between the tendons at slab edges extending from the slab edge for a distance equal to the tendon spacing.

COMMENTARY

R7.12.3 — Prestressed reinforcement requirements have been selected to provide an effective force on the slab approximately equal to the yield strength force for nonprestressed shrinkage and temperature reinforcement. This amount of prestressing, 100 psi on the gross concrete area, has been used successfully on a large number of projects. When the spacing of prestressing tendons used for shrinkage and temperature reinforcement exceeds 54 in., additional bonded reinforcement is required at slab edges where the prestressing forces are applied in order to adequately reinforce the area between the slab edge and the point where compressive stresses behind individual anchorages have "spread" sufficiently such that the slab is uniformly in compression. Application of the provisions of 7.12.3 to monolithic cast-in-place post-tensioned beam and slab construction is illustrated in Fig. R7.12.3.

Tendons used for shrinkage and temperature reinforcement should be positioned vertically in the slab as close as practicable to the center of the slab. In cases where the shrinkage and temperature tendons are used for supporting the principal tendons, variations from the slab centroid are permissible; however, the resultant of the shrinkage and temperature tendons should not fall outside the kern area of the slab.

The designer should evaluate the effects of slab shortening to assure proper action. In most cases, the low level of prestressing recommended should not cause difficulties in a properly detailed structure. Special attention may be required where thermal effects become significant.



For shrinkage and temperature stresses, provide a minimum of 100 psi prestressing in this section parallel to beam webs as an alternate to deformed reinforcement.

In positive moment areas, reinforcement in accordance with 7.12.2 should be provided unless an average compressive stress of 100 psi is maintained under prestress plus service dead load.

* Width of slab effective as a T-beam other than b_w + 16h (see 8.10) may be applicable for prestressed concrete T-beam construction.

Fig. R7.12.3—Prestressing used for shrinkage and temperature

CODE

7.13 — Requirements for structural integrity

7.13.1 — In the detailing of reinforcement and connections, members of a structure shall be effectively tied together to improve integrity of the overall structure.

7.13.2 — For cast-in-place construction, the following shall constitute minimum requirements:

7.13.2.1 — In joist construction, at least one bottom bar shall be continuous or shall be spliced over the support with a Class A tension splice and at noncontinuous supports be terminated with a standard hook.

7.13.2.2 — Beams at the perimeter of the structure shall have at least one-sixth of the tension reinforcement required for negative moment at the support and one-quarter of the positive moment reinforcement required at midspan made continuous around the perimeter and tied with closed stirrups, or stirrups anchored around the negative moment reinforcement with a hook having a bend of at least 135 deg. Stirrups need not be extended through any joints. When splices are needed, the required continuity shall be provided with top reinforcement spliced at midspan and bottom reinforcement spliced at or near the support with Class A tension splices.

7.13.2.3 — In other than perimeter beams, when closed stirrups are not provided, at least one-quarter of the positive moment reinforcement required at midspan shall be continuous or shall be spliced over the support with a Class A tension splice and at noncontinuous supports be terminated with a standard hook.

7.13.2.4 — For two-way slab construction, see 13.3.8.5.

7.13.3 — For precast concrete construction, tension ties shall be provided in the transverse, longitudinal, and vertical directions and around the perimeter of the structure to effectively tie elements together. The provisions of 16.5 shall apply.

7.13.4 — For lift-slab construction, see 13.3.8.6 and 18.12.6.

COMMENTARY

R7.13 — Requirements for structural integrity

Experience has shown that the overall integrity of a structure can be substantially enhanced by minor changes in detailing of reinforcement. It is the intent of this section of the code to improve the redundancy and ductility in structures so that in the event of damage to a major supporting element or an abnormal loading event, the resulting damage may be confined to a relatively small area and the structure will have a better chance to maintain overall stability.

R7.13.2 — With damage to a support, top reinforcement which is continuous over the support, but not confined by stirrups, will tend to tear out of the concrete and will not provide the catenary action needed to bridge the damaged support. By making a portion of the bottom reinforcement continuous, catenary action can be provided.

Requiring continuous top and bottom reinforcement in perimeter or spandrel beams provides a continuous tie around the structure. It is not the intent to require a tensile tie of continuous reinforcement of constant size around the entire perimeter of a structure, but simply to require that one half of the top flexural reinforcement required to extend past the point of inflection by 12.12.3 be further extended to lap splice at midspan. Similarly, the bottom reinforcement required to extend into the support by 12.11.1 must be made continuous or spliced with bottom reinforcement from the adjacent span. If the depth of a continuous beam changes at a support, the bottom reinforcement in the deeper member should be terminated with a standard hook and bottom reinforcement in the shallower member should be extended into and fully developed in the deeper member.

R7.13.3 — The code requires tension ties for precast concrete buildings of all heights. Details should provide connections to resist applied loads. Connection details that rely solely on friction caused by gravity forces are not permitted.

Connection details should be arranged so as to minimize the potential for cracking due to restrained creep, shrinkage and temperature movements. For information on connections and detailing requirements, see Reference 7.13.

Reference 7.14 recommends minimum tie requirements for precast concrete bearing wall buildings.

PART 4 — GENERAL REQUIREMENTS

CHAPTER 8 — ANALYSIS AND DESIGN — GENERAL CONSIDERATIONS

CODE

8.0 — Notation

A_s = area of nonprestressed tension reinforcement, in.^{2}
A_s' = area of compression reinforcement, in.^{2}
b = width of compression face of member, in.
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
E_c = modulus of elasticity of concrete, psi. See 8.5.1
E_s = modulus of elasticity of reinforcement, psi. See 8.5.2 and 8.5.3
f_c' = specified compressive strength of concrete, psi
f_y = specified yield strength of nonprestressed reinforcement, psi
ℓ_n = clear span for positive moment or shear and average of adjacent clear spans for negative moment
V_c = nominal shear strength provided by concrete
w_c = unit weight of concrete, lb/ft^3
w_u = factored load per unit length of beam or per unit area of slab
β_1 = factor defined in 10.2.7.3
ε_t = net tensile strain in extreme tension steel at nominal strength
ρ = ratio of nonprestressed tension reinforcement
= A_s/bd
ρ' = ratio of nonprestressed compression reinforcement
= A_s'/bd
ρ_b = reinforcement ratio producing balanced strain conditions. See 10.3.2
ϕ = strength reduction factor. See 9.3

8.1 — Design methods

8.1.1 — In design of structural concrete, members shall be proportioned for adequate strength in accordance with provisions of this code, using load factors and strength reduction factors ϕ specified in Chapter 9.

8.1.2 — Design of nonprestressed reinforced concrete members using Appendix A, Alternate Design Method, shall be permitted.

COMMENTARY

R8.0 — Notation

The definition of net tensile strain in 2.1 excludes strains due to effective prestress, creep, shrinkage, and temperature.

R8.1 — Design methods

R8.1.1 — The strength design method requires service loads or related internal moments and forces to be increased by specified load factors (required strength) and computed nominal strengths to be reduced by specified strength reduction factors ϕ (design strength).

R8.1.2 — The alternate method of design, outlined in Appendix A, is similar to the working stress design method of the 1963 ACI Building Code. The general serviceability requirements of the code, such as the requirements for

CODE

COMMENTARY

deflection and crack control must be met whether the strength design method of the code or the alternate design method of Appendix A is used.

Although prestressed members may not be designed under the provisions of the alternate design method, Chapter 18 requires linear stress-strain assumptions for computing service load stresses and prestress transfer stresses for investigation of behavior at service conditions, while using the strength design method for computing flexural strength (see 18.7).

An appendix may be judged not to be an official part of a legal document unless specifically adopted. Therefore specific reference is made to Appendix A in the main body of the code, to make it a legal part of the code.

8.1.3 — Design of reinforced concrete using the provisions of Appendix B, Unified Design Provisions for Reinforced and Prestressed Concrete Flexural and Compression Members, shall be permitted.

R8.1.3 — Designs made in accordance with Appendix B are equally acceptable, provided the provisions of Appendix B are used in their entirety.

An appendix may be judged not to be an official part of a legal document unless specifically adopted. Therefore, specific reference is made to Appendix B in the main body of the code, to make it a legal part of the code.

8.2 — Loading*

8.2.1 — Design provisions of this code are based on the assumption that structures shall be designed to resist all applicable loads.

8.2.2 — Service loads shall be in accordance with the general building code of which this code forms a part, with such live load reductions as are permitted in the general building code.

R8.2 — Loading

The provisions in the code are for live, wind, and earthquake loads such as those recommended in **"Minimum Design Loads for Buildings and Other Structures,"** (ASCE 7), of the American Society of Civil Engineers (ASCE)(formerly ANSI A58.1). If the service loads specified by the general building code (of which ACI 318 forms a part) differ from those of ASCE 7, the general building code governs. However, if the nature of the loads contained in a general building code differ considerably from ASCE 7 loads, some provisions of this code may need modification to reflect the difference.

Roofs should be designed with sufficient slope or camber to ensure adequate drainage accounting for any long-term deflection of the roof due to the dead loads, or the loads should be increased to account for all likely accumulations of water. If deflection of roof members may result in ponding of water accompanied by increased deflection and additional ponding, the design must ensure that this process is self-limiting.

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* Provisions in this code are suitable for live, wind, and earthquake loads, such as those recommended in "Minimum Design Loads for Buildings and Other Structures," ASCE 7, of the American Society of Civil Engineers (formerly ANSI A58.1).

CODE

8.2.3 — In design for wind and earthquake loads, integral structural parts shall be designed to resist the total lateral loads.*

8.2.4 — Consideration shall be given to effects of forces due to prestressing, crane loads, vibration, impact, shrinkage, temperature changes, creep, expansion of shrinkage-compensating concrete, and unequal settlement of supports.

8.3 — Methods of analysis

8.3.1 — All members of frames or continuous construction shall be designed for the maximum effects of factored loads as determined by the theory of elastic analysis, except as modified according to 8.4. It shall be permitted to simplify design by using the assumptions specified in 8.6 through 8.9.

8.3.2 — Except for prestressed concrete, approximate methods of frame analysis shall be permitted for buildings of usual types of construction, spans, and story heights.

8.3.3 — As an alternate to frame analysis, the following approximate moments and shears shall be permitted for design of continuous beams and one-way slabs (slabs reinforced to resist flexural stresses in only one direction), provided:

(a) There are two or more spans,

(b) Spans are approximately equal, with the larger of two adjacent spans not greater than the shorter by more than 20 percent,

(c) Loads are uniformly distributed,

(d) Unit live load does not exceed three times unit dead load, and

(e) Members are prismatic.

Positive moment

End spans
Discontinuous end
unrestrained.................................... $w_u \ell_n^2/11$

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*Special provisions for seismic design are given in Chapter 21.

COMMENTARY

R8.2.3 — Any reinforced concrete wall that is monolithic with other structural elements is considered to be an "integral part." Partition walls may or may not be integral structural parts. If partition walls may be removed, the primary lateral load resisting system must provide all of the required resistance without contribution of the removable partition. However, the effects of all partition walls attached to the structure must be considered in the analysis of the structure because they may lead to increased design forces in some or all elements.

R8.2.4 — Information is accumulating on the magnitudes of these various effects, especially the effects of column creep and shrinkage in tall structures,[8.1] and on procedures for including the forces resulting from these effects in design.

R8.3 — Methods of analysis

R8.3.1 — Factored loads are service loads multiplied by appropriate load factors. If the alternate design method of Appendix A is used, the loads used in design are service loads (load factors of unity). For both the strength design method and the alternate design method, elastic analysis is used to obtain moments, shears, reactions, etc.

R8.3.3 — The approximate moments and shears give reasonably conservative values for the stated conditions if the flexural members are part of a frame or continuous construction. Because the load patterns that produce critical values for moments in columns of frames differ from those for maximum negative moments in beams, column moments must be evaluated separately.

Discontinuous end integral with support	$w_u \ell_n^2/14$
Interior spans	$w_u \ell_n^2/16$
Negative moments at exterior face of first interior support	
Two spans	$w_u \ell_n^2/9$
More than two spans	$w_u \ell_n^2/10$
Negative moment at other faces of interior supports	$w_u \ell_n^2/11$
Negative moment at face of all supports for	
Slabs with spans not exceeding 10 ft; and beams where ratio of sum of column stiffnesses to beam stiffness exceeds eight at each end of the span	$w_u \ell_n^2/12$
Negative moment at interior face of exterior support for members built integrally with supports	
Where support is spandrel beam	$w_u \ell_n^2/24$
Where support is a column	$w_u \ell_n^2/16$
Shear in end members at face of first interior support	$1.15\ w_u \ell_n/2$
Shear at face of all other supports	$w_u \ell_n/2$

8.4 — Redistribution of negative moments in continuous nonprestressed flexural members*

8.4.1 — Except where approximate values for moments are used, it shall be permitted to increase or decrease negative moments calculated by elastic theory at supports of continuous flexural members for any assumed loading arrangement by not more than

$$20\left(1 - \frac{\rho - \rho'}{\rho_b}\right) \text{ percent}$$

8.4.2 — The modified negative moments shall be used for calculating moments at sections within the spans.

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* For criteria on moment redistribution for prestressed concrete members, see 18.10.4.

R8.4 — Redistribution of negative moments in continuous nonprestressed flexural members

Moment redistribution is dependent on adequate ductility in plastic hinge regions. These plastic hinge regions develop at points of maximum moment and cause a shift in the elastic moment diagram. The usual result is a reduction in the values of negative moments in the plastic hinge region and an increase in the values of positive moments from those computed by elastic analysis. Since negative moments are determined for one loading arrangement and positive moments for another, each section has a reserve capacity that is not fully utilized for any one loading condition. The plastic hinges permit the utilization of the full capacity of more cross sections of a flexural member at ultimate loads.

CODE

8.4.3 — Redistribution of negative moments shall be made only when the section at which moment is reduced is so designed that ρ or $\rho - \rho'$ is not greater than **0.50** ρ_b, where

$$\rho_b = \frac{0.85\beta_1 f_c'}{f_y}\left(\frac{87{,}000}{87{,}000 + f_y}\right) \qquad (8\text{-}1)$$

8.5 — Modulus of elasticity

8.5.1 — Modulus of elasticity E_c for concrete shall be permitted to be taken as $w_c^{1.5}\ 33\sqrt{f_c'}$ (in psi) for values of w_c between 90 and 155 lb/ft.^{3} For normal weight concrete, E_c shall be permitted to be taken as $57{,}000\sqrt{f_c'}$.

COMMENTARY

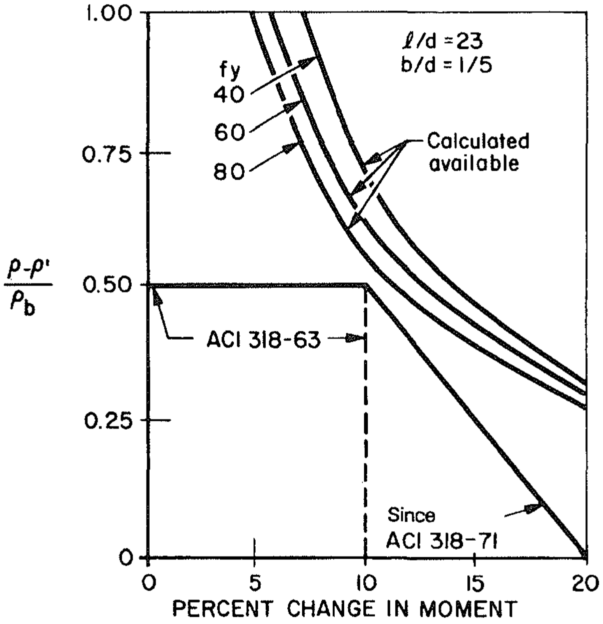


Fig R8.4—Permissible moment redistribution for minimum rotation capacity

Using conservative values of ultimate concrete strains and lengths of plastic hinges derived from extensive tests, flexural members with small rotation capacity were analyzed for moment redistribution varying from 10 to 20 percent, depending on the reinforcement ratio. The results were found to be conservative (see Fig. R8.4). Studies by Cohn[8.2] and Mattock[8.3] support this conclusion and indicate that cracking and deflection of beams designed for moment redistribution are not significantly greater at service loads than for beams designed by the elastic theory distribution of moments. Also, these studies indicated that adequate rotation capacity for the moment redistribution allowed by the code is available if the members satisfy the code requirements. This code maintains the same limit on redistribution as the 1971 and 1977 code editions.

Moment redistribution does not apply to members designed by the alternate design method of Appendix A; nor may it be used for slab systems designed by the Direct Design Method (see 13.6.1.7).

R8.5 — Modulus of elasticity

R8.5.1 — Studies leading to the expression for modulus of elasticity of concrete in 8.5.1 are summarized in Reference 8.4 where E_c was defined as the slope of the line drawn from a stress of zero to a compressive stress of $\mathbf{0.45} f_c'$. The modulus for concrete is sensitive to the modulus of the aggregate and may differ from the specified value. Measured values range typically from 120 to 80 percent of the specified value. Methods for determining Young's modulus for concrete are described in Reference 8.5.

CODE

8.5.2 — Modulus of elasticity E_s for nonprestressed reinforcement shall be permitted to be taken as 29,000,000 psi.

8.5.3 — Modulus of elasticity E_s for prestressing tendons shall be determined by tests or supplied by the manufacturer.

8.6 — Stiffness

8.6.1 — Use of any set of reasonable assumptions shall be permitted for computing relative flexural and torsional stiffnesses of columns, walls, floors, and roof systems. The assumptions adopted shall be consistent throughout analysis.

8.6.2 — Effect of haunches shall be considered both in determining moments and in design of members.

8.7 — Span length

8.7.1 — Span length of members not built integrally with supports shall be considered the clear span plus depth of member but need not exceed distance between centers of supports.

8.7.2 — In analysis of frames or continuous construction for determination of moments, span length shall be taken as the distance center-to-center of supports.

8.7.3 — For beams built integrally with supports, design on the basis of moments at faces of support shall be permitted.

COMMENTARY

R8.6 — Stiffness

R8.6.1 — Ideally, the member stiffnesses EI and GJ should reflect the degree of cracking and inelastic action which has occurred along each member before yielding. However, the complexities involved in selecting different stiffnesses for all members of a frame would make frame analyses inefficient in design offices. Simpler assumptions are required to define flexural and torsional stiffnesses.

For braced frames, relative values of stiffness are important. Two usual assumptions are to use gross EI values for all members or, to use half the gross EI of the beam stem for beams and the gross EI for the columns.

For frames that are free to sway, a realistic estimate of EI is desirable and should be used if second-order analyses are carried out. Guidance for the choice of EI for this case is given in the commentary to 10.11.1.

Two conditions determine whether it is necessary to consider torsional stiffness in the analysis of a given structure: (1) the relative magnitude of the torsional and flexural stiffnesses, and (2) whether torsion is required for equilibrium of the structure (equilibrium torsion) or is due to members twisting to maintain deformation compatibility (compatibility torsion). In the case of compatibility torsion, the torsional stiffness may be neglected. For cases involving equilibrium torsion, torsional stiffness should be considered.

R8.6.2 — Stiffness and fixed-end moment coefficients for haunched members may be obtained from Reference 8.6.

R8.7 — Span length

Beam moments calculated at support centers may be reduced to the moments at support faces for design of beams. Reference 8.7 provides an acceptable method of reducing moments at support centers to those at support faces.

CODE

8.7.4 — It shall be permitted to analyze solid or ribbed slabs built integrally with supports, with clear spans not more than 10 ft, as continuous slabs on knife edge supports with spans equal to the clear spans of the slab and width of beams otherwise neglected.

8.8 — Columns

8.8.1 — Columns shall be designed to resist the axial forces from factored loads on all floors or roof and the maximum moment from factored loads on a single adjacent span of the floor or roof under consideration. Loading condition giving the maximum ratio of moment to axial load shall also be considered.

8.8.2 — In frames or continuous construction, consideration shall be given to the effect of unbalanced floor or roof loads on both exterior and interior columns and of eccentric loading due to other causes.

8.8.3 — In computing gravity load moments in columns, it shall be permitted to assume far ends of columns built integrally with the structure to be fixed.

8.8.4 — Resistance to moments at any floor or roof level shall be provided by distributing the moment between columns immediately above and below the given floor in proportion to the relative column stiffnesses and conditions of restraint.

8.9 — Arrangement of live load

8.9.1 — It shall be permitted to assume that:

(a) The live load is applied only to the floor or roof under consideration, and

(b) The far ends of columns built integrally with the structure are considered to be fixed.

8.9.2 — It shall be permitted to assume that the arrangement of live load is limited to combinations of:

(a) Factored dead load on all spans with full factored live load on two adjacent spans, and

(b) Factored dead load on all spans with full factored live load on alternate spans.

COMMENTARY

R8.8 — Columns

Section 8.8 has been developed with the intent of making certain that the most demanding combinations of axial load and moments be identified for design.

Section 8.8.4 has been included to make certain that moments in columns are recognized in design if the girders have been proportioned using 8.3.3. The "moment" in 8.8.4 refers to the difference between the moments in a given vertical plane, exerted at column centerline by members framing into that column.

R8.9 — Arrangement of live load

For determining column, wall, and beam moments and shears caused by gravity loads, the code permits the use of a model limited to the beams in the level considered and the columns above and below that level. Far ends of columns are to be considered as fixed for the purpose of analysis under gravity loads. This assumption does not apply to lateral load analysis. However in analysis for lateral loads, simplified methods (such as the portal method) may be used to obtain the moments, shears, and reactions for structures that are symmetrical and satisfy the assumptions used for such simplified methods. For unsymmetrical and high-rise structures, rigorous methods recognizing all structural displacements should be used.

The engineer is expected to establish the most demanding sets of design forces by investigating the effects of live load placed in various critical patterns.

Most approximate methods of analysis neglect effects of deflections on geometry and axial flexibility. Therefore, beam and column moments may have to be amplified for column slenderness in accordance with 10.11, 10.12, and 10.13.

CODE

8.10 — T-beam construction

8.10.1 — In T-beam construction, the flange and web shall be built integrally or otherwise effectively bonded together.

8.10.2 — Width of slab effective as a T-beam flange shall not exceed one-quarter of the span length of the beam, and the effective overhanging flange width on each side of the web shall not exceed:

(a) eight times the slab thickness, and

(b) one-half the clear distance to the next web.

8.10.3 — For beams with a slab on one side only, the effective overhanging flange width shall not exceed:

(a) one-twelfth the span length of the beam,

(b) six times the slab thickness, and

(c) one-half the clear distance to the next web.

8.10.4 — Isolated beams, in which the T-shape is used to provide a flange for additional compression area, shall have a flange thickness not less than one-half the width of web and an effective flange width not more than four times the width of web.

8.10.5 — Where primary flexural reinforcement in a slab that is considered as a T-beam flange (excluding joist construction) is parallel to the beam, reinforcement perpendicular to the beam shall be provided in the top of the slab in accordance with the following:

8.10.5.1 — Transverse reinforcement shall be designed to carry the factored load on the overhanging slab width assumed to act as a cantilever. For isolated beams, the full width of overhanging flange shall be considered. For other T-beams, only the effective overhanging slab width need be considered.

8.10.5.2 — Transverse reinforcement shall be spaced not farther apart than five times the slab thickness, nor 18 in.

8.11 — Joist construction

8.11.1 — Joist construction consists of a monolithic combination of regularly spaced ribs and a top slab arranged to span in one direction or two orthogonal directions.

8.11.2 — Ribs shall be not less than 4 in. in width, and shall have a depth of not more than $3^1/_2$ times the minimum width of rib.

COMMENTARY

R8.10 — T-beam construction

This section contains provisions identical to those of previous ACI Building Codes for limiting dimensions related to stiffness and flexural calculations. Special provisions related to T-beams and other flanged members are stated in 11.6.1 with regard to torsion.

R8.11 — Joist construction

The size and spacing limitations for concrete joist construction meeting the limitations of 8.11.1 through 8.11.3 are based on successful performance in the past.

CODE

8.11.3 — Clear spacing between ribs shall not exceed 30 in.

8.11.4 — Joist construction not meeting the limitations of 8.11.1 through 8.11.3 shall be designed as slabs and beams.

8.11.5 — When permanent burned clay or concrete tile fillers of material having a unit compressive strength at least equal to that of the specified strength of concrete in the joists are used:

8.11.5.1 — For shear and negative moment strength computations, it shall be permitted to include the vertical shells of fillers in contact with the ribs. Other portions of fillers shall not be included in strength computations.

8.11.5.2 — Slab thickness over permanent fillers shall be not less than one-twelfth the clear distance between ribs, nor less than $1^{1}/_{2}$ in.

8.11.5.3 — In one-way joists, reinforcement normal to the ribs shall be provided in the slab as required by 7.12.

8.11.6 — When removable forms or fillers not complying with 8.11.5 are used:

8.11.6.1 — Slab thickness shall be not less than one-twelfth the clear distance between ribs, nor less than 2 in.

8.11.6.2 — Reinforcement normal to the ribs shall be provided in the slab as required for flexure, considering load concentrations, if any, but not less than required by 7.12.

8.11.7 — Where conduits or pipes as permitted by 6.3 are embedded within the slab, slab thickness shall be at least 1 in. greater than the total overall depth of the conduits or pipes at any point. Conduits or pipes shall not impair significantly the strength of the construction.

8.11.8 — For joist construction, contribution of concrete to shear strength V_c shall be permitted to be 10 percent more than that specified in Chapter 11. It shall be permitted to increase shear strength using shear reinforcement or by widening the ends of ribs.

8.12 — Separate floor finish

8.12.1 — A floor finish shall not be included as part of a structural member unless placed monolithically with

COMMENTARY

R8.11.3 — A limit on the maximum spacing of ribs is required because of the special provisions permitting higher shear strengths and less concrete protection for the reinforcement for these relatively small, repetitive members.

R8.11.8 — The increase in shear strength permitted by 8.11.8 is justified on the basis of: (1) satisfactory performance of joist construction with higher shear strengths, designed under previous ACI Building Codes, which allowed comparable shear stresses, and (2) redistribution of local overloads to adjacent joists.

R8.12 — Separate floor finish

The code does not specify an additional thickness for wearing surfaces subjected to unusual conditions of wear. The

CODE

the floor slab or designed in accordance with requirements of Chapter 17.

8.12.2 — It shall be permitted to consider all concrete floor finishes as part of required cover or total thickness for nonstructural considerations.

COMMENTARY

need for added thickness for unusual wear is left to the discretion of the designer.

As in previous editions of the code, a floor finish may be considered for strength purposes only if it is cast monolithically with the slab. Permission is given to include a separate finish in the structural thickness if composite action is provided for in accordance with Chapter 17.

All floor finishes may be considered for nonstructural purposes such as cover for reinforcement, fire protection, etc. Provisions should be made, however, to ensure that the finish will not spall off, thus causing decreased cover. Furthermore, development of reinforcement considerations require minimum monolithic concrete cover according to 7.7.

CHAPTER 9 — STRENGTH AND SERVICEABILITY REQUIREMENTS

CODE

9.0 — Notation

A_g = gross area of section, in.^{2}
A_s' = area of compression reinforcement, in.^{2}
b = width of compression face of member, in.
c = distance from extreme compression fiber to neutral axis, in.
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
d' = distance from extreme compression fiber to centroid of compression reinforcement, in.
d_s = distance from extreme tension fiber to centroid of tension reinforcement, in.
d_t = distance from extreme compression fiber to extreme tension steel, in.
D = dead loads, or related internal moments and forces
E = load effects of earthquake, or related internal moments and forces
E_c = modulus of elasticity of concrete, psi. See 8.5.1
f_c' = specified compressive strength of concrete, psi
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi
f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi
f_r = modulus of rupture of concrete, psi
f_y = specified yield strength of nonprestressed reinforcement, psi
F = loads due to weight and pressures of fluids with well-defined densities and controllable maximum heights, or related internal moments and forces
h = overall thickness of member, in.
H = loads due to weight and pressure of soil, water in soil, or other materials, or related internal moments and forces
I_{cr} = moment of inertia of cracked section transformed to concrete
I_e = effective moment of inertia for computation of deflection
I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement
ℓ = span length of beam or one-way slab, as defined in 8.7; clear projection of cantilever, in.
ℓ_n = length of clear span in long direction of two-way construction, measured face-to-face of

COMMENTARY

R9. 0 — Notation

CODE

supports in slabs without beams and face-to-face of beams or other supports in other cases

L = live loads, or related internal moments and forces

M_a = maximum moment in member at stage deflection is computed

M_{cr} = cracking moment. See 9.5.2.3

P_b = nominal axial load strength at balanced strain conditions. See 10.3.2

P_n = nominal axial load strength at given eccentricity

T = cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage-compensating concrete

U = required strength to resist factored loads or related internal moments and forces

W = wind load, or related internal moments and forces

w_c = weight of concrete, lb/ft^3

y_t = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension

α = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of beam. See Chapter 13

α_m = average value of α for all beams on edges of a panel

β = ratio of clear spans in long to short direction of two-way slabs

ε_t = net tensile strain in extreme tension steel at nominal strength

λ = multiplier for additional long-term deflection as defined in 9.5.2.5

ξ = time-dependent factor for sustained load. See 9.5.2.5

ρ = ratio of nonprestressed tension reinforcement, A_s/bd

ρ' = reinforcement ratio for nonprestressed compression reinforcement, A_s'/bd

ρ_b = reinforcement ratio producing balanced strain conditions. See B.10.3.2

ϕ = strength reduction factor. See 9.3

9.1 — General

9.1.1 — Structures and structural members shall be designed to have design strengths at all sections at least equal to the required strengths calculated for the factored loads and forces in such combinations as are stipulated in this code.

9.1.2 — Members also shall meet all other requirements of this code to ensure adequate performance at service load levels.

COMMENTARY

The definition of net tensile strain in 2.1 excludes strains due to effective prestress, creep, shrinkage, and temperature.

R9.1 — General

R9.1.1 — Chapter 9 defines the basic strength and serviceability conditions for proportioning reinforced concrete members.

The basic requirement for strength design may be expressed as follows:

$$\text{Design Strength} \geq \text{Required Strength}$$

CODE

COMMENTARY

$$\phi \text{ (Nominal Strength)} \geq U$$

In the strength design procedure, the margin of safety is provided by multiplying the service load by a load factor and the nominal strength by a strength reduction factor as described below.

1. The "required strength" U is computed by multiplying the service loads by load factors. Thus, for example, the factored moment M_u or "required moment strength" for dead and live load is computed as:

$$U = 1.4D + 1.7L$$

or

$$M_u = 1.4M_d + 1.7M_\ell$$

where M_d and M_ℓ are the moments caused by service dead and live loads. The required strength is discussed in detail in the commentary for 9.2. The definition and notations for required strength are discussed in the commentary for Chapter 2.

2. The "design strength" of a structural element is computed by multiplying the "nominal strength" by a strength reduction factor ϕ which is less than one. The strength reduction factor accounts for uncertainties in design computations and the relative importance of various types of members. This factor also reflects the effect of variations in material strengths, workmanship, and dimensions that may combine to result in understrength. The "nominal strength" is computed by the code procedures assuming the member will have the exact dimensions and material properties used in the computations.[9.1] For example, the design strength in flexure of a cross section (without compression reinforcement) may be expressed as:

$$\phi M_n = \phi\left[A_s f_y\left(d - \frac{a}{2}\right)\right]$$

The design strength and the strength reduction factor ϕ are discussed in detail in the commentary for 9.3.

Combining these two safety provisions, the basic requirement for the design of a beam cross section can be stated as:

$$\text{Design Strength} \geq \text{Required Strength}$$

$$\phi M_n \geq M_u$$

$$\phi\left[A_s f_y\left(d - \frac{a}{2}\right)\right] \geq 1.4M_d + 1.7\,M_\ell$$

All notations with the subscript **u** such as M_u, P_u, and V_u, refer only to the required strength values. The design strength values are noted by ϕ times nominal strength, such as ϕM_n, ϕP_n, and ϕV_n.

CODE

9.2 — Required strength

9.2.1 — Required strength ***U*** to resist dead load ***D*** and live load ***L*** shall be at least equal to

$$U = 1.4D + 1.7L \qquad (9\text{-}1)$$

9.2.2 — If resistance to structural effects of a specified wind load ***W*** are included in design, the following combinations of ***D***, ***L***, and ***W*** shall be investigated to determine the greatest required strength ***U***

$$U = 0.75\,(1.4D + 1.7L + 1.7W) \qquad (9\text{-}2)$$

where load combinations shall include both full value and zero value of ***L*** to determine the more severe condition, and

$$U = 0.9D + 1.3W \qquad (9\text{-}3)$$

but for any combination of ***D***, ***L***, and ***W***, required strength ***U*** shall not be less than Eq. (9-1).

9.2.3 — If resistance to specified earthquake loads or forces ***E*** are included in design, load combinations of 9.2.2 shall apply, except that **1.1*E*** shall be substituted for ***W***.

9.2.4 — If resistance to earth pressure ***H*** is included in design, required strength ***U*** shall be at least equal to

$$U = 1.4D + 1.7L + 1.7H \qquad (9\text{-}4)$$

except that where ***D*** or ***L*** reduce the effect of ***H***, **0.9*D***

COMMENTARY

R9.2 — Required strength

The required strength ***U*** is expressed in terms of factored loads, or related internal moments and forces. Factored loads are the loads specified in the general building code multiplied by appropriate load factors.

The factor assigned to each load is influenced by the degree of accuracy to which the load effect usually can be calculated and the variation that might be expected in the load during the lifetime of the structure. Dead loads, because they are more accurately determined and less variable, are assigned a lower load factor than live loads. Load factors also account for variability in the structural analysis used to compute moments and shears.

The code gives load factors for specific combinations of loads. In assigning factors to combinations of loading, some consideration is given to the probability of simultaneous occurrence. While most of the usual combinations of loadings are included, the designer should not assume that all cases are covered.

Due regard is to be given to sign in determining ***U*** for combinations of loadings, as one type of loading may produce effects of opposite sense to that produced by another type. The load combinations with **0.9*D*** are specifically included for the case where a higher dead load reduces the effects of other loads.

Consideration must be given to various combinations of loading to determine the most critical design condition. This is particularly true when strength is dependent on more than one load effect, such as strength for combined flexure and axial load or shear strength in members with axial load.

If special circumstances require greater reliance on the strength of particular members than encountered in usual practice, some reduction in the stipulated strength reduction factors ϕ or increase in the stipulated load factors ***U*** may be appropriate for such members.

R9.2.3 — If earthquake effects must be considered in design, Eq. (9-2) and (9-3) become:

$$U = 1.05D + 1.28L + 1.40E$$

and

$$U = 0.90D + 1.43E$$

R9.2.4 — If effects ***H*** caused by earth pressure, groundwater pressure, or pressure caused by granular materials are included in design, the required strength equations become:

$$U = 1.4D + 1.7L + 1.7H$$

and where ***D*** or ***L*** reduce the effect of ***H***

CODE

shall be substituted for **1.4*D*** and zero value of ***L*** shall be used to determine the greatest required strength ***U***. For any combination of ***D***, ***L***, and ***H***, required strength **U** shall not be less than Eq. (9-1).

9.2.5 — If resistance to loadings due to weight and pressure of fluids with well-defined densities and controllable maximum heights ***F*** is included in design, such loading shall have a load factor of 1.4, and be added to all loading combinations that include live load.

9.2.6 — If resistance to impact effects is taken into account in design, such effects shall be included with live load ***L***.

9.2.7 — Where structural effects ***T*** of differential settlement, creep, shrinkage, expansion of shrinkage-compensating concrete, or temperature change are significant in design, required strength ***U*** shall be at least equal to

$$U = 0.75(1.4D + 1.4T + 1.7L) \qquad (9\text{-}5)$$

but required strength ***U*** shall not be less than

$$U = 1.4(D + T) \qquad (9\text{-}6)$$

Estimations of differential settlement, creep, shrinkage, expansion of shrinkage-compensating concrete, or temperature change shall be based on a realistic assessment of such effects occurring in service.

9.3 — Design strength

9.3.1 — Design strength provided by a member, its connections to other members, and its cross sections, in terms of flexure, axial load, shear, and torsion, shall

COMMENTARY

$$U = 0.9D + 1.7H$$

but for any combination of ***D***, ***L***, or ***H***

$$U = 1.4D + 1.7L$$

R9.2.5 — This section addresses the need to consider loading due to weight of liquid or liquid pressure. It specifies a load factor for such loadings with well-defined densities and controllable maximum heights equivalent to that used for dead load. Such reduced factors would not be appropriate where there is considerable uncertainty of pressures, as with groundwater pressures or uncertainty as to the possible maximum liquid depth as in ponding of water. See discussion on ponding in R8.2.

For well-defined fluid pressures, the required strength equations become:

$$U = 1.4D + 1.7L + 1.4F$$

and where ***D*** or ***L*** reduce the effect of ***F***

$$U = 0.9D + 1.4F$$

but for any combination of ***D***, ***L***, or ***F***

$$U = 1.4D + 1.7L$$

R9.2.6 — If the live load is applied rapidly, as may be the case for parking structures, loading docks, warehouse floors, elevator shafts, etc., impact effects should be considered. In all equations substitute (***L*** + impact) for ***L*** when impact must be considered.

R9.2.7 — The designer should consider the effects of differential settlement, creep, shrinkage, temperature, and shrinkage-compensating concrete. The term "realistic assessment" is used to indicate that the most probable values rather than the upper bound values of the variables should be used.

Eq. (9-6) is to prevent a design for load

$$U = 0.75\,(1.4D + 1.4T + 1.7L)$$

to approach

$$U = 1.05\,(D + T)$$

when live load is negligible.

R9.3 — Design strength

R9.3.1 — The term "design strength" of a member, refers to the nominal strength calculated in accordance with the requirements stipulated in this code multiplied by a strength

CODE

be taken as the nominal strength calculated in accordance with requirements and assumptions of this code, multiplied by the strength reduction factors ϕ in 9.3.2 and 9.3.4.

9.3.1.1 — If the structural framing includes primary members of other materials proportioned to satisfy the load factor combinations in Section 2.4 of ASCE 7, it shall be permitted to proportion the concrete members using the set of strength reduction factors ϕ listed in Appendix C and the load factor combinations in ASCE 7.

9.3.2 — Strength reduction factor ϕ shall be as follows:

9.3.2.1 — Flexure, without axial load 0.90

9.3.2.2 — Axial load, and axial load with flexure. (For axial load with flexure, both axial load and moment nominal strength shall be multiplied by appropriate single value of ϕ)

(a) Axial tension, and axial tension with flexure 0.90

(b) Axial compression, and axial compression with flexure:
Members with spiral reinforcement conforming to 10.9.3 .... 0.75
Other reinforced members 0.70

except that for low values of axial compression ϕ shall be permitted to be increased in accordance with the following:

For members in which f_y does not exceed 60,000 psi, with symmetric reinforcement, and with $(h-d'-d_s)/h$ not less than 0.70, ϕ shall be permitted to be increased linearly to 0.90 as ϕP_n decreases from $0.10 f_c' A_g$ to zero.

For other reinforced members, ϕ shall be permitted to be increased linearly to 0.90 as ϕP_n decreases from $0.10 f_c' A_g$ or ϕP_b, whichever is smaller, to zero.

9.3.2.3 — Shear and torsion 0.85

9.3.2.4 — Bearing on concrete
(See also 18.13) 0.70

COMMENTARY

reduction factor ϕ, which is always less than one.

The purposes of the strength reduction factor ϕ are (1) to allow for the probability of understrength members due to variations in material strengths and dimensions, (2) to allow for inaccuracies in the design equations, (3) to reflect the degree of ductility and required reliability of the member under the load effects being considered, and (4) to reflect the importance of the member in the structure.[9.2,9.3] For example, a lower ϕ is used for columns than for beams because columns generally have less ductility, are more sensitive to variations in concrete strength, and generally support larger loaded areas than beams. Furthermore, spiral columns are assigned a higher ϕ than tied columns since they have greater ductility or toughness.

R9.3.1.1 — Appendix C has been included in order to facilitate computations for buildings with substantial portions of their structural framing provided by elements other than concrete. If the strength reduction factors in Appendix C are used for the concrete elements, the required strengths are to be determined using the load factor combinations in Section 2.4 of ASCE 7.

R9.3.2.2 — For members subjected to axial load with flexure, design strengths are determined by multiplying both P_n and M_n by the appropriate single value of ϕ. For members subjected to flexure and relatively small axial compression loads, failure is initiated by yielding of the tension reinforcement and takes place in an increasingly more ductile manner as the ratio of axial load to moment decreases. At the same time the variability of the strength also decreases. For small axial loads the value of ϕ may be increased from that for compression members to 0.90 permitted for flexure as the design axial load strength ϕP_n decreases from a specified value to zero.

For members meeting the limitations specified for $(h-d'-d_s)/h$ and f_y, the transition starts at a design axial load strength, ϕP_n of $0.10 f_c' A_g$. For other conditions, P_b must be calculated to determine the upper value of design axial load strength ϕP_n (the smaller of $0.10 f_c' A_g$ and ϕP_b) below which an increase in ϕ can be made.

The ϕ-factor for bearing on concrete in this section does not apply to post-tensioning anchorage bearing plates (see commentary on 18.13).

CODE

9.3.3 — Development lengths specified in Chapter 12 do not require a ϕ-factor.

9.3.4 — In regions of high seismic risk, strength reduction factors ϕ shall be given as above except for the following:

9.3.4.1 — Except for determining the strength of joints, the shear strength reduction factor shall be 0.6 for any structural member if its nominal shear strength is less than the shear corresponding to the development of the nominal flexural strength of the member. The nominal flexural strength shall be determined considering the most critical factored axial loads and including earthquake effects. Shear strength reduction factor for joints shall be 0.85.

9.3.5 — Strength reduction factor ϕ for flexure, compression, shear, and bearing of structural plain concrete in Chapter 22 shall be 0.65.

9.4 — Design strength for reinforcement

Designs shall not be based on a yield strength of reinforcement f_y in excess of 80,000 psi, except for prestressing tendons.

9.5 — Control of deflections

9.5.1 — Reinforced concrete members subjected to flexure shall be designed to have adequate stiffness to limit deflections or any deformations that affect strength or serviceability of a structure adversely.

COMMENTARY

R9.3.4 — Strength reduction factors in 9.3.4 are intended to compensate for uncertainties in estimation of strength of structural members in buildings. They are based primarily on experience with constant or steadily increasing applied load. For construction in regions of high seismic risk, some of the strength reduction factors have been modified in 9.3.4 to account for the effects on strength of displacements into the nonlinear range of response.

Section 9.3.4.1 refers to brittle members such as low-rise walls or portions of walls between openings of which proportions are such that it becomes impractical to reinforce them to raise their nominal shear strength above the shear corresponding to nominal flexural strength for the pertinent loading conditions. This requirement does not apply to calculations for evaluating the shear strength of connections.

R9.3.5 — The strength reduction factor ϕ for structural plain concrete design is made the same for all strength conditions. Since both flexural tension strength and shear strength for plain concrete depend on the tensile strength characteristics of the concrete, with no reserve strength or ductility possible due to the absence of reinforcement, equal strength reduction factors for both bending and shear are considered appropriate.

R9.4 — Design strength for reinforcement

Reinforcing bars with a yield strength of 75,000 psi in sizes No. 11, 14, and 18 and yield measured at a strain of 0.0035 and so meeting the requirements of this code were first included in ASTM A 615-87.

In addition to the upper limit of 80,000 psi for yield strength of nonprestressed reinforcement, there are limitations on yield strength in other sections of the code:

Sections 11.5.2, 11.6.3.4, and 11.7.6: the maximum f_y that may be used in design for shear and torsion reinforcement is 60,000 psi, except that f_y up to 80,000 psi may be used for shear reinforcement meeting the requirements of ASTM A 497.

Sections 19.3.2 and 21.2.5: the maximum specified f_y is 60,000 psi in shells, folded plates, and structures governed by the special seismic provisions of Chapter 21.

The deflection provisions of 9.5 and the limitations on distribution of flexural reinforcement of 10.6 become increasingly critical as f_y increases.

R9.5 — Control of deflections[9.4]

R9.5.1 — The provisions of 9.5 are concerned only with deflections or deformations which may occur at service load levels. Where long-term deflections are computed, only the dead load and that portion of the live load which is sustained need be considered.

CODE

9.5.2 — One-way construction (nonprestressed)

9.5.2.1 — Minimum thickness stipulated in Table 9.5(a) shall apply for one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections, unless computation of deflection indicates a lesser thickness can be used without adverse effects.

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED*

		Minimum thickness, h		
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

* Span length ℓ is in inches.

Values given shall be used directly for members with normal weight concrete ($w_c = 145$ lb/ft^3) and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:

a) For structural lightweight concrete having unit weight in the range 90-120 lb/ft^3, the values shall be multiplied by $(1.65 - 0.005 w_c)$ but not less than 1.09, where w_c is the unit weight in lb/ft^3.

b) For f_y other than 60,000 psi, the values shall be multiplied by $(0.4 + f_y/100{,}000)$.

9.5.2.2 — Where deflections are to be computed, deflections that occur immediately on application of load shall be computed by usual methods or formulas for elastic deflections, considering effects of cracking and reinforcement on member stiffness.

COMMENTARY

Two methods are given for controlling deflections. For nonprestressed beams and one-way slabs, and for composite members, provision of a minimum overall thickness as required by Table 9.5(a) will satisfy the requirements of the code for members not supporting or attached to partitions or other construction likely to be damaged by large deflections. For nonprestressed two-way construction, minimum thickness as required by 9.5.3.1, 9.5.3.2, and 9.5.3.3 will satisfy the requirements of the code.

For nonprestressed members which do not meet these minimum thickness requirements or which support or are attached to partitions or other construction likely to be damaged by large deflections, and for all prestressed concrete flexural members, deflections must be calculated by the procedures described or referred to in the appropriate sections of the code, and are limited to the values in Table 9.5(b).

R9.5.2 — One-way construction (nonprestressed)

R9.5.2.1 — The minimum thicknesses of Table 9.5(a) apply for nonprestressed beams and one-way slabs (see 9.5.2), and for composite members (see 9.5.5).

It should be emphasized that these minimum thicknesses apply only to members not supporting or attached to partitions and other construction likely to be damaged by deflection.

Values of minimum thickness must be modified if other than normal weight concrete and Grade 60 reinforcement are used. The notes beneath the table are essential to its use for reinforced concrete members constructed with structural lightweight concrete and/or with reinforcement having a yield strength other than 60,000 psi. If both of these conditions exist, the corrections in footnotes (a) and (b) shall both be applied.

The modification for lightweight concrete in footnote (a) is based on studies of the results and discussions in Reference 9.5. No correction is specified for concretes weighing between 120 and 145 lb/ft^3 because the correction term would be close to unity in this range.

The modification for yield strength in footnote (b) is approximate but should yield conservative results for the type of members considered in the table, for typical reinforcement ratios, and for values of f_y between 40,000 and 80,000 psi.

R9.5.2.2 — For calculation of immediate deflections of uncracked prismatic members, the usual methods or formulas for elastic deflections may be used with a constant value of $E_c I_g$ along the length of the member. However, if the member is cracked at one or more sections, or if its depth

CODE

9.5.2.3 — Unless stiffness values are obtained by a more comprehensive analysis, immediate deflection shall be computed with the modulus of elasticity $\boldsymbol{E_c}$ for concrete as specified in 8.5.1 (normal weight or lightweight concrete) and with the effective moment of inertia as follows, but not greater than $\boldsymbol{I_g}$.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \qquad (9\text{-}7)$$

where

$$M_{cr} = \frac{f_r I_g}{y_t} \qquad (9\text{-}8)$$

and for normal weight concrete,

$$f_r = 7.5\sqrt{f_c'} \qquad (9\text{-}9)$$

When lightweight aggregate concrete is used, one of the following modifications shall apply:

(a) When $\boldsymbol{f_{ct}}$ is specified and concrete is proportioned in accordance with 5.2, $\boldsymbol{f_r}$ shall be modified by substituting $\boldsymbol{f_{ct}/6.7}$ for $\sqrt{f_c'}$, but the value of $\boldsymbol{f_{ct}/6.7}$ shall not exceed $\sqrt{f_c'}$.

(b) When $\boldsymbol{f_{ct}}$ is not specified, $\boldsymbol{f_r}$ shall be multiplied by 0.75 for "all-lightweight" concrete, and 0.85 for "sand-lightweight" concrete. Linear interpolation shall be permitted if partial sand replacement is used.

9.5.2.4 — For continuous members, effective moment of inertia shall be permitted to be taken as the average of values obtained from Eq. (9-7) for the critical positive and negative moment sections. For prismatic members, effective moment of inertia shall be permitted to be taken as the value obtained from Eq. (9-7) at midspan for simple and continuous spans, and at support for cantilevers.

9.5.2.5 — Unless values are obtained by a more comprehensive analysis, additional long-term deflection resulting from creep and shrinkage of flexural members (normal weight or lightweight concrete) shall be determined by multiplying the immediate deflection caused by the sustained load considered, by the factor

$$\lambda = \frac{\xi}{1 + 50\rho'} \qquad (9\text{-}10)$$

where ρ' shall be the value at midspan for simple and

COMMENTARY

varies along the span, a more exact calculation becomes necessary.

R9.5.2.3 — The effective moment of inertia procedure described in the code and developed in Reference 9.6 was selected as being sufficiently accurate for use to control deflections.$^{9.7\text{-}9.9}$ The effective $\boldsymbol{I_e}$ was developed to provide a transition between the upper and lower bounds of $\boldsymbol{I_g}$ and $\boldsymbol{I_{cr}}$ as a function of the ratio $\boldsymbol{M_{cr}/M_a}$. For most practical cases $\boldsymbol{I_e}$ will be less than $\boldsymbol{I_g}$.

R9.5.2.4 — For continuous members, the code procedure suggests a simple averaging of $\boldsymbol{I_e}$ values for the positive and negative moment sections. The use of the midspan section properties for continuous prismatic members is considered satisfactory in approximate calculations primarily because the midspan rigidity (including the effect of cracking) has the dominant effect on deflections, as shown by ACI Committee 435$^{9.10,9.11}$ and SP-43.$^{9.4}$

R9.5.2.5 — Shrinkage and creep due to sustained loads cause additional "long-term deflections" over and above those which occur when loads are first placed on the structure. Such deflections are influenced by temperature, humidity, curing conditions, age at time of loading, quantity of compression reinforcement, magnitude of the sustained load, and other factors. The expression given in this section is considered satisfactory for use with the code procedures for the calculation of immediate deflections, and with the limits given in Table 9.5(b). It should also be noted that the deflection computed in accordance with this section is the

CODE

continuous spans, and at support for cantilevers. It shall be permitted to assume the time-dependent factor ξ for sustained loads to be equal to

5 years or more	2.0
12 months	1.4
6 months	1.2
3 months	1.0

9.5.2.6 — Deflection computed in accordance with 9.5.2.2 through 9.5.2.5 shall not exceed limits stipulated in Table 9.5(b).

COMMENTARY

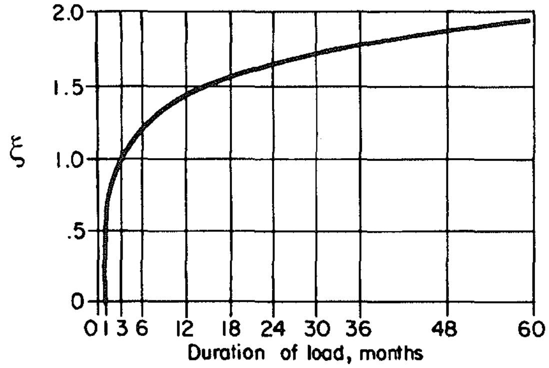


Fig. R9.5.2.5—Multipliers for long-term deflections

additional long-term deflection due to the dead load and that portion of the live load which will be sustained for a sufficient period to cause significant time-dependent deflections.

Equation (9-10) was developed in Reference 9.13. In Eq. (9-10) the multiplier on ξ accounts for the effect of compression reinforcement in reducing long-term deflections, and $\xi = 2.0$ represents a nominal time-dependent factor for 5 years duration of loading. The curve in Fig. R9.5.2.5 may be used to estimate values of ξ for loading periods less than five years.

If it is desired to consider creep and shrinkage separately, approximate equations provided in References 9.6, 9.7, 9.13, and 9.14 may be used.

R9.5.2.6 — It should be noted that the limitations given in this table relate only to supported or attached nonstructural elements. For those structures in which structural members are likely to be affected by deflection or deformation of members to which they are attached in such a manner as to affect adversely the strength of the structure, these deflections and the resulting forces should be considered

TABLE 9.5(b)—MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load ***L***	$\frac{\ell^*}{180}$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load ***L***	$\frac{\ell}{360}$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)[†]	$\frac{\ell^\ddagger}{480}$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\frac{\ell^\S}{240}$

[*] Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[†] Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.2, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[‡] Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

[§] But not greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

CODE

9.5.3 — Two-way construction (nonprestressed)

9.5.3.1 — Section 9.5.3 shall govern the minimum thickness of slabs or other two-way construction designed in accordance with the provisions of Chapter 13 and conforming with the requirements of 13.6.1.2. The thickness of slabs without interior beams spanning between the supports on all sides shall satisfy the requirements of 9.5.3.2 or 9.5.3.4. The thickness of slabs with beams spanning between the supports on all sides shall satisfy requirements of 9.5.3.3 or 9.5.3.4.

9.5.3.2 — For slabs without interior beams spanning between the supports and having a ratio of long to short span not greater than 2, the minimum thickness shall be in accordance with the provisions of Table 9.5(c) and shall not be less than the following values:

(a) Slabs without drop panels as defined in 13.3.7.1 and 13.3.7.2........................ 5 in.

(b) Slabs with drop panels as defined in 13.3.7.1 and 13.3.7.2 .... 4 in.

TABLE 9.5(c)—MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS

	Without drop panels†			With drop panels†		
	Exterior panels		Interior panels	Exterior panels		Interior panels
Yield strength, f_y, psi*	Without edge beams	With edge beams‡		Without edge beams	With edge beams‡	
40,000	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{40}$	$\frac{\ell_n}{40}$
60,000	$\frac{\ell_n}{30}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$
75,000	$\frac{\ell_n}{28}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{34}$	$\frac{\ell_n}{34}$

* For values of reinforcement yield strength between the values given in the table, minimum thickness shall be determined by linear interpolation.

† Drop panel is defined in 13.3.7.1 and 13.3.7.2.

‡ Slabs with beams between columns along exterior edges. The value of α for the edge beam shall not be less than 0.8.

9.5.3.3 — For slabs with beams spanning between the supports on all sides, the minimum thickness shall be as follows:

(a) For α_m equal to or less than 0.2, the provisions of 9.5.3.2 shall apply.

COMMENTARY

explicitly in the analysis and design of the structures as required by 9.5.1. (See Reference 9.9.)

Where long-term deflections are computed, the portion of the deflection before attachment of the nonstructural elements may be deducted. In making this correction use may be made of the curve in Fig. R9.5.2.5 for members of usual sizes and shapes.

R9.5.3 — Two-way construction (nonprestressed)

R9.5.3.2 — The minimum thicknesses in Table 9.5(c) are those that have evolved through the years in building codes. It is assumed that slabs conforming to those limits have not resulted in systematic problems related to stiffness for short- and long-term loads. Naturally, this conclusion applies in only the domain of previous experience in loads, environment, materials, boundary conditions, and spans.

EQ 9.13 has been dropped from '89 code.

R9.5.3.3 — For panels having a ratio of long to short span greater than 2, the use of Eq. (9-11) and (9-12), which express the minimum thickness as a fraction of the long span, may give unreasonable results. For such panels, the rules applying to one-way construction in 9.5.2 should be used.

CODE

(b) For α_m greater than 0.2 but not greater than 2.0, the thickness shall not be less than

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{200{,}000}\right)}{36 + 5\beta\,(\alpha_m - 0.2)} \qquad \text{(9-11)}$$

and not less than 5 in.

(c) For α_m greater than 2.0, the thickness shall not be less than

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{200{,}000}\right)}{36 + 9\beta} \qquad \text{(9-12)}$$

and not less than 3.5 in.

(d) At discontinuous edges, an edge beam shall be provided with a stiffness ratio α not less than 0.80 or the minimum thickness required by Eq. (9-11) or (9-12) shall be increased by at least 10 percent in the panel with a discontinuous edge.

9.5.3.4 — Slab thickness less than the minimum thickness required by 9.5.3.1, 9.5.3.2, and 9.5.3.3 shall be permitted to be used if shown by computation that the deflection will not exceed the limits stipulated in Table 9.5(b). Deflections shall be computed taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. The modulus of elasticity of concrete E_c shall be as specified in 8.5.1. The effective moment of inertia shall be that given by Eq. (9-7); other values shall be permitted to be used if they result in computed deflections in reasonable agreement with results of comprehensive tests. Additional long-term deflection shall be computed in accordance with 9.5.2.5.

9.5.4 — Prestressed concrete construction

9.5.4.1 — For flexural members designed in accordance with provisions of Chapter 18, immediate deflection shall be computed by usual methods or formulas for elastic deflections, and the moment of inertia of the gross concrete section shall be permitted to be used for uncracked sections.

COMMENTARY

The requirement in 9.5.3.3(a) for α_m equal to 0.2 makes it possible to eliminate Eq. (9-13) of ACI 318-89. That equation gave values essentially the same as those in Table 9.5(c), as does Eq. (9-11) at a value of α_m equal to 0.2.

In '89 $+0.2$ was β.
β was found to have little
effect on h

R9.5.3.4 — The calculation of deflections for slabs is complicated even if linear elastic behavior can be assumed. For immediate deflections, the values of E_c and I_e specified in 9.5.2.3 may be used.[9.9] However, other procedures and other values of the stiffness EI may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests.

Since available data on long-term deflections of slabs are too limited to justify more elaborate procedures, the additional long-term deflection for two-way construction is required to be computed using the multipliers given in 9.5.2.5.

R9.5.4 — Prestressed concrete construction

The code requires deflections for all prestressed concrete flexural members to be computed and compared with the allowable values in Table 9.5(b).

R9.5.4.1 — Immediate deflections of prestressed concrete members may be calculated by the usual methods or formulas for elastic deflections using the moment of inertia of the gross (uncracked) concrete section and the modulus of elasticity for concrete specified in 8.5.1. Since this method assumes that the concrete is uncracked, it may be unconservative for members having a relatively high concrete tensile stress as permitted by 18.4.2(d). Hence, 18.4.2(d) requires calculation of deflection based on the transformed cracked section for members designed for a tensile stress in the precompressed tension zone equal to $12\sqrt{f_c'}$.

CODE

COMMENTARY

It has also been shown in Reference 9.15 that the I_e method can be used to compute deflections of partially prestressed members loaded above the cracking load. For this case, the cracking moment must take into account the effect of prestress. A method for predicting the effect of nonprestressed tension steel in reducing creep camber is also given in Reference 9.15 with approximate forms referred to in References 9.9 and 9.16.

9.5.4.2 — Additional long-term deflection of prestressed concrete members shall be computed taking into account stresses in concrete and steel under sustained load and including effects of creep and shrinkage of concrete and relaxation of steel.

R9.5.4.2 — Calculation of long-term deflections of prestressed concrete flexural members is complicated. The calculations must consider not only the increased deflections due to flexural stresses, but also the additional long-term deflections resulting from time-dependent shortening of the flexural member.

Prestressed concrete generally shortens more with time than similar nonprestressed members. This is due to the precompression in the slab or beam which causes axial creep. This creep together with shrinkage of the concrete results in significant shortening of the flexural members which continues for several years after construction and must be considered in design. The shortening tends to reduce the tension in the prestressing tendons, thus reducing the precompression in the member and thereby causing increased long-term deflections.

Another factor that can influence long-term deflections of prestressed flexural members is adjacent concrete or masonry nonprestressed in the direction of the prestressed member. This can be a slab nonprestressed in the beam direction adjacent to a prestressed beam or a nonprestressed slab system. As the prestressed member tends to shrink and creep more than the adjacent nonprestressed concrete, the structure will tend to reach a compatibility of the shortening effects. This results in a reduction of the precompression in the prestressed member as the adjacent concrete absorbs the compression. This reduction in precompression of the prestressed member can occur over a period of years and will result in additional long-term deflections, and in increased stresses in the prestressed member.

Any suitable method for calculating long-term deflections of prestressed members may be used, provided all effects are considered. Guidance may be found in References 9.9, 9.12, 9.15, 9.17, and 9.18.

9.5.4.3 — Deflection computed in accordance with 9.5.4.1 and 9.5.4.2 shall not exceed limits stipulated in Table 9.5(b).

9.5.5 — Composite construction

9.5.5.1 — Shored construction

If composite flexural members are supported during construction so that, after removal of temporary sup-

R9.5.5 — Composite construction

Since few tests have been made to study the immediate and long-term deflections of composite members, the rules given in 9.5.5.1 and 9.5.5.2 are based on the judgment of

CODE

ports, dead load is resisted by the full composite section, it shall be permitted to consider the composite member equivalent to a monolithically cast member for computation of deflection. For nonprestressed members, the portion of the member in compression shall determine whether values in Table 9.5(a) for normal weight or lightweight concrete shall apply. If deflection is computed, account shall be taken of curvatures resulting from differential shrinkage of precast and cast-in-place components, and of axial creep effects in a prestressed concrete member.

9.5.5.2 — Unshored construction

If the thickness of a nonprestressed precast flexural member meets the requirements of Table 9.5(a), deflection need not be computed. If the thickness of a nonprestressed composite member meets the requirements of Table 9.5(a), it is not required to compute deflection occurring after the member becomes composite, but the long-term deflection of the precast member shall be investigated for magnitude and duration of load prior to beginning of effective composite action.

9.5.5.3 — Deflection computed in accordance with 9.5.5.1 and 9.5.5.2 shall not exceed limits stipulated in Table 9.5(b).

COMMENTARY

ACI Committee 318 and on experience.

If any portion of a composite member is prestressed or if the member is prestressed after the components have been cast, the provisions of 9.5.4 apply and deflections must be calculated. For nonprestressed composite members, deflections need to be calculated and compared with the limiting values in Table 9.5(b) only when the thickness of the member or the precast part of the member is less than the minimum thickness given in Table 9.5(a). In unshored construction the thickness of concern depends on whether the deflection before or after the attainment of effective composite action is being considered. (In Chapter 17, it is stated that distinction need not be made between shored and unshored members. This refers to strength calculations, not to deflections.)

CHAPTER 10 — FLEXURE AND AXIAL LOADS

CODE

10.0 — Notation

a = depth of equivalent rectangular stress block as defined in 10.2.7.1
A = effective tension area of concrete surrounding the flexural tension reinforcement and having the same centroid as that reinforcement, divided by the number of bars or wires, in.^{2} When the flexural reinforcement consists of different bar or wire sizes the number of bars or wires shall be computed as the total area of reinforcement divided by the area of the largest bar or wire used
A_c = area of core of spirally reinforced compression member measured to outside diameter of spiral, in.^{2}
A_g = gross area of section, in.^{2}
A_s = area of nonprestressed tension reinforcement, in.^{2}
A_{sk} = area of skin reinforcement per unit height in one side face, in.^{2}/ft. See 10.6.7
$A_{s,min}$ = minimum amount of flexural reinforcement, in.^{2} See 10.5
A_{st} = total area of longitudinal reinforcement, (bars or steel shapes), in.^{2}
A_t = area of structural steel shape, pipe, or tubing in a composite section, in.^{2}
A_1 = loaded area
A_2 = the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal
b = width of compression face of member, in.
b_w = web width, in.
c = distance from extreme compression fiber to neutral axis, in.
C_m = a factor relating actual moment diagram to an equivalent uniform moment diagram
d = distance from extreme compression fiber to centroid of tension reinforcement, in.
d_c = thickness of concrete cover measured from extreme tension fiber to center of bar or wire located closest thereto, in.
d_t = distance from extreme compression fiber to extreme tension steel, in.
E_c = modulus of elasticity of concrete, psi. See 8.5.1
E_s = modulus of elasticity of reinforcement, psi. See 8.5.2 or 8.5.3
EI = flexural stiffness of compression member. See Eq. (10-12) and Eq. (10-13)

COMMENTARY

R10.0 — Notation

CODE

f_c' = specified compressive strength of concrete, psi

f_s = calculated stress in reinforcement at service loads, ksi

f_y = specified yield strength of nonprestressed reinforcement, psi

h = overall thickness of member, in.

I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement

I_{se} = moment of inertia of reinforcement about centroidal axis of member cross section

I_t = moment of inertia of structural steel shape, pipe, or tubing about centroidal axis of composite member cross section

k = effective length factor for compression members

ℓ_c = length of compression member in a frame, measured from center to center of the joints in the frame

ℓ_u = unsupported length of compression member

M_c = factored moment to be used for design of compression member

M_s = moment due to loads causing appreciable sway

M_u = factored moment at section

M_1 = smaller factored end moment on a compression member, positive if member is bent in single curvature, negative if bent in double curvature

M_{1ns} = factored end moment on a compression member at the end at which M_1 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis

M_{1s} = factored end moment on compression member at the end at which M_1 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis

M_2 = larger factored end moment on compression member, always positive

$M_{2,min}$ = minimum value of M_2

M_{2ns} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis

M_{2s} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis

P_b = nominal axial load strength at balanced strain conditions. See 10.3.2

P_c = critical load. See Eq. (10-11)

COMMENTARY

CODE

P_n = nominal axial load strength at given eccentricity

P_o = nominal axial load strength at zero eccentricity

P_u = factored axial load at given eccentricity $\leq \phi P_n$

Q = stability index for a story. See 10.11.4

r = radius of gyration of cross section of a compression member

V_u = factored horizontal shear in a story

z = quantity limiting distribution of flexural reinforcement. See 10.6

β_1 = factor defined in 10.2.7.3

β_d = (a) for non-sway frames, β_d is the ratio of the maximum factored axial dead load to the total factored axial load
(b) for sway frames, except as required in (c), β_d is the ratio of the maximum factored sustained shear within a story to the total factored shear in that story
(c) for stability checks of sway frames carried out in accordance with 10.13.6, β_d is the ratio of the maximum factored sustained axial load to the total factored axial load

δ_{ns} = moment magnification factor for frames braced against sidesway, to reflect effects of member curvature between ends of compression member

δ_s = moment magnification factor for frames not braced against sidesway, to reflect lateral drift resulting from lateral and gravity loads

Δ_o = relative lateral deflection between the top and bottom of a story due to V_u, computed using a first-order elastic frame analysis and stiffness values satisfying 10.11.1

ε_t = net tensile strain in extreme tension steel at nominal strength

ρ = ratio of nonprestressed tension reinforcement
= A_s/bd

ρ_b = reinforcement ratio producing balanced strain conditions. See 10.3.2

ρ_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of a spirally reinforced compression member

ϕ = strength reduction factor. See 9.3

ϕ_K = stiffness reduction factor. See R10.12.3

10.1 — Scope

Provisions of Chapter 10 shall apply for design of members subject to flexure or axial loads or to combined flexure and axial loads.

COMMENTARY

The definition of net tensile strain in 2.1 excludes strains due to effective prestress, creep, shrinkage, and temperature.

CODE

10.2 — Design assumptions

10.2.1 — Strength design of members for flexure and axial loads shall be based on assumptions given in 10.2.2 through 10.2.7, and on satisfaction of applicable conditions of equilibrium and compatibility of strains.

10.2.2 — Strain in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis, except, for deep flexural members with overall depth to clear span ratios greater than $^2/_5$ for continuous spans and $^4/_5$ for simple spans, a nonlinear distribution of strain shall be considered. See 10.7.

10.2.3 — Maximum usable strain at extreme concrete compression fiber shall be assumed equal to 0.003.

10.2.4 — Stress in reinforcement below specified yield strength f_y for grade of reinforcement used shall be taken as E_s times steel strain. For strains greater than that corresponding to f_y, stress in reinforcement shall be considered independent of strain and equal to f_y.

10.2.5 — Tensile strength of concrete shall be neglected in axial and flexural calculations of reinforced concrete, except when meeting requirements of 18.4.

COMMENTARY

R10.2 — Design assumptions

R10.2.1 — The strength of a member computed by the strength design method of the code requires that two basic conditions be satisfied: (1) static equilibrium and (2) compatibility of strains. Equilibrium between the compressive and tensile forces acting on the cross section at nominal strength must be satisfied. Compatibility between the stress and strain for the concrete and the reinforcement at nominal strength conditions must also be established within the design assumptions allowed by 10.2.

R10.2.2—Many tests have confirmed that the distribution of strain is essentially linear across a reinforced concrete cross section, even near ultimate strength.

Both the strain in reinforcement and in concrete are assumed to be directly proportional to the distance from the neutral axis. This assumption is of primary importance in design for determining the strain and corresponding stress in the reinforcement.

R10.2.3 — The maximum concrete compressive strain at crushing of the concrete has been observed in tests of various kinds to vary from 0.003 to higher than 0.008 under special conditions. However, the strain at which ultimate moments are developed is usually about 0.003 to 0.004 for members of normal proportions and materials.

R10.2.4 — For deformed reinforcement, it is reasonably accurate to assume that the stress in reinforcement is proportional to strain below the yield strength f_y. The increase in strength due to the effect of strain hardening of the reinforcement is neglected for strength computations. In strength computations, the force developed in tensile or compressive reinforcement is computed as,

when $\varepsilon_s < \varepsilon_y$ (yield strain)

$$A_s f_s = A_s E_s \varepsilon_s$$

when $\varepsilon_s \geq \varepsilon_y$

$$A_s f_s = A_s f_y$$

where ε_s is the value from the strain diagram at the location of the reinforcement. For design, the modulus of elasticity of steel reinforcement E_s may be taken as 29,000,000 psi (see 8.5.2).

R10.2.5 — The tensile strength of concrete in flexure (modulus of rupture) is a more variable property than the compressive strength and is about 10 to 15 percent of the compressive strength. Tensile strength of concrete in flexure is neglected in strength design. For members with normal

CODE

10.2.6 — Relationship between concrete compressive stress distribution and concrete strain shall be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.

10.2.7 — Requirements of 10.2.6 are satisfied by an equivalent rectangular concrete stress distribution defined by the following:

10.2.7.1 — Concrete stress of $\mathbf{0.85 f_c'}$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain.

10.2.7.2 — Distance c from fiber of maximum strain to the neutral axis shall be measured in a direction perpendicular to that axis.

10.2.7.3 — Factor β_1 shall be taken as 0.85 for concrete strengths f_c' up to and including 4000 psi. For strengths above 4000 psi, β_1 shall be reduced continuously at a rate of 0.05 for each 1000 psi of strength in excess of 4000 psi, but β_1 shall not be taken less than 0.65.

COMMENTARY

percentages of reinforcement, this assumption is in good agreement with tests. For very small percentages of reinforcement, neglect of the tensile strength at ultimate is usually correct.

The strength of concrete in tension, however, is important in cracking and deflection considerations at service loads.

R10.2.6 — This assumption recognizes the inelastic stress distribution of concrete at high stress. As maximum stress is approached, the stress-strain relationship for concrete is not a straight line but some form of a curve (stress is not proportional to strain). The general shape of a stress-strain curve is primarily a function of concrete strength and consists of a rising curve from zero to a maximum at a compressive strain between 0.0015 and 0.002 followed by a descending curve to an ultimate strain (crushing of the concrete) from 0.003 to higher than 0.008. As discussed under R10.2.3. the code sets the maximum usable strain at 0.003 for design.

The actual distribution of concrete compressive stress in a practical case is complex and usually not known explicitly. However, research has shown that the important properties of the concrete stress distribution can be approximated closely using any one of several different assumptions as to the form of stress distribution. The code permits any particular stress distribution to be assumed in design if shown to result in predictions of ultimate strength in reasonable agreement with the results of comprehensive tests. Many stress distributions have been proposed. The three most common are the parabola, trapezoid, and rectangle.

R10.2.7 — For practical design, the code allows the use of a rectangular compressive stress distribution (stress block) to replace the more exact concrete stress distributions. In the equivalent rectangular stress block, an average stress of $\mathbf{0.85 f_c'}$ is used with a rectangle of depth $a = \beta_1 c$. The β_1 of 0.85 for concrete with $f_c' \leq 4000$ psi and 0.05 less for each 1000 psi of f_c' in excess of 4000 was determined experimentally.

In the 1976 supplement to ACI 318-71, a lower limit of β_1 equal to 0.65 was adopted for concrete strengths greater than 8000 psi. Research data from tests with high strength concretes[10.1,10.2] supported the equivalent rectangular stress block for concrete strengths exceeding 8000 psi, with a β_1 equal to 0.65. Use of the equivalent rectangular stress distribution specified in ACI 318-71, with no lower limit on β_1, resulted in inconsistent designs for high strength concrete for members subject to combined flexure and axial load.

The rectangular stress distribution does not represent the actual stress distribution in the compression zone at ultimate, but does provide essentially the same results as those obtained in tests.[10.3]

CODE

10.3 — General principles and requirements

10.3.1 — Design of cross section subject to flexure or axial loads or to combined flexure and axial loads shall be based on stress and strain compatibility using assumptions in 10.2.

10.3.2 — Balanced strain conditions exist at a cross section when tension reinforcement reaches the strain corresponding to its specified yield strength f_y just as concrete in compression reaches its assumed ultimate strain of 0.003.

10.3.3 — For flexural members, and for members subject to combined flexure and compressive axial load when the design axial load strength ϕP_n is less than the smaller of $0.10 f_c' A_g$ or ϕP_b, the ratio of reinforcement ρ provided shall not exceed 0.75 of the ratio ρ_b that would produce balanced strain conditions for the section under flexure without axial load. For members with compression reinforcement, the portion of ρ_b equalized by compression reinforcement need not be reduced by the 0.75 factor.

COMMENTARY

R10.3 — General principles and requirements

R10.3.1 — Design strength equations for members subject to flexure or combined flexure and axial load are derived in the paper, "Rectangular Concrete Stress Distribution in Ultimate Strength Design."[10.3] Reference 10.3 and previous editions of this commentary also give the derivations of strength equations for cross sections other than rectangular.

R10.3.2 — A balanced strain condition exists at a cross section when the maximum strain at the extreme compression fiber just reaches 0.003 simultaneously with the first yield strain f_y/E_s in the tension reinforcement. The reinforcement ratio ρ_b, which produces balanced conditions under flexure, depends on the shape of the cross section and the location of the reinforcement.

R10.3.3 — The maximum amount of tension reinforcement in flexural members is limited to ensure a level of ductile behavior.

The ultimate flexural strength of a member is reached when the strain in the extreme compression fiber reaches the ultimate (crushing) strain of the concrete. At ultimate strain of the concrete, the strain in the tension reinforcement could just reach the strain at first yield, be less than the yield strain (elastic), or exceed the yield strain (inelastic). Which steel strain condition exists at ultimate concrete strain depends on the relative proportion of steel to concrete and material strengths f_c' and f_y. If $\rho(f_y/f_c')$ is sufficiently low, the strain in the tension steel will greatly exceed the yield strain when the concrete strain reaches its ultimate, with large deflection and ample warning of impending failure (ductile failure condition). With a larger $\rho(f_y/f_c')$, the strain in the tension steel may not reach the yield strain when the concrete strain reaches its ultimate, with consequent small deflection and little warning of impending failure (brittle failure condition). For design it is considered more conservative to restrict the ultimate strength condition so that a ductile failure mode can be expected.

Unless unusual amounts of ductility are required, the **0.75** ρ_b limitation will provide ductile behavior for most designs. One condition where greater ductile behavior is required is in design for redistribution of moments in continuous members and frames. Code Section 8.4 permits negative moment redistribution. Since moment redistribution is dependent on adequate ductility in hinge regions, the amount of tension reinforcement in hinging regions is limited to $\mathbf{0.5}\rho_b$.

For ductile behavior of beams with compression reinforcement, only that portion of the total tension steel balanced by compression in the concrete need be limited; that portion of the total tension steel where force is balanced by compression reinforcement need not be limited by the 0.75 factor.

CODE

10.3.4 — Use of compression reinforcement shall be permitted in conjunction with additional tension reinforcement to increase the strength of flexural members.

10.3.5 — Design axial load strength $\phi \boldsymbol{P_n}$ of compression members shall not be taken greater than the following:

10.3.5.1 — For nonprestressed members with spiral reinforcement conforming to 7.10.4 or composite members conforming to 10.16:

$$\phi P_{n(max)} = 0.85\phi\left[0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right] \quad (10\text{-}1)$$

10.3.5.2—For nonprestressed members with tie reinforcement conforming to 7.10.5:

$$\phi P_{n(max)} = 0.80\phi\left[0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right] \quad (10\text{-}2)$$

10.3.5.3 — For prestressed members, design axial load strength $\phi \boldsymbol{P_n}$ shall not be taken greater than 0.85 (for members with spiral reinforcement) or 0.80 (for members with tie reinforcement) of the design axial load strength at zero eccentricity $\phi \boldsymbol{P_o}$.

10.3.6 — Members subject to compressive axial load shall be designed for the maximum moment that can accompany the axial load. The factored axial load $\boldsymbol{P_u}$ at given eccentricity shall not exceed that given in 10.3.5. The maximum factored moment $\boldsymbol{M_u}$ shall be magnified for slenderness effects in accordance with 10.10.

COMMENTARY

R10.3.5 and R10.3.6 — The minimum design eccentricities included in the 1963 and 1971 codes were deleted from the 1977 code except for consideration of slenderness effects in compression members with small or zero computed end moments (see 10.12.3.2). The specified minimum eccentricities were originally intended to serve as a means of reducing the axial load design strength of a section in pure compression to account for accidental eccentricities not considered in the analysis that may exist in a compression member, and to recognize that concrete strength may be less than f_c' under sustained high loads. The primary purpose of the minimum eccentricity requirement was to limit the maximum design axial load strength of a compression member. This is now accomplished directly in 10.3.5 by limiting the design axial load strength of a section in pure compression to 85 or 80 percent of the nominal strength. These percentage values approximate the axial load strengths at e/h ratios of 0.05 and 0.10, specified in the earlier codes for the spirally reinforced and tied members respectively. The same axial load limitation applies to both cast-in-place and precast compression members. Design aids and computer programs based on the minimum eccentricity requirement of the 1963 and 1971 codes are equally applicable for usage.

For prestressed members, the design axial load strength in pure compression is computed by the strength design methods of Chapter 10, including the effect of the prestressing force.

Compression member end moments must be considered in the design of adjacent flexural members. In braced frames, the effects of magnifying the end moments need not be considered in the design of the adjacent beams. In frames which are not braced against sidesway, the magnified end moments must be considered in designing the flexural members, as required in 10.13.7.

Corner and other columns exposed to known moments about each axis simultaneously should be designed for biaxial bending and axial load. Satisfactory methods are available in the ACI *Design Handbook*[10.4] and the CRSI *Handbook.*[10.5] The reciprocal load method[10.6] and the load contour method[10.7] are the methods used in those two handbooks. Research[10.8,10.9] indicates that using the rectangular stress block provisions of 10.2.7 produces satisfactory strength estimates for doubly symmetric sections. A simple and somewhat conservative estimate of nominal strength $\boldsymbol{P_{ni}}$ can be obtained from the reciprocal load relationship[10.6]

$$\frac{1}{P_{ni}} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$$

CODE

10.4 — Distance between lateral supports of flexural members

10.4.1 — Spacing of lateral supports for a beam shall not exceed 50 times the least width ***b*** of compression flange or face.

10.4.2 — Effects of lateral eccentricity of load shall be taken into account in determining spacing of lateral supports.

10.5 — Minimum reinforcement of flexural members

10.5.1 — At every section of a flexural member where tensile reinforcement is required by analysis, except as provided in 10.5.2, 10.5.3, and 10.5.4, the area A_s provided shall not be less than that given by

$$A_{s,min} = \frac{3\sqrt{f_c'}}{f_y} b_w d \qquad (10\text{-}3)$$

f_c' in psi

and not less than $200\, b_w d / f_y$.

10.5.2 — For a statically determinate T-section with flange in tension, the area $A_{s,min}$ shall be equal to or greater than the smaller value given either by

$$A_{s,min} = \frac{6\sqrt{f_c'}}{f_y} b_w d \qquad (10\text{-}4)$$

or Eq. (10-3) with b_w set equal to the width of the flange.

COMMENTARY

where

P_{ni} = nominal axial load strength at given eccentricity along both axes
P_o = nominal axial load strength at zero eccentricity
P_{nx} = nominal axial load strength at given eccentricity along x-axis
P_{ny} = nominal axial load strength at given eccentricity along y-axis

This relationship is most suitable when values P_{nx} and P_{ny} are greater than the balanced axial force P_b for the particular axis.

R10.4 — Distance between lateral supports of flexural members

Tests have shown that laterally unbraced reinforced concrete beams of any reasonable dimensions, even when very deep and narrow, will not fail prematurely by lateral buckling provided the beams are loaded without lateral eccentricity that could cause torsion[10.10,10.11]

Laterally unbraced beams are frequently loaded off center (lateral eccentricity) or with slight inclination. Stresses and deformations set up by such loading become detrimental for narrow, deep beams, the more so as the unsupported length increases. Lateral supports spaced closer than **50*b*** may be required by actual loading conditions.

R10.5 — Minimum reinforcement of flexural members

The provision for a minimum amount of reinforcement applies to flexural members, which for architectural or other reasons, are larger in cross section than required for strength. With a very small amount of tensile reinforcement, the computed moment strength as a reinforced concrete section using cracked section analysis becomes less than that of the corresponding unreinforced concrete section computed from its modulus of rupture. Failure in such a case can be sudden.

To prevent such a failure, a minimum amount of tensile reinforcement is required by 10.5.1. This is required in both positive and negative moment regions. The $200/f_y$ value formerly used was originally derived to provide the same 0.5 percent minimum (for mild grade steel) as required in earlier editions of the ACI Building Code. When concrete strength higher than about 5000 psi is used, the $200/f_y$ value previously used may not be sufficient. The value given by Eq. (10-3) gives the same amount as $200/f_y$ when f_c' equals 4440 psi. When the flange of a T-section is in tension, the amount of tensile reinforcement needed to make the

CODE

COMMENTARY

strength of a reinforced concrete section equal that of an unreinforced section is about twice that for a rectangular section or that of a T-section with the flange in compression. It was concluded that this higher amount is necessary, particularly for cantilevers and other statically determinate situations where the flange is in tension.

10.5.3 — The requirements of 10.5.1 and 10.5.2 need not be applied if at every section the area of tensile reinforcement provided is at least one-third greater than that required by analysis.

R10.5.3 — The minimum reinforcement required by Eq. (10-3) or (10-4) must be provided wherever reinforcement is needed, except where such reinforcement is at least one-third greater than that required by analysis. This exception provides sufficient additional reinforcement in large members where the amount required by 10.5.1 or 10.5.2 would be excessive.

10.5.4 — For structural slabs and footings of uniform thickness the minimum area of tensile reinforcement in the direction of the span shall be the same as that required by 7.12. Maximum spacing of this reinforcement shall not exceed the lesser of three times the thickness and 18 in.

R10.5.4 — The minimum reinforcement required for slabs should be equal to the same amount as that required by 7.12 for shrinkage and temperature reinforcement.

Soil-supported slabs such as slabs on grade are not considered to be structural slabs in the context of this section, unless they transmit vertical loads from other parts of the structure to the soil. Reinforcement, if any, in soil-supported slabs should be proportioned with due consideration of all design forces. Mat foundations and other slabs which help support the structure vertically should meet the requirements of this section.

In reevaluating the overall treatment of 10.5, the maximum spacing for reinforcement in structural slabs (including footings) was reduced from the ***5h*** for temperature and shrinkage reinforcement to the compromise value of ***3h***, which is somewhat larger than the ***2h*** limit of 13.3.2 for two-way slab systems.

10.6 — Distribution of flexural reinforcement in beams and one-way slabs

10.6.1 — This section prescribes rules for distribution of flexural reinforcement to control flexural cracking in beams and in one-way slabs (slabs reinforced to resist flexural stresses in only one direction).

R10.6 — Distribution of flexural reinforcement in beams and one-way slabs

R10.6.1 — Many structures designed by working stress methods and with low steel stress served their intended functions with very limited flexural cracking. When high strength reinforcing steels are used at high service load stresses, however, visible cracks must be expected, and steps must be taken in detailing of the reinforcement to control cracking. To assure protection of reinforcement against corrosion, and for aesthetic reasons, many fine hairline cracks are preferable to a few wide cracks.

Control of cracking is particularly important when reinforcement with a yield strength in excess of 40,000 psi is used. Current good detailing practices will usually lead to adequate crack control even when reinforcement of 60,000 psi yield is used.

Extensive laboratory work[10.12-10.14] involving modern deformed bars has confirmed that crack width at service

CODE

10.6.2 — Distribution of flexural reinforcement in two-way slabs shall be as required by 13.3.

10.6.3 — Flexural tension reinforcement shall be well distributed within maximum flexural tension zones of a member cross section as required by 10.6.4.

10.6.4 — When design yield strength f_y for tension reinforcement exceeds 40,000 psi, cross sections of maximum positive and negative moment shall be so proportioned that the quantity z given by

$$z = f_s \sqrt[3]{d_c A} \qquad (10\text{-}5)$$

does not exceed 175 kips/in. for interior exposure and 145 kips/in. for exterior exposure. Calculated stress in reinforcement at service load f_s (kips/in.^{2}) shall be computed as the moment divided by the product of steel area and internal moment arm. Alternatively, it shall be permitted to take f_s as 60 percent of specified yield strength f_y.

COMMENTARY

loads is proportional to steel stress. However, the significant variables reflecting steel detailing were found to be thickness of concrete cover and the area of concrete in the zone of maximum tension surrounding each individual reinforcing bar.

Crack width is inherently subject to wide scatter even in careful laboratory work and is influenced by shrinkage and other time-dependent effects. The best crack control is obtained when the steel reinforcement is well distributed over the zone of maximum concrete tension.

R10.6.3 — Several bars at moderate spacing are much more effective in controlling cracking than one or two larger bars of equivalent area.

R10.6.4 — Eq. (10-5) will provide a distribution that will reasonably control flexural cracking. The equation is written in a form emphasizing reinforcement details rather than crack width w, per se. It is based on the Gergely-Lutz expression:

$$w = 0.076\beta f_s \sqrt[3]{d_c A}$$

in which w is in units of 0.001 in. To simplify practical design, an approximate value of 1.2 is used for β (ratio of distances to the neutral axis from the extreme tension fiber and from the centroid of the main reinforcement). Laboratory tests[10.15] have shown that the Gergely-Lutz expression

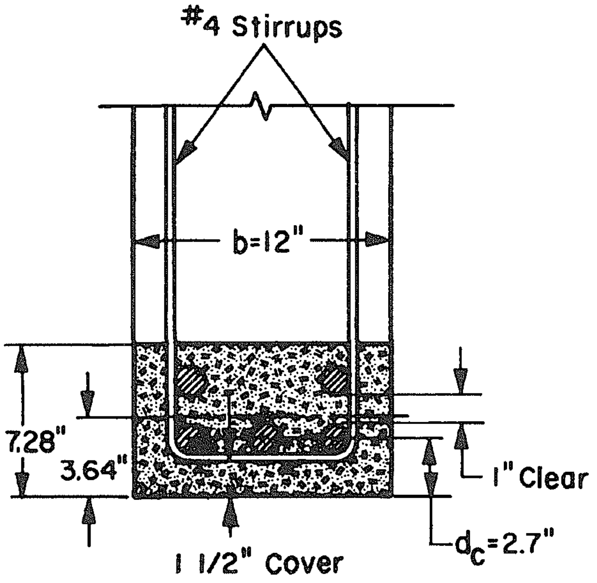


Fig. R10.6.4—Effective tension area of concrete (beam with five No. 11 bars)

CODE

10.6.5 — Provisions of 10.6.4 are not sufficient for structures subject to very aggressive exposure or designed to be watertight. For such structures, special investigations and precautions are required.

10.6.6 — Where flanges of T-beam construction are in tension, part of the flexural tension reinforcement shall be distributed over an effective flange width as defined in 8.10, or a width equal to $^1/_{10}$ the span, whichever is smaller. If the effective flange width exceeds $^1/_{10}$ the span, some longitudinal reinforcement shall be provided in the outer portions of the flange.

10.6.7 — If the effective depth $\boldsymbol{d}$ of a beam or joist exceeds 36 in., longitudinal skin reinforcement shall be uniformly distributed along both side faces of the member for a distance $\boldsymbol{d/2}$ nearest the flexural tension reinforcement. The area of skin reinforcement $\boldsymbol{A_{sk}}$ per foot of height on each side face shall be $\geq 0.012(\boldsymbol{d}-\mathbf{30})$. The maximum spacing of the skin reinforcement shall not exceed the lesser of $\boldsymbol{d/6}$ and 12 in. It shall be permitted to include such reinforcement in strength computations if a strain compatibility analysis is made to determine stress in the individual bars or wires. The total area of longitudinal skin reinforcement in both faces need not exceed one-half of the required flexural tensile reinforcement.

COMMENTARY

applies reasonably to one-way slabs. The average ratio β is about 1.35 for floor slabs, rather than the value 1.2 used for beams. Accordingly it would be consistent to reduce the maximum values for z by the factor 1.2/1.35.

The numerical limitations of z = **175 and 145** kips/in. for interior and exterior exposure, respectively, correspond to limiting crack widths of 0.016 and 0.013 in.

The effective tension area of concrete surrounding the principal reinforcement is defined as having the same centroid as the reinforcement. Moreover, this area is to be bounded by the surfaces of the cross section and a straight line parallel to the neutral axis. Computation of the effective area per bar, A (see notation definition), is illustrated by the example shown in Fig. R10.6.4 in which the centroid of the main reinforcement is located 3.64 in. from the bottom of the beam. The effective tension area is then taken as twice 3.64 in. times the beam width $\boldsymbol{b}$. Divided by the number of bars, this gives 17.6 in.^{2} per bar.

R10.6.5 — Although a number of studies have been conducted, clear experimental evidence is not available regarding the crack width beyond which a corrosion danger exists. Exposure tests indicate that concrete quality, adequate compaction, and ample concrete cover may be of greater importance for corrosion protection than crack width at the concrete surface. The limiting values for z were, therefore, chosen primarily to give reasonable reinforcement details in terms of practical experiences with existing structures.

R10.6.6 — In major T-beams, distribution of the negative reinforcement for control of cracking must take into account two considerations: (1) wide spacing of the reinforcement across the full effective width of flange may cause some wide cracks to form in the slab near the web and, (2) close spacing near the web leaves the outer regions of the flange unprotected. The $^1/_{10}$ limitation is to guard against too wide a spacing, with some additional reinforcement required to protect the outer portions of the flange.

R10.6.7 — For relatively deep flexural members, some reinforcement should be placed near the vertical faces in the tension zone to control cracking in the web. Without such auxiliary steel, the width of the cracks in the web may greatly exceed the crack widths at the level of the flexural tension reinforcement.

The requirements for skin reinforcement were modified in the 1989 edition of the code, as the previous requirements were found to be inadequate in some cases. See Reference 10.16. For lightly reinforced members, these requirements may be reduced to one-half of the main flexural reinforcement. Where the provisions for deep beams, walls, or precast panels require more steel, those provisions (along with their spacing requirements) will govern.

CODE

10.7 — Deep flexural members

10.7.1 — Flexural members with overall depth to clear span ratios greater than $^2/_5$ for continuous spans, or $^4/_5$ for simple spans, shall be designed as deep flexural members taking into account nonlinear distribution of strain and lateral buckling. (See also 12.10.6.)

10.7.2 — Shear strength of deep flexural members shall be in accordance with 11.8.

10.7.3 — Minimum flexural tension reinforcement shall conform to 10.5.

10.7.4 — Minimum horizontal and vertical reinforcement in the side faces of deep flexural members shall be the greater of the requirements of 11.8.8, 11.8.9, and 11.8.10 or 14.3.2 and 14.3.3.

10.8 — Design dimensions for compression members

10.8.1 — Isolated compression member with multiple spirals

Outer limits of the effective cross section of a compression member with two or more interlocking spirals shall be taken at a distance outside the extreme limits of the spirals equal to the minimum concrete cover required by 7.7.

10.8.2 — Compression member built monolithically with wall

Outer limits of the effective cross section of a spirally reinforced or tied reinforced compression member built monolithically with a concrete wall or pier shall be taken not greater than $1^1/_2$ in. outside the spiral or tie reinforcement.

10.8.3 — Equivalent circular compression member

As an alternative to using the full gross area for design of a compression member with a square, octagonal, or other shaped cross section, it shall be permitted to use a circular section with a diameter equal to the least lateral dimension of the actual shape. Gross area considered, required percentage of reinforcement, and design strength shall be based on that circular section.

10.8.4 — Limits of section

For a compression member with a cross section larger than required by considerations of loading, it shall be permitted to base the minimum reinforcement and

COMMENTARY

R10.7 — Deep flexural members

The code does not contain detailed requirements for designing deep beams for flexure except that nonlinearity of strain distribution and lateral buckling must be considered.

Suggestions for the design of deep beams for flexure are given in References 10.17, 10.18, and 10.19.

R10.8 — Design dimensions for compression members

With the 1971 edition of the ACI Building Code, minimum sizes for compression members were eliminated to allow wider utilization of reinforced concrete compression members in smaller size and lightly loaded structures, such as low rise residential and light office buildings. The engineer should recognize the need for careful workmanship, as well as the increased significance of shrinkage stresses with small sections.

R10.8.2, R10.8.3, R10.8.4 — For column design,[10.20] the code provisions for quantity of reinforcement, both vertical and spiral, are based on the gross column area and core area, and the design strength of the column is based on the gross area of the column section. In some cases, however, the gross area is larger than necessary to carry the factored load. The basis of 10.8.2, 10.8.3, and 10.8.4 is that it is satisfactory to design a column of sufficient size to carry the factored load and then simply add concrete around the designed section without increasing the reinforcement to meet the minimum percentages required by 10.9.1. The additional concrete must not be considered as carrying load; however, the effects of the additional concrete on member stiffness must be included in the structural analysis. The effects of the additional concrete also must be considered in design of the other parts of the structure that interact with the oversize member.

CODE

strength on a reduced effective area A_g not less than one-half the total area. This provision shall not apply in regions of high seismic risk.

10.9 — Limits for reinforcement of compression members

10.9.1 — Area of longitudinal reinforcement for noncomposite compression members shall be not less than 0.01 nor more than 0.08 times gross area A_g of section.

COMMENTARY

UBC ZONES 3,4

R10.9 — Limits for reinforcement of compression members

R10.9.1 — This section prescribes the limits on the amount of longitudinal reinforcement for noncomposite compression members. If the use of high reinforcement ratios would involve practical difficulties in the placing of concrete, a lower percentage and hence a larger column, or higher strength concrete or reinforcement (see R9.4) should be considered. The percentage of reinforcement in columns should usually not exceed 4 percent if the column bars are required to be lap spliced.

Minimum reinforcement. Since the design methods for columns incorporate separate terms for the load carried by concrete and by reinforcement, it is necessary to specify some minimum amount of reinforcement to ensure that only reinforced concrete columns are designed by these procedures. Reinforcement is necessary to provide resistance to bending, which may exist whether or not computations show that bending exists, and to reduce the effects of creep and shrinkage of the concrete under sustained compressive stresses. Tests have shown that creep and shrinkage tend to transfer load from the concrete to the reinforcement, with a consequent increase in stress in the reinforcement, and that this increase is greater as the ratio of reinforcement decreases. Unless a lower limit is placed on this ratio, the stress in the reinforcement may increase to the yield level under sustained service loads. This phenomenon was emphasized in the report of ACI Committee 105[10.21] and minimum reinforcement ratios of 0.01 and 0.005 were recommended for spiral and tied columns, respectively. However, in all editions of the code since 1936, the minimum ratio has been 0.01 for both types of laterally reinforced columns.

Maximum reinforcement. Extensive tests of the ACI column investigation[10.21] included reinforcement ratios no greater than 0.06. Although other tests with as much as 17 percent reinforcement in the form of bars produced results similar to those obtained previously, it is necessary to note that the loads in these tests were applied through bearing plates on the ends of the columns and the problem of transferring a proportional amount of the load to the bars was thus minimized or avoided. Maximum ratios of 0.08 and 0.03 were recommended by ACI Committee 105[10.21] for spiral and tied columns, respectively. In the 1936 ACI Building Code, these limits were made 0.08 and 0.04, respectively. In the 1956 code, the limit for tied columns with bending was raised to 0.08. Since the 1963 code, it has been required that bending be considered in the design of all

CODE

10.9.2 — Minimum number of longitudinal bars in compression members shall be 4 for bars within rectangular or circular ties, 3 for bars within triangular ties, and 6 for bars enclosed by spirals conforming to 10.9.3.

10.9.3 — Ratio of spiral reinforcement ρ_s shall be not less than the value given by

$$\rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_y} \qquad (10\text{-}6)$$

where f_y is the specified yield strength of spiral reinforcement but not more than 60,000 psi.

10.10 — Slenderness effects in compression members

*COMPLETE REWRITE

10.10.1 — Except as allowed in 10.10.2, the design of compression members, restraining beams, and other supporting members shall be based on the factored forces and moments from a second-order analysis considering material nonlinearity and cracking, as well as the effects of member curvature and lateral drift,

COMMENTARY

columns, and the maximum ratio of 0.08 has been applied to both types of columns. This limit can be considered a practical maximum for reinforcement in terms of economy and requirements for placing.

R10.9.2 — For compression members, a minimum of four longitudinal bars are required when bars are enclosed by rectangular or circular ties. For other shapes, one bar should be provided at each apex or corner and proper lateral reinforcement provided. For example, tied triangular columns require three longitudinal bars, one at each apex of the triangular ties. For bars enclosed by spirals, six bars are required.

When the number of bars in a circular arrangement is less than eight, the orientation of the bars will affect the moment strength of eccentrically loaded columns and must be considered in design.

R10.9.3 — The effect of spiral reinforcement in increasing the load-carrying strength of the concrete within the core is not realized until the column has been subjected to a load and deformation sufficient to cause the concrete shell outside the core to spall off. The amount of spiral reinforcement required by Eq. (10-6) is intended to provide additional load-carrying strength for concentrically loaded columns equal to or slightly greater than the strength lost when the shell spalls off. This principle was recommended by ACI Committee 105[10.21] and has been a part of the code since 1963. The derivation of Eq. (10-6) is given in the ACI Committee 105 report. Tests and experience show that columns containing the amount of spiral reinforcement required by this section exhibit considerable toughness and ductility.

R10.10 — Slenderness effects in compression members

Provisions for slenderness effects in compression members and frames were revised in the 1995 code to better recognize the use of second-order analyses and to improve the arrangement of the provisions dealing with braced and sway frames.[10.22] The use of a refined nonlinear second-order analysis is permitted in 10.10.1. Sections 10.11, 10.12, and 10.13 present an approximate design method based on the traditional moment magnifier method. For sway frames, the magnified sway moment $\delta_s M_s$ may be calculated using a second-order elastic analysis, by an approximation to such an analysis, or by the traditional sway moment magnifier.

R10.10.1 — Two limits are placed on the use of the refined second-order analysis. First, the structure which is analyzed must have members similar to those in the final structure. If the members in the final structure have cross-sectional dimensions more than 10 percent different from those assumed in the analysis, new member properties should be

CODE

duration of the loads, shrinkage and creep, and interaction with the supporting foundation. The dimensions of each member cross section used in the analysis shall be within 10 percent of the dimensions of the members shown on the design drawings or the analysis shall be repeated. The analysis procedure shall have been shown to result in prediction of strength in substantial agreement with the results of comprehensive tests of columns in statically indeterminate reinforced concrete structures.

Analysis Procedure Predicts test Results
Within 15%

10.10.2 — As an alternate to the procedure prescribed in 10.10.1, it shall be permitted to base the design of compression members, restraining beams, and other supporting members on axial forces and moments from the analyses described in 10.11.

10.11 — Magnified moments — General

10.11.1 — The factored axial forces P_u, the factored moments M_1 and M_2 at the ends of the column, and, where required, the relative lateral story deflections Δ_o shall be computed using an elastic first-order frame analysis with the section properties determined taking into account the influence of axial loads, the presence of cracked regions along the length of the member, and effects of duration of the loads. Alternatively, it shall be permitted to use the following properties for the members in the structure:

(a) Modulus of elasticity	E_c from 8.5.1
(b) Moments of inertia	
Beams	0.35 I_g
Columns	0.70 I_g
Walls—Uncracked	0.70 I_g
—Cracked	0.35 I_g
Flat plates and flat slabs	0.25 I_g
(c) Area	1.0 A_g

COMMENTARY

computed and the analysis repeated. Second, the refined second-order analysis procedure should have been shown to predict ultimate loads within 15 percent of those reported in tests of indeterminate reinforced concrete structures. At the very least, the comparison should include tests of columns in planar braced frames, sway frames, and frames with varying column stiffnesses. To allow for variability in the actual member properties and in the analysis, the member properties used in analysis should be multiplied by a stiffness reduction factor ϕ_K less than one. For consistency with the second-order analysis in 10.13.4.1, the stiffness reduction factor ϕ_K can be taken as 0.80. The concept of a stiffness reduction factor ϕ_K is discussed in R10.12.3.

R10.10.2 — As an alternate to the refined second-order analysis of 10.10.1, design may be based on elastic analyses and the moment magnifier approach.[10.23,10.24] For sway frames the magnified sway moments may be calculated using a second-order elastic analysis based on realistic stiffness values. See R10.13.4.1.

R10.11 — Magnified moments — General

This section describes an approximate design procedure which uses the moment magnifier concept to account for slenderness effects. Moments computed using an ordinary first-order frame analysis are multiplied by a "moment magnifier" which is a function of the factored axial load P_u and the critical buckling load P_c for the column. Nonsway and sway frames are treated separately in 10.12 and 10.13. Provisions applicable to both non-sway and sway columns are given in 10.11. A first-order frame analysis is an elastic analysis which does not include the internal force effects resulting from deflections.

R10.11.1 — The stiffnesses EI used in an elastic analysis used for strength design should represent the stiffnesses of the members immediately prior to failure. This is particularly true for a second-order analysis which should predict the lateral deflections at loads approaching ultimate. The EI values should not be based totally on the moment-curvature relationship for the most highly loaded section along the length of each member. Instead, they should correspond to the moment-end rotation relationship for a complete member.

The alternative values of E, I, and A given in 10.11.1 have been chosen from the results of frame tests and analyses and include an allowance for the variability of the computed deflections. The modulus of elasticity E is based on the specified concrete strength while the sway deflections are a function of the average concrete strength which is higher. The moments of inertia were taken as $^7/_8$ of those in Reference 10.25. These two effects result in an overestimation of the second-order deflections in the order of 20 to 25 percent, corresponding to an implicit stiffness reduction factor ϕ_K of

CODE

The moments of inertia shall be divided by $(1 + \beta_d)$

(a) When sustained lateral loads act, or

(b) For stability checks made in accordance with 10.13.6.

10.11.2 — It shall be permitted to take the radius of gyration r equal to 0.30 times the overall dimension in the direction stability is being considered for rectangular compression members and 0.25 times the diameter for circular compression members. For other shapes, it shall be permitted to compute the radius of gyration for the gross concrete section.

COMMENTARY

0.80 to 0.85 on the stability calculation. The concept of a stiffness reduction factor ϕ_K is discussed in R10.12.3

The moment of inertia of T-beams should be based on the effective flange width defined in 8.10. It is generally sufficiently accurate to take I_g of a T-beam as two times the I_g for the web, $2(b_w h^3/12)$.

If the factored moments and shears from an analysis based on the moment of inertia of a wall taken equal to $0.70I_g$ indicate that the wall will crack in flexure, based on the modulus of rupture, the analysis should be repeated with $I = 0.35I_g$ in those stories where cracking is predicted at factored loads.

The alternative values of the moments of inertia given in 10.11.1 were derived for nonprestressed members. For prestressed members, the moments of inertia may differ from the values in 10.11.1 depending on the amount, location, and type of the reinforcement and the degree of cracking prior to ultimate. The stiffness values for prestressed concrete members should include an allowance for the variability of the stiffnesses.

Sections 10.11 through 10.13 provide requirements for strength and assume frame analyses will be carried out using factored loads. Analyses of deflections, vibrations, and building periods are needed at various service (unfactored) load levels[10.26,10.27] to determine the serviceability of the structure and to estimate the wind forces in wind tunnel laboratories. The seismic base shear is also based on the service load periods of vibration. The magnified service loads and deflections by a second-order analysis should also be computed using service loads. The moments of inertia of the structural members in the service load analyses should, therefore, be representative of the degree of cracking at the various service load levels investigated. Unless a more accurate estimate of the degree of cracking at design service load level is available, it is satisfactory to use $1/0.70 = 1.43$ times the moments of inertia given in 10.11.1 for service load analyses.

The last sentence in 10.11.1 refers to the unusual case of sustained lateral loads. Such a case might exist, for example, if there were permanent lateral loads resulting from unequal earth pressures on two sides of a building.

CODE

10.11.3 — Unsupported length of compression members

10.11.3.1 — The unsupported length ℓ_u of a compression member shall be taken as the clear distance between floor slabs, beams, or other members capable of providing lateral support in the direction being considered.

10.11.3.2 — Where column capitals or haunches are present, the unsupported length shall be measured to the lower extremity of the capital or haunch in the plane considered.

10.11.4 — Columns and stories in structures shall be designated as non-sway or sway columns or stories. The design of columns in non-sway frames or stories shall be based on 10.12. The design of columns in sway frames or stories shall be based on 10.13.

10.11.4.1 — It shall be permitted to assume a column in a structure is non-sway if the increase in column end moments due to second-order effects does not exceed 5 percent of the first-order end moments.

10.11.4.2 — It also shall be permitted to assume a story within a structure is non-sway if:

$$Q = \frac{\Sigma P_u \Delta_o}{V_u \ell_c} \qquad (10\text{-}7)$$

is less than or equal to 0.05, where ΣP_u and V_u are the total vertical load and the story shear, respectively, in the story in question and Δ_o is the first-order relative deflection between the top and bottom of that story due to V_u.

10.11.5 — Where an individual compression member in the frame has a slenderness $k\ell_u/r$ of more than 100, 10.10.1 shall be used to compute the forces and moments in the frame.

10.11.6 — For compression members subject to bending about both principal axes, the moment about each axis shall be magnified separately based on the conditions of restraint corresponding to that axis.

COMMENTARY

	Non Sway	Sway
Col	$\frac{M_{2nd}-M_{1st}}{M_{1st}} \leq 0.05$	>0.05
Story	$Q = \frac{\sum P_u \Delta_o}{V_c \ell_c} \leq 0.05$	>0.05

R10.11.4 — The moment magnifier design method requires the designer to distinguish between non-sway frames which are designed according to 10.12 and sway frames which are designed according to 10.13. Frequently this can be done by inspection by comparing the total lateral stiffness of the columns in a story to that of the bracing elements. A compression member may be assumed braced by inspection if it is located in a story in which the bracing elements (shearwalls, shear trusses, or other types of lateral bracing) have such substantial lateral stiffness to resist the lateral deflections of the story that any resulting lateral deflection is not large enough to affect the column strength substantially. If not readily apparent by inspection, 10.11.4.1 and 10.11.4.2 give two possible ways of doing this. In 10.11.4.1, a story in a frame is said to be non-sway if the increase in the lateral load moments resulting from $P\Delta$ effects does not exceed 5 percent of the first-order moments.[10.25] Section 10.11.4.2 gives an alternative method of determining this based on the stability index for a story Q. In computing Q, ΣP_u should correspond to the lateral loading case for which ΣP_u is greatest. It should be noted that a frame may contain both non-sway and sway stories. This test would not be suitable if V_u were zero.

If the lateral load deflections of the frame have been computed using service loads and the service load moments of inertia given in 10.11.1, it is permissible to compute Q in Eq. (10-7) using 1.2 times the sum of the service gravity loads, the service load story shear, and 1.43 times the first-order service load story deflections.

R10.11.5 — An upper limit is imposed on the slenderness ratio of columns designed by the moment magnifier method of 10.11 to 10.13. No similar limit is imposed if design is carried out according to 10.10.1. The limit of $k\ell_u/r$ = **100** represents the upper range of actual tests of slender compression members in frames.

R10.11.6 — When biaxial bending occurs in a compression member, the computed moments about each of the principal axes must be magnified. The magnification factors δ are computed considering the buckling load P_c about each axis

CODE

10.12 — Magnified moments — Non-sway frames

10.12.1 — For compression members in non-sway frames, the effective length factor ***k*** shall be taken as 1.0, unless analysis shows that a lower value is justified. The calculation of ***k*** shall be based on the ***E*** and ***I*** values used in 10.11.1.

COMMENTARY

separately based on the appropriate effective length $k\ell_u$ and the stiffness ***EI***. If the buckling capacities are different about the two axes different magnification factors will result.

R10.12 — Magnified moments — Non-sway frames

R10.12.1 — The moment magnifier equations were derived for hinged end columns and must be modified to account for the effect of end restraints. This is done by using an "effective length" $k\ell_u$ in the computation of P_c.

The primary design aid to estimate the effective length factor ***k*** is the Jackson and Moreland Alignment Charts (Fig. R10.12.1) which allow a graphical determination of ***k*** for a column of constant cross section in a multibay frame.[10.28,10.29]

The effective length is a function of the relative stiffness at each end of the compression member. Studies have indicated that the effects of varying beam and column reinforcement percentages and beam cracking should be considered in determining the relative end stiffnesses. In determining ψ for use in evaluating the effective length factor ***k***, the rigidity of the flexural members may be calculated on the basis of $0.35I_g$ for flexural members to account for the effect of cracking and reinforcement on relative stiffness, and $0.70I_g$ for compression members.

The following simplified equations for computing the effective length factors for braced and unbraced members may be used. Eq. (A), (B), and (E) are taken from the 1972 British Standard Code of Practice.[10.30,10.31] Eq. (C) and (D) for unbraced members were developed in Reference 10.29.

For braced compression members, an upper bound to the effective length factor may be taken as the smaller of the following two expressions:

$$k = 0.7 + 0.05\,(\psi_A + \psi_B) \leq 1.0 \qquad \text{(A)}$$

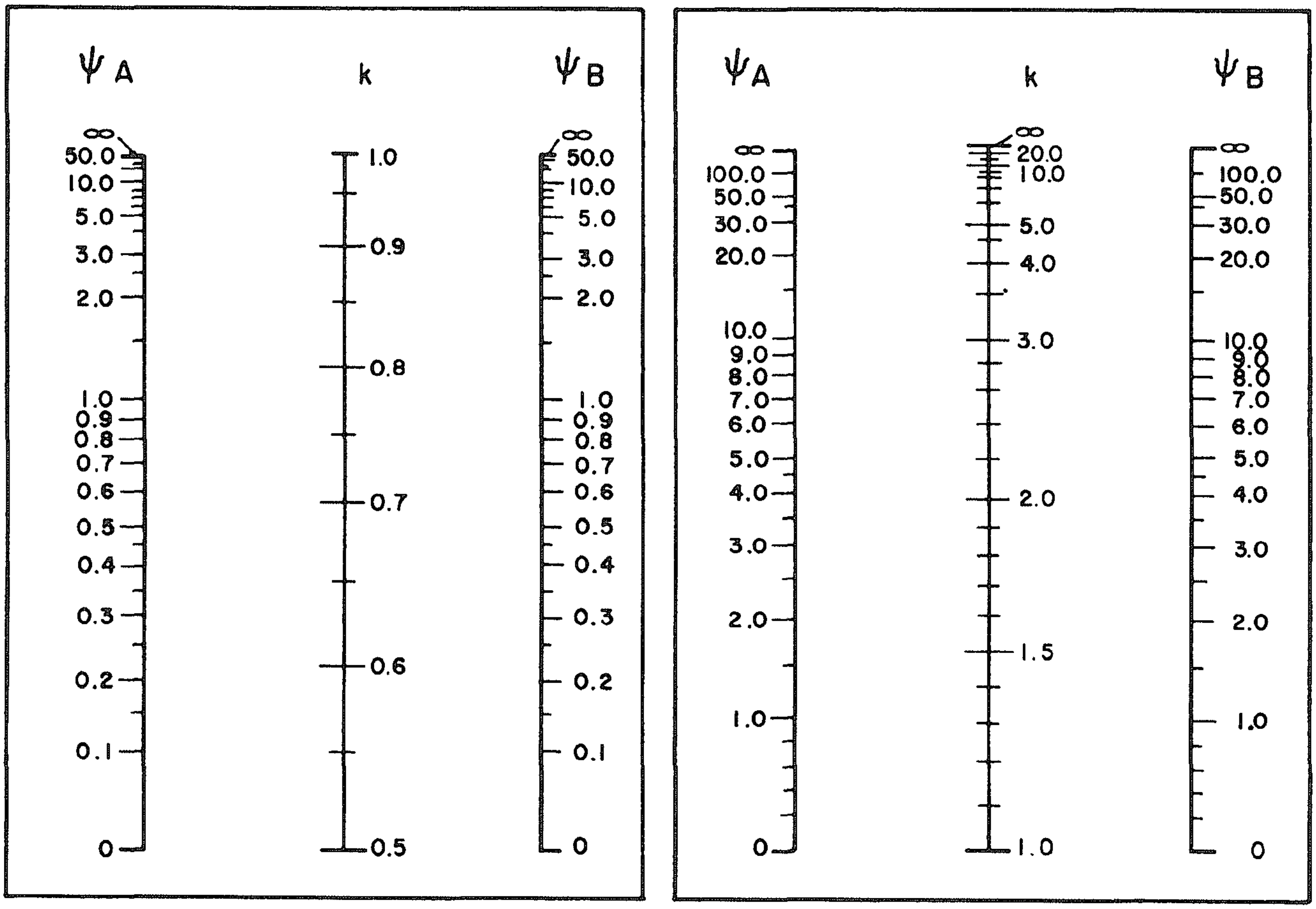
$$k = 0.85 + 0.05\psi_{min} \leq 1.0 \qquad \text{(B)}$$

where ψ_A and ψ_B are the values of ψ at the two ends of the column and ψ_{min} is the smaller of the two values.

For unbraced compression members restrained at both ends, the effective length factor may be taken as:

For $\psi_m < 2$

$$k = \frac{20 - \psi_m}{20}\sqrt{1 + \psi_m} \qquad \text{(C)}$$



(a)

Braced Frames

(b)

Unbraced Frames

ψ = ratio of $\Sigma(EI/\ell_c)$ of compression members to $\Sigma(EI/\ell)$ of flexural members in a plane at one end of a compression member

ℓ = span length of flexural member measured center-to-center of joints

Fig. R10.12.1—Effective length factors, k

For $\psi_m \geq 2$

$$k = 0.9\sqrt{1+\psi_m} \qquad \text{(D)}$$

where ψ_m is the average of the ψ-values at the two ends of the compression member.

For unbraced compression members hinged at one end, the effective length factor may be taken as:

$$k = 2.0 + 0.3\psi \qquad \text{(E)}$$

where ψ is the value at the restrained end.

The use of the charts in Fig. R10.12.1, or the equations in this section, may be considered as satisfying the requirements of the code to justify k less than 1.0.

CODE

10.12.2 — In non-sway frames it shall be permitted to ignore slenderness effects for compression members which satisfy:

$$\frac{k\ell_u}{r} \le 34 - 12\,(M_1/M_2) \qquad (10\text{-}8)$$

where M_1/M_2 is not taken less than –0.5. The term M_1/M_2 is positive if the column is bent in single curvature.

10.12.3 — Compression members shall be designed for the factored axial load P_u and the moment amplified for the effects of member curvature M_c as follows:

$$M_c = \delta_{ns} M_2 \qquad (10\text{-}9)$$

where

δ_{ns} - Non Sway Magnifier

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \ge 1.0 \qquad (10\text{-}10)$$

Euler's Buckling Eq

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \qquad (10\text{-}11)$$

EI shall be taken as

$$EI = \frac{(0.2E_cI_g + E_sI_{se})}{1 + \beta_d} \qquad (10\text{-}12)$$

or

$$EI = \frac{0.4E_cI_g}{1 + \beta_d} \qquad (10\text{-}13)$$

Optional 10-12 OR 10-13

$$\beta_d = \frac{\text{Max Fact' Axial DL}}{\text{Total Fact Axial LOAD}}$$

COMMENTARY

R10.12.2 — Eq. (10-8) is derived from Eq. (10-10) assuming that a 5 percent increase in moments due to slenderness is acceptable.[10.23] The derivation did not include ϕ in the calculation of the moment magnifier. As a first approximation, k may be taken equal to 1.0 in Eq. (10-8).

R10.12.3 — The ϕ-factors used in the design of slender columns represent two different sources of variability. First, the stiffness reduction ϕ-factors in the magnifier equations in the 1989 and earlier codes were intended to account for the variability in the stiffness EI and the moment magnification analysis. Second, the variability of the strength of the cross section is accounted for by strength reduction ϕ-factors of 0.70 for tied columns and 0.75 for spiral columns. Studies reported in Reference 10.32 indicate that the stiffness reduction factor ϕ_K, and the cross-sectional strength reduction ϕ-factors do not have the same values, contrary to the assumption in the 1989 and earlier codes. These studies suggest the stiffness reduction factor ϕ_K for an isolated column should be 0.75 for both tied and spiral columns. The 0.75 factors in Eq. (10-10) and (10-19) are stiffness reduction factors ϕ_K and replace the ϕ-factors in these equations in the 1989 and earlier codes. This has been done to avoid confusion between a stiffness reduction factor ϕ_K in Eq. (10-10) and (10-19), and the cross-sectional strength reduction ϕ-factors.

In defining the critical load, the main problem is the choice of a stiffness EI which reasonably approximates the variations in stiffness due to cracking, creep, and the nonlinearity of the concrete stress-strain curve. Eq. (10-12) was derived for small eccentricity ratios and high levels of axial load where the slenderness effects are most pronounced.

Creep due to sustained load will increase the lateral deflections of a column and hence the moment magnification. This is approximated for design by reducing the stiffness EI used to compute P_c and hence δ_{ns} by dividing EI by $(1 + \beta_d)$. Both the concrete and steel terms in Eq. (10-12) are divided by $(1 + \beta_d)$. This reflects the premature yielding of steel in columns subjected to sustained load.

Either Eq. (10-12) or (10-13) may be used to compute EI. Eq. (10-13) is a simplified approximation to Eq. (10-12). It is less accurate than Eq. (10-12).[10.33] Eq. (10-13) may be simplified further by assuming $\beta_d = 0.6$. When this is done Eq. (10-13) becomes

$$EI = 0.25E_cI_g \qquad (F)$$

The term β_d is defined differently for non-sway and sway

CODE

10.12.3.1 — For members without transverse loads between supports, C_m shall be taken as

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4 \qquad (10\text{-}14)$$

where M_1/M_2 is positive if the column is bent in single curvature. For members with transverse loads between supports, C_m shall be taken as 1.0.

10.12.3.2 — The factored moment M_2 in Eq. (10-9) shall not be taken less than

$$M_{2,min} = P_u\,(0.6 + 0.03h) \qquad (10\text{-}15)$$

h is not height

about each axis separately, where 0.6 and h are in inches. For members for which $M_{2,min}$ exceeds M_2, the value of C_m in Eq. (10-14) shall either be taken equal to 1.0, or shall be based on the ratio of the computed end moments M_1 and M_2.

10.13 — Magnified moments — Sway frames

COMMENTARY

frames. See 10.0. For non-sway frames, β_d is the ratio of the maximum factored axial dead load to the total factored axial load.

R10.12.3.1 — The factor C_m is an equivalent moment correction factor. The derivation of the moment magnifier assumes that the maximum moment is at or near midheight of the column. If the maximum moment occurs at one end of the column, design must be based on an "equivalent uniform moment" C_mM_2 which would lead to the same maximum moment when magnified.[10.23]

In the case of compression members that are subjected to transverse loading between supports, it is possible that the maximum moment will occur at a section away from the end of the member. If this occurs, the value of the largest calculated moment occurring anywhere along the member should be used for the value of M_2 in Eq. (10-9). In accordance with the last sentence of 10.12.3.1, C_m must be taken as 1.0 for this case.

R10.12.3.2 — In this code, slenderness is accounted for by magnifying the column end moments. If the factored column moments are very small or zero, the design of slender columns must be based on the minimum eccentricity given in this section. It is not intended that the minimum eccentricity be applied about both axes simultaneously.

The factored column end moments from the structural analysis are used in Eq. (10-14) in determining the ratio M_1/M_2 for the column when the design must be based on minimum eccentricity. This eliminates what would otherwise be a discontinuity between columns with computed eccentricities less than the minimum eccentricity and columns with computed eccentricities equal to or greater than the minimum eccentricity.

R10.13 — Magnified moments — Sway frames

The design of sway frames for slenderness has been revised in the 1995 ACI Building Code. The revised procedure consists of three steps:

(1) The magnified sway moments δ_sM_s are computed. This should be done in one of three ways. First, a second-order elastic frame analysis may be used (10.13.4.1). Second, an approximation to such analysis (10.13.4.2) may be used. The third option is to use the sway magnifier δ_s from previous editions of the ACI Building Code (10.13.4.3).

(2) The magnified sway moments δ_sM_s are added to the unmagnified non-sway moment M_{ns} at each end of each column (10.13.3). The non-sway moments may be computed using a first-order elastic analysis.

CODE

10.13.1 — For compression members not braced against sidesway, the effective length factor k shall be determined using E and I values in accordance with 10.11.1 and shall be greater than 1.0.

10.13.2 — For compression members not braced against sidesway, effects of slenderness may be neglected when $k\ell_u/r$ is less than 22.

10.13.3 — The moments M_1 and M_2 at the ends of an individual compression member shall be taken as

$$M_1 = M_{1ns} + \delta_s M_{1s} \qquad (10\text{-}16)$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \qquad (10\text{-}17)$$

where $\delta_s M_{1s}$ and $\delta_s M_{2s}$ shall be computed according to 10.13.4.

10.13.4 — Calculation of $\delta_s M_s$

10.13.4.1 — The magnified sway moments $\delta_s M_s$ shall be taken as the column end moments calculated using a second-order elastic analysis based on the member stiffnesses given in 10.11.1.

COMMENTARY

(3) If the column is slender and the loads on it are high, it is checked to see whether the moments at points between the ends of the column exceed those at the ends of the column. As specified in 10.13.5 this is done using the non-sway frame magnifier δ_{ns} with P_c computed assuming $k = 1.0$ or less.

R10.13.1 — See R10.12.1.

R10.13.3 — The analysis described in this section deals only with plane frames subjected to loads causing deflections in that plane. If torsional displacements are significant, a three-dimensional second-order analysis should be used.

10.13.4 — Calculation of $\delta_s M_s$

R10.13.4.1 — A second-order analysis is a frame analysis which includes the internal force effects resulting from deflections. When a second-order elastic analysis is used to compute $\delta_s M_s$ the deflections must be representative of the stage immediately prior to the ultimate load. For this reason the reduced EI values given in 10.11.1 must be used in the second-order analysis.

The term β_d is defined differently for non-sway and sway frames. See 10.0. Sway deflections due to short-term loads such as wind or earthquake are a function of the short-term stiffness of the columns following a period of sustained gravity load. For this case the definition of β_d in 10.0 gives $\beta_d = 0$. In the unusual case of a sway frame where the lateral loads are sustained, β_d will not be zero. This might occur if a building on a sloping site is subjected to earth pressure on one side but not on the other.

In a second-order analysis the axial loads in all columns which are not part of the lateral load resisting elements and depend on these elements for stability must be included.

In the 1989 and earlier codes, the moment magnifier equations for δ_b and δ_s included a stiffness reduction factor ϕ_K to cover the variability in the stability calculation. The second-order analysis method is based on the values of E and I

CODE

10.13.4.2 — Alternatively it shall be permitted to calculate $\delta_s M_s$ as

$$\delta_s M_s = \frac{M_s}{1-Q} \geq M_s \qquad (10\text{-}18)$$

Q = Stability Index

If δ_s calculated in this way exceeds 1.5, $\delta_s M_s$ shall be calculated using 10.13.4.1 or 10.13.4.3.

10.13.4.3 — Alternatively it shall be permitted to calculate the magnified sway moment $\delta_s M_s$ as

$$\delta_s M_s = \frac{M_s}{1 - \frac{\Sigma P_u}{0.75 \Sigma P_c}} \geq M_s \qquad (10\text{-}19)$$

where ΣP_u is the summation for all the vertical loads in a story and ΣP_c is the summation for all sway resisting columns in a story. P_c is calculated using Eq. (10-11) using k from 10.13.1 and EI from Eq. (10-12) or Eq. (10-13).

COMMENTARY

from 10.11.1. These lead to a 20 to 25 percent overestimation of the lateral deflections which corresponds to a stiffness reduction factor ϕ_K between 0.80 and 0.85 on the $P\Delta$ moments. No additional ϕ-factor is needed in the stability calculation. Once the moments are established, selection of the cross sections of the columns involves the strength reduction factors ϕ from 9.3.2.2.

R10.13.4.2 — The iterative $P\Delta$ analysis for second-order moments can be represented by an infinite series. The solution of this series is given by Eq. (10-18).[10.25] Reference 10.34 shows that Eq. (10-18) closely predicts the second-order moments in an unbraced frame until δ_s exceeds 1.5.

The $P\Delta$ moment diagrams for deflected columns are curved, with Δ related to the deflected shape of the columns. Eq. (10-18) and most commercially available second-order frame analyses have been derived assuming that the $P\Delta$ moments result from equal and opposite forces of $P\Delta/\ell_c$ applied at the bottom and top of the story. These forces give a straight line $P\Delta$ moment diagram. The curved $P\Delta$ moment diagrams lead to lateral displacements in the order of 15 percent larger than those from the straight line $P\Delta$ moment diagrams. This effect can be included in Eq. (10-18) by writing the denominator as $(1-1.15Q)$ rather than $(1-Q)$. The 1.15 factor has been left out of Eq. (10-18) to maintain consistency with commercially available computer programs.

If deflections have been calculated using service loads, Q in Eq. (10-18) should be calculated in the manner explained in R10.11.4.

In the 1989 and earlier codes, the moment magnifier equations for δ_b and δ_s included a stiffness reduction factor ϕ_K to cover the variability in the stability calculation. The Q factor analysis is based on deflections calculated using the values of E and I from 10.11.1 which include the equivalent of a stiffness reduction factor ϕ_K as explained in R10.13.4.1. As a result, no additional ϕ-factor is needed in the stability calculation. Once the moments are established using Eq. (10-18), selection of the cross sections of the columns involves the strength reduction factors ϕ from 9.3.2.2.

R10.13.4.3 — To check the effects of story stability, δ_s is computed as an averaged value for the entire story based on use of $\Sigma P_u / \Sigma P_c$. This reflects the interaction of all sway resisting columns in the story in the $P\Delta$ effects since the lateral deflection of all columns in the story must be equal in the absence of torsional displacements about a vertical axis. In addition, it is possible that a particularly slender individual column in an unbraced frame could have substantial midheight deflections even if adequately braced against lateral end deflections by other columns in the story. Such a column will have ℓ_u/r greater than the value given in Eq. (10-20) and would have to be checked using 10.13.5.

CODE

10.13.5 — If an individual compression member has

$$\frac{\ell_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f_c' A_g}}} \qquad (10\text{-}20)$$

it shall be designed for the factored axial load P_u and the moment M_c calculated using 10.12.3 in which M_1 and M_2 are computed in accordance with 10.13.3, β_d as defined for the load combination under consideration, and k as defined in 10.12.1.

10.13.6 — In addition to load cases involving lateral loads, the strength and stability of the structure as a whole under factored gravity loads shall be considered.

(a) When $\delta_s M_s$ is computed from 10.13.4.1, the ratio of second-order lateral deflections to first-order lateral deflections for 1.4 dead load and 1.7 live load plus lateral load applied to the structure shall not exceed 2.5.

(b) When $\delta_s M_s$ is computed according to 10.13.4.2, the value of Q computed using ΣP_u for 1.4 dead load plus 1.7 live load shall not exceed 0.60.

(c) When $\delta_s M_s$ is computed from 10.13.4.3, δ_s computed using ΣP_u and ΣP_c corresponding to the factored dead and live loads shall be positive and shall not exceed 2.5.

In cases (a), (b), and (c) above, β_d shall be taken as the ratio of the maximum factored sustained axial load to the total factored axial load.

COMMENTARY

If the lateral load deflections involve a significant torsional displacement, the moment magnification in the columns farthest from the center of twist may be underestimated by the moment magnifier procedure. In such cases a three-dimensional second-order analysis should be considered.

The 0.75 in the denominator of Eq. (10-19) is a stiffness reduction factor ϕ_K as explained in R10.12.3.

In the calculation of EI, β_d will normally be zero for an unbraced frame because the lateral loads are generally of short duration. (See R10.13.4.1).

R10.13.5 — The unmagnified non-sway moments at the ends of the columns are added to the magnified sway moments at the same points. Generally one of the resulting end moments is the maximum moment in the column. However, for slender columns with high axial loads the point of maximum moment may be between the ends of the column so that the end moments are no longer the maximum moments. If ℓ_u/r is less than the value given by Eq. (10-20) the maximum moment at any point along the height of such a column will be less than 1.05 times the maximum end moment. When ℓ_u/r exceeds the value given by Eq. (10-20), the maximum moment will occur at a point between the ends of the column and will exceed the maximum end moment by more than 5 percent.[10.22] In such a case the maximum moment is calculated by magnifying the end moments using Eq. (10-9).

R10.13.6 — The possibility of sidesway instability under gravity loads alone must be investigated. When using second-order analyses to compile $\delta_s M_s$ (10.13.4.1), the frame should be analyzed twice for the case of factored gravity loads plus a lateral load applied to the frame. This load may be the lateral load used in design or it may be a single lateral load applied to the top of the frame. The first analysis should be a first-order analysis, the second analysis should be a second-order analysis. The deflection from the second-order analysis should not exceed 2.5 times the deflection from the first-order analysis. If one story is much more flexible than the others the deflection ratio should be computed in that story. The lateral load should be large enough to give deflections of size that can be compared accurately. In unsymmetrical frames which deflect laterally under gravity loads alone, the lateral load should act in the direction for which it will increase the lateral deflections.

When using 10.13.4.2 to compute $\delta_s M_s$, the value of Q evaluated using factored gravity loads should not exceed 0.60. This is equivalent to δ_s = **2.5**. The values of V_u and Δ_o used to compute Q can result from assuming any real or arbitrary set of lateral loads provided that V_u and Δ_o are both from the same loading. If Q as computed in 10.11.4.2 is 0.2 or less, the stability check in 10.13.6 is satisfied.

CODE

10.13.7 — In sway frames, flexural members shall be designed for the total magnified end moments of the compression members at the joint.

10.14 — Axially loaded members supporting slab system

Axially loaded members supporting a slab system included within the scope of 13.1 shall be designed as provided in Chapter 10 and in accordance with the additional requirements of Chapter 13.

10.15 — Transmission of column loads through floor system

When the specified compressive strength of concrete in a column is greater than 1.4 times that specified for a floor system, transmission of load through the floor system shall be provided by one of the following.

10.15.1 — Concrete of strength specified for the column shall be placed in the floor at the column location. Top surface of the column concrete shall extend 2 ft into the slab from face of column. Column concrete shall be well integrated with floor concrete, and shall be placed in accordance with 6.4.5 and 6.4.6.

COMMENTARY

When $\delta_s M_s$ is computed using Eq. (10-19), an upper limit of 2.5 is placed on δ_s. For higher δ_s values the frame will be very susceptible to variations in EI, foundation rotations, and the like. If δ_s exceeds 2.5 the frame must be stiffened to reduce δ_s. ΣP_u shall include the axial load in all columns and walls including columns which are not part of the lateral load resisting system. The value $\delta_s = 2.5$ is a very high magnifier. It has been chosen to offset the conservatism inherent in the moment magnifier procedure.

The value of β_d should be an overall value for each story calculated as the ratio of the maximum factored sustained axial load in that story to the total factored axial load in that story.

R10.13.7 — The strength of a laterally unbraced frame is governed by the stability of the columns and by the degree of end restraint provided by the beams in the frame. If plastic hinges form in the restraining beam, the structure approaches a mechanism and its axial load capacity is drastically reduced. Section 10.13.7 provides that the designer make certain that the restraining flexural members have the capacity to resist the magnified column moments.

R10.15 — Transmission of column loads through floor system

The requirements of this section are based on a paper on the effect of floor concrete strength on column strength.[10.35] The provisions mean that where the column concrete strength does not exceed the floor concrete strength by more than 40 percent, no special precautions need be taken. For higher column concrete strengths, methods in 10.15.1 or 10.15.2 must be used for corner or edge columns and methods in 10.15.1, 10.15.2, or 10.15.3 for interior columns with adequate restraint on all four sides.

R10.15.1 — Application of the concrete placement procedure described in 10.15.1 requires the placing of two different concrete mixes in the floor system. The lower strength mix must be placed while the higher strength concrete is still plastic and must be adequately vibrated to ensure the concretes are well integrated. This requires careful coordination of the concrete deliveries and possible use of retarders. In some cases, additional inspection services will be required when this procedure is used. It is important that the

CODE

10.15.2 — Strength of a column through a floor system shall be based on the lower value of concrete strength with vertical dowels and spirals as required.

10.15.3 — For columns laterally supported on four sides by beams of approximately equal depth or by slabs, strength of the column may be based on an assumed concrete strength in the column joint equal to 75 percent of column concrete strength plus 35 percent of floor concrete strength.

10.16 — Composite compression members

10.16.1 — Composite compression members shall include all such members reinforced longitudinally with structural steel shapes, pipe, or tubing with or without longitudinal bars.

10.16.2 — Strength of a composite member shall be computed for the same limiting conditions applicable to ordinary reinforced concrete members.

10.16.3 — Any axial load strength assigned to concrete of a composite member shall be transferred to the concrete by members or brackets in direct bearing on the composite member concrete.

10.16.4 — All axial load strength not assigned to concrete of a composite member shall be developed by direct connection to the structural steel shape, pipe, or tube.

COMMENTARY

higher strength concrete in the floor in the region of the column be placed before the lower strength concrete in the remainder of the floor to prevent accidental placing of the low strength concrete in the column area. It is the designer's responsibility to indicate on the drawings where the high and low strength concretes are to be placed.

With the 1983 code, the amount of column concrete to be placed within the floor is expressed as a simple 2-ft extension from face of column. Since the concrete placement requirement must be carried out in the field, it is now expressed in a way that is directly evident to workers. The new requirement will also locate the interface between column and floor concrete farther out into the floor, away from regions of very high shear.

R10.16 — Composite compression members

R10.16.1 — Composite columns are defined without reference to classifications of combination, composite, or concrete-filled pipe column. Reference to other metals used for reinforcement has been omitted because they are seldom used with concrete in construction.

R10.16.2 — The same rules used for computing the load-moment interaction strength for reinforced concrete sections can be applied to composite sections. Interaction charts for concrete-filled tubing would have a form identical to those of ACI SP-7[10.36] and the *Design Handbook*, V.2, Columns,[10.29] but with γ slightly greater than 1.0.

R10.16.3 and R10.16.4 — Direct bearing or direct connection for transfer of forces between steel and concrete can be developed through lugs, plates, or reinforcing bars welded to the structural shape or tubing before the concrete is cast. Flexural compressive stress need not be considered a part of direct compression load to be developed by bearing. A concrete encasement around a structural steel shape may stiffen the shape, but it would not necessarily increase its strength.

CODE

10.16.5 — For evaluation of slenderness effects, radius of gyration of a composite section shall be not greater than the value given by

$$r = \sqrt{\frac{(E_c I_g/5) + E_s I_t}{(E_c A_g/5) + E_s A_t}} \qquad (10\text{-}21)$$

and, as an alternative to a more accurate calculation, ***EI*** in Eq. (10-11) shall be taken either as Eq. (10-12) or

$$EI = \frac{(E_c I_g/5)}{1 + \beta_d} + E_s I_t \quad (10\text{-}22)$$

10.16.6 — Structural steel encased concrete core

10.16.6.1 — For a composite member with concrete core encased by structural steel, thickness of the steel encasement shall be not less than

$$b\sqrt{\frac{f_y}{3E_s}} \text{ for each face of width } b$$

nor

$$h\sqrt{\frac{f_y}{8E_s}} \text{ for circular sections of diameter } h$$

10.16.6.2 — Longitudinal bars located within the encased concrete core shall be permitted to be used in computing A_t and I_t.

10.16.7 — Spiral reinforcement around structural steel core

A composite member with spirally reinforced concrete around a structural steel core shall conform to the following.

10.16.7.1 — Specified compressive strength of concrete f_c' shall be not less than 2500 psi.

10.16.7.2 — Design yield strength of structural steel core shall be the specified minimum yield strength for grade of structural steel used but not to exceed 50,000 psi.

COMMENTARY

R10.16.5 — Eq. (10-21) is given because the rules of 10.11.2 for estimating the radius of gyration are overly conservative for concrete filled tubing and are not applicable for members with enclosed structural shapes.

In reinforced concrete columns subject to sustained loads, creep transfers some of the load from the concrete to the steel thus increasing the steel stresses. In the case of lightly reinforced columns, this load transfer may cause the compression steel to yield prematurely, resulting in a loss in the effective ***EI***. Accordingly, both the concrete and steel terms in Eq. (10-12) are reduced to account for creep. For heavily reinforced columns or for composite columns in which the pipe or structural shape makes up a large percentage of the cross section, the load transfer due to creep is not significant. Accordingly, Eq. (10-22) was revised in the 1980 code supplement so that only the ***EI*** of the concrete is reduced for sustained load effects.

R10.16.6 — Structural steel encased concrete core

Steel encased concrete sections should have a metal wall thickness large enough to attain longitudinal yield stress before buckling outward.

R10.16.7 — Spiral reinforcement around structural steel core

Concrete that is laterally contained by a spiral has increased load-carrying strength, and the size of spiral required can be regulated on the basis of the strength of the concrete outside the spiral by means of the same reasoning that applies for columns reinforced only with longitudinal bars. The radial pressure provided by the spiral ensures interaction between concrete, reinforcing bars, and steel core such that longitudinal bars will both stiffen and strengthen the cross section.

CODE

10.16.7.3 — Spiral reinforcement shall conform to 10.9.3.

10.16.7.4 — Longitudinal bars located within the spiral shall be not less than 0.01 nor more than 0.08 times net area of concrete section.

10.16.7.5 — Longitudinal bars located within the spiral shall be permitted to be used in computing A_t and I_t.

10.16.8 — Tie reinforcement around structural steel core

A composite member with laterally tied concrete around a structural steel core shall conform to the following.

10.16.8.1 — Specified compressive strength of concrete f_c' shall be not less than 2500 psi.

10.16.8.2 — Design yield strength of structural steel core shall be the specified minimum yield strength for grade of structural steel used but not to exceed 50,000 psi.

10.16.8.3 — Lateral ties shall extend completely around the structural steel core.

10.16.8.4 — Lateral ties shall have a diameter not less than $^1/_{50}$ times the greatest side dimension of composite member, except that ties shall not be smaller than No. 3 and are not required to be larger than No. 5. Welded wire fabric of equivalent area shall be permitted.

10.16.8.5 — Vertical spacing of lateral ties shall not exceed 16 longitudinal bar diameters, 48 tie bar diameters, or $^1/_2$ times the least side dimension of the composite member.

10.16.8.6 — Longitudinal bars located within the ties shall be not less than 0.01 nor more than 0.08 times net area of concrete section.

10.16.8.7 — A longitudinal bar shall be located at every corner of a rectangular cross section, with other longitudinal bars spaced not farther apart than one-half the least side dimension of the composite member.

10.16.8.8 — Longitudinal bars located within the ties shall be permitted to be used in computing A_t for strength but not in computing I_t for evaluation of slenderness effects.

COMMENTARY

R10.16.8 — Tie reinforcement around structural steel core

Concrete that is laterally contained by tie bars is likely to be rather thin along at least one face of a steel core section, and complete interaction between the core, the concrete, and any longitudinal reinforcement should not be assumed. Concrete will probably separate from smooth faces of the steel core. To maintain the concrete around the structural steel core, it is reasonable to require more lateral ties than needed for ordinary reinforced concrete columns. Because of probable separation at high strains between the steel core and the concrete, longitudinal bars will be ineffective in stiffening cross sections even though they would be useful in sustaining compression forces. Finally, the yield strength of the steel core should be limited to that which exists at strains below those that can be sustained without spalling of the concrete. It has been assumed that axially-compressed concrete will not spall at strains less than 0.0018. The yield strength of 0.0018 x 29,000,000, or 52,000 psi, represents an upper limit of the useful maximum steel stress.

CODE

10.17 — Bearing strength

10.17.1 — Design bearing strength on concrete shall not exceed ϕ **(0.85$f_c'A_1$)**, except when the supporting surface is wider on all sides than the loaded area, design bearing strength on the loaded area shall be permitted to be multiplied by $\sqrt{A_2/A_1}$ but not more than 2.

COMMENTARY

R10.17 — Bearing strength

R10.17.1 — This section deals with bearing strength on concrete supports. The permissible bearing stress of **0.85f_c'** is based on tests reported in Reference 10.37. (See also 15.8).

When the supporting area is wider than the loaded area on all sides, the surrounding concrete confines the bearing area, resulting in an increase in bearing strength. No minimum depth is given for a supporting member. The minimum depth of support will be controlled by the shear requirements of 11.11.

When the top of the support is sloped or stepped, advantage may still be taken of the condition that the supporting member is larger than the loaded area, provided the supporting member does not slope at too great an angle. Fig. R10.17 illustrates the application of the frustum to find A_2. The frustum should not be confused with the path by which a

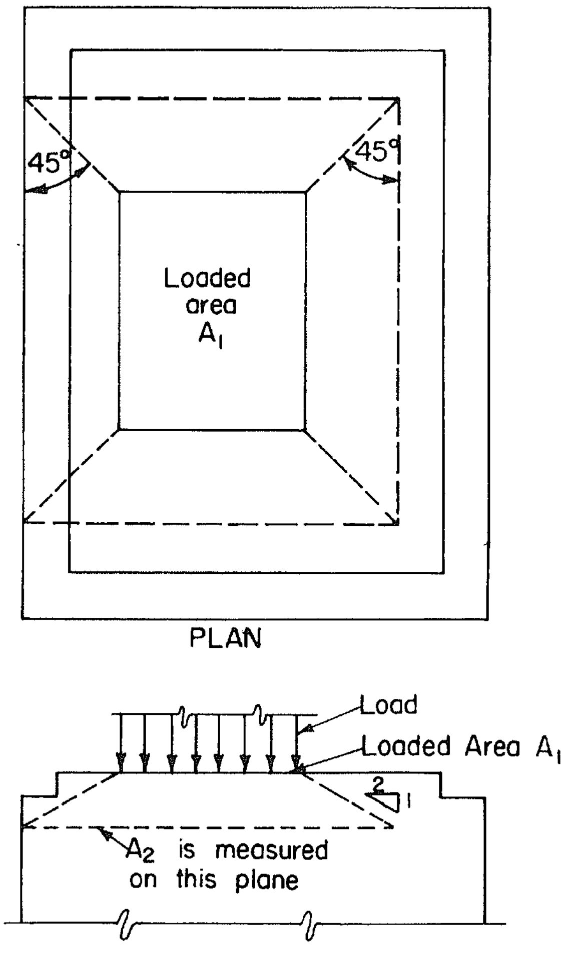


Fig. R10.17—Application of frustum to find A_2 *in stepped or sloped supports*

CODE

COMMENTARY

load spreads out as it travels downward through the support. Such a load path would have steeper sides. However, the frustum described has somewhat flat side slopes to ensure that there is concrete immediately surrounding the zone of high stress at the bearing. A_1 is the loaded area but not greater than the bearing plate or bearing cross-sectional area.

10.17.2 — Section 10.17 does not apply to post-tensioning anchorages.

R10.17.2 — Post-tensioning anchorages are normally laterally reinforced, in accordance with 18.13.

CHAPTER 11 — SHEAR AND TORSION

CODE

11.0 — Notation

a = shear span, distance between concentrated load and face of support

A_c = area of concrete section resisting shear transfer, in.^{2}

A_{cp} = area enclosed by outside perimeter of concrete cross section, in.^{2} See 11.6.1

A_f = area of reinforcement in bracket or corbel resisting factored moment, $[V_u a + N_{uc} (h - d)]$, in.^{2}

A_g = gross area of section, in.^{2}

A_h = area of shear reinforcement parallel to flexural tension reinforcement, in.^{2}

A_l = total area of longitudinal reinforcement to resist torsion, in.^{2}

A_n = area of reinforcement in bracket or corbel resisting tensile force N_{uc}, in.^{2}

A_o = gross area enclosed by shear flow path, in.^{2}

A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement, in.^{2}

A_{ps} = area of prestressed reinforcement in tension zone, in.^{2}

A_s = area of nonprestressed tension reinforcement, in.^{2}

A_t = area of one leg of a closed stirrup resisting torsion within a distance s, in.^{2}

A_v = area of shear reinforcement within a distance s, or area of shear reinforcement perpendicular to flexural tension reinforcement within a distance s for deep flexural members, in.^{2}

A_{vf} = area of shear-friction reinforcement, in.^{2}

A_{vh} = area of shear reinforcement parallel to flexural tension reinforcement within a distance s_2, in.^{2}

b = width of compression face of member, in.

b_o = perimeter of critical section for slabs and footings, in.

b_t = width of that part of cross section containing the closed stirrups resisting torsion

b_w = web width, or diameter of circular section, in.

COMMENTARY

This chapter includes shear and torsion provisions for both nonprestressed and prestressed concrete members. The shear-friction concept (11.7) is particularly applicable to design of reinforcement details in precast structures. Special provisions are included for deep flexural members (11.8), brackets and corbels (11.9), and shearwalls (11.10). Shear provisions for slabs and footings are given in 11.12.

R11.0 — Notation

Tests[11.1] have indicated that the average shear over the full effective section also may be applied for circular sections. Note the special definition of d for such sections.

CODE

b_1 = width of the critical section defined in 11.12.1.2 measured in the direction of the span for which moments are determined, in.

b_2 = width of the critical section defined in 11.12.1.2 measured in the direction perpendicular to b_1, in.

c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in.

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction of the span for which moments are being determined, in.

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, but need not be less than $0.80h$ for prestressed members, in. (For circular sections, d need not be less than the distance from extreme compression fiber to centroid of tension reinforcement in opposite half of member.)

f_c' = specified compressive strength of concrete, psi

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi

f_d = stress due to unfactored dead load, at extreme fiber of section where tensile stress is caused by externally applied loads, psi

f_{pc} = compressive stress in concrete (after allowance for all prestress losses) at centroid of cross section resisting externally applied loads or at junction of web and flange when the centroid lies within the flange, psi. (In a composite member, f_{pc} is resultant compressive stress at centroid of composite section, or at junction of web and flange when the centroid lies within the flange, due to both prestress and moments resisted by precast member acting alone)

f_{pe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads, psi

f_{pu} = specified tensile strength of prestressing tendons, psi

f_y = specified yield strength of nonprestressed reinforcement, psi

f_{yv} = yield strength of closed transverse torsional reinforcement, psi

$f_{y\ell}$ = yield strength of longitudinal torsional reinforcement, psi

h = overall thickness of member, in.

h_v = total depth of shearhead cross section, in.

COMMENTARY

Although the value of d may vary along the span of a prestressed beam, studies[11.2] showed that, for prestressed concrete members, d need not be taken less than $0.8h$. The beams considered had some straight tendons or reinforcing bars at the bottom of the section and had stirrups which enclosed those tendons.

CODE

COMMENTARY

h_w = total height of wall from base to top, in.

I = moment of inertia of section resisting externally applied factored loads

ℓ_n = clear span measured face-to-face of supports

ℓ_v = length of shearhead arm from centroid of concentrated load or reaction, in.

ℓ_w = horizontal length of wall, in.

M_{cr} = moment causing flexural cracking at section due to externally applied loads. See 11.4.2.1

M_m = modified moment

M_{max} = maximum factored moment at section due to externally applied loads

M_p = required plastic moment strength of shearhead cross section

M_u = factored moment at section

M_v = moment resistance contributed by shearhead reinforcement

N_u = factored axial load normal to cross section occurring simultaneously with V_u; to be taken as positive for compression, negative for tension, and to include effects of tension due to creep and shrinkage

N_{uc} = factored tensile force applied at top of bracket or corbel acting simultaneously with V_u, to be taken as positive for tension

p_{cp} = outside perimeter of the concrete cross section, in. See 11.6.1.

p_h = perimeter of centerline of outermost closed transverse torsional reinforcement, in.

s = spacing of shear or torsion reinforcement in direction parallel to longitudinal reinforcement, in.

s_1 = spacing of vertical reinforcement in wall, in.

s_2 = spacing of shear or torsion reinforcement in direction perpendicular to longitudinal reinforcement—or spacing of horizontal reinforcement in wall, in.

t = thickness of a wall of a hollow section, in.

T_n = nominal torsional moment strength

T_u = factored torsional moment at section

V_c = nominal shear strength provided by concrete

V_{ci} = nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment

V_{cw} = nominal shear strength provided by concrete when diagonal cracking results from excessive principal tensile stress in web

V_d = shear force at section due to unfactored dead load

V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}

V_n = nominal shear strength

V_p = vertical component of effective prestress force at section

V_s = nominal shear strength provided by shear reinforcement

CODE

V_u = factored shear force at section

v_n = nominal shear stress, psi. See 11.12.6.2

y_t = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension

α = angle between inclined stirrups and longitudinal axis of member

α_f = angle between shear-friction reinforcement and shear plane

α_s = constant used to compute V_c in slabs and footings

α_v = ratio of stiffness of shearhead arm to surrounding composite slab section. See 11.12.4.5

β_c = ratio of long side to short side of concentrated load or reaction area

β_p = constant used to compute V_c in prestressed slabs

γ_f = fraction of unbalanced moment transferred by flexure at slab-column connections. See 13.5.3.2

γ_v = fraction of unbalanced moment transferred by eccentricity of shear at slab-column connections. See 11.12.6.1

= $1 - \gamma_f$

η = number of identical arms of shearhead

θ = angle of compression diagonals in truss analogy for torsion

λ = correction factor related to unit weight of concrete

μ = coefficient of friction. See 11.7.4.3

ρ = ratio of nonprestressed tension reinforcement

= A_s/bd

ρ_h = ratio of horizontal shear reinforcement area to gross concrete area of vertical section

ρ_n = ratio of vertical shear reinforcement area to gross concrete area of horizontal section

ρ_w = A_s/b_wd

ϕ = strength reduction factor. See 9.3

11.1 — Shear strength

11.1.1 — Design of cross sections subject to shear shall be based on:

$$\phi V_n \geq V_u \qquad (11\text{-}1)$$

where V_u is factored shear force at section considered and V_n is nominal shear strength computed by:

$$V_n = V_c + V_s \qquad (11\text{-}2)$$

where V_c is nominal shear strength provided by concrete in accordance with 11.3 or 11.4, and V_s is nominal shear strength provided by shear reinforcement in accordance with 11.5.6.

COMMENTARY

R11.1 — Shear strength

The shear strength is based on an average shear stress on the full effective cross section b_wd. In a member without shear reinforcement, shear is assumed to be carried by the concrete web. In a member with shear reinforcement, a portion of the shear is assumed to be provided by the concrete and the remainder by the shear reinforcement.

The shear strength provided by concrete V_c is assumed to be the same for beams with and without shear reinforcement and is taken as the shear causing significant inclined cracking. These assumptions are discussed in the ACI-ASCE Committee 426 reports[11.1,11.3] and in Reference 11.2.

CODE

11.1.1.1 — In determining shear strength V_n, effect of any openings in members shall be considered.

11.1.1.2 — In determining shear strength V_c, whenever applicable, effects of axial tension due to creep and shrinkage in restrained members shall be considered and effects of inclined flexural compression in variable depth members shall be permitted to be included.

11.1.2 — The values of $\sqrt{f_c'}$ used in this chapter shall not exceed 100 psi except as allowed in 11.1.2.1.

11.1.2.1 — Values of $\sqrt{f_c'}$ greater than 100 psi shall be permitted in computing V_c, V_{ci}, and V_{cw} for reinforced or prestressed concrete beams and concrete joist construction having minimum web reinforcement equal to $f_c'/5000$ times, but not more than three times, the amounts required by 11.5.5.3, 11.5.5.4, or 11.6.5.2.

11.1.3 — Computation of maximum factored shear force V_u at supports in accordance with 11.1.3.1 or 11.1.3.2 shall be permitted when both of the following conditions are satisfied:

(a) Support reaction, in direction of applied shear, introduces compression into the end regions of member, and

(b) No concentrated load occurs between face of support and location of critical section defined in 11.1.3.1 or 11.1.3.2.

11.1.3.1 — For nonprestressed members, sections located less than a distance d from face of support shall be permitted to be designed for the same shear V_u as that computed at a distance d.

COMMENTARY

R11.1.1.1 — Openings in the web of a member can reduce its shear strength. The effects of openings are discussed in Section 4.7 of Reference 11.1 and in References 11.4 and 11.5.

R11.1.1.2 — In a member of variable depth, the internal shear at any section is increased or decreased by the vertical component of the inclined flexural stresses. Computation methods are outlined in various textbooks and in the 1940 Joint Committee Report.[11.6]

R11.1.2 — A limited number of tests[11.7,11.8] of reinforced concrete beams made with high strength concrete (f_c' greater than about 8000 psi) suggest that the inclined cracking load increases less rapidly than Eq. (11-3) or (11-5) would suggest. This was offset by an increased effectiveness of the stirrups compared to the strength predicted by Eq. (11-15), (11-16), and (11-17). Other unpublished tests of high strength concrete girders with minimum web reinforcement indicated that this amount of web reinforcement was inadequate to prevent brittle shear failures when inclined cracking occurs. There are no test data on the two-way shear strength of high strength concrete slabs or torsional strength. Until more practical experience is obtained with beams and slabs built with concretes with strengths greater than 10,000 psi, the committee thought it was prudent to limit $\sqrt{f_c'}$ to 100 psi in calculations of shear strength and development length. For beams with enough stirrups to allow post-cracking capacity this limit is not imposed.

R11.1.3 — The closest inclined crack to the support of the beam in Fig. R11.1.3 will extend upwards from the face of the support reaching the compression zone about d from the face of the support. If loads are applied to the top of this beam, the stirrups across this crack are stressed by loads acting on the lower freebody in Fig. R11.1.3. The loads applied to the beam between the face of the column and the point d away from the face are transferred directly to the support by compression in the web above the crack. Accordingly, the code permits design for a maximum factored shear force V_u at a distance d from the support for nonprestressed members, and at a distance $h/2$ for prestressed members. Two things must be emphasized: first, stirrups are required across the potential crack designed for the shear at d from the support, and second, a tension force exists in the longitudinal reinforcement at the face of the support.

Typical support conditions where the shear force at a distance d from the support may be used include: (1) members supported by bearing at the bottom of the member, such as shown in Fig. R11.1.3.1(a); and (2) members framing monolithically into another member as illustrated in Fig. R11.1.3.1(b).

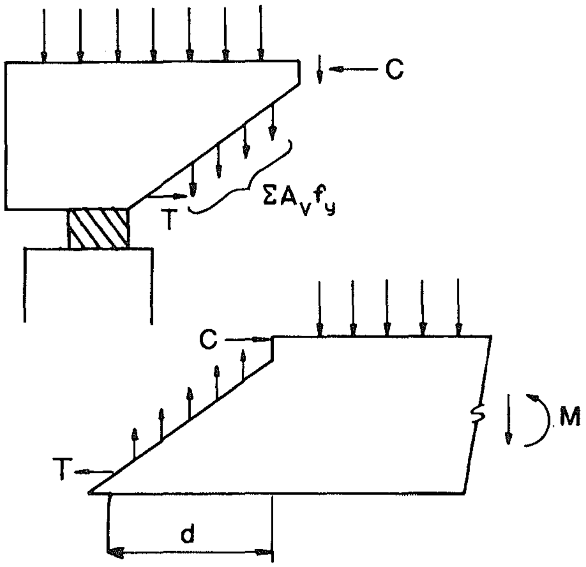


Fig. R11.1.3—Free body diagrams of the end of a beam

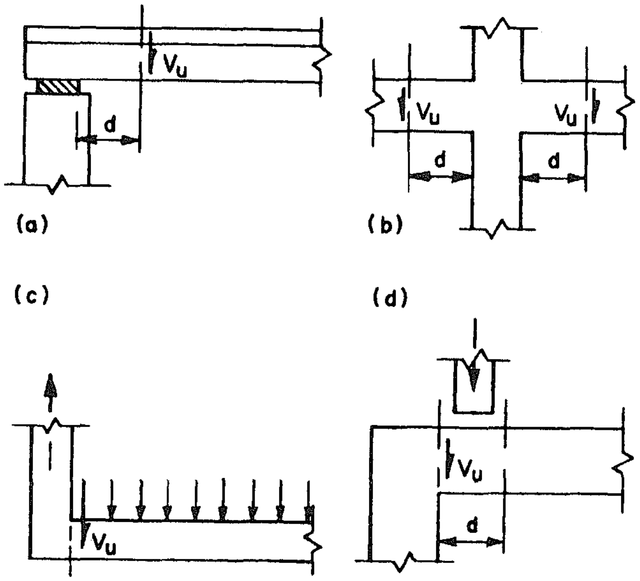


Fig. R11.1.3.1—Typical support conditions for locating factored shear force V_u

Support conditions where this provision should not be applied include: (1) Members framing into a supporting member in tension, such as shown in Fig. R11.1.3.1(c). For this case, the critical section for shear should be taken at the face of the support, shear within the connection should also be investigated, and special corner reinforcement should be provided. (2) Members loaded such that the shear at sections between the support and a distance d differs radically from the shear at distance d. This commonly occurs in brackets and in beams where a concentrated load is located close to the support, as shown in Fig. R11.1.3.1(d) or in footings supported on piles. In this case the shear at the face of the support should be used.

CODE

11.1.3.2 — For prestressed members, sections located less than a distance $h/2$ from face of support shall be permitted to be designed for the same shear V_u as that computed at a distance $h/2$.

11.1.4 — For deep flexural members, brackets and corbels, walls, and slabs and footings, the special provisions of 11.8 through 11.12 shall apply.

11.2 — Lightweight concrete

11.2.1 — Provisions for shear strength V_c apply to normal weight concrete. When lightweight aggregate concrete is used, one of the following modifications shall apply:

11.2.1.1 — When f_{ct} is specified and concrete is proportioned in accordance with 5.2, provisions for V_c shall be modified by substituting $f_{ct}/6.7$ for $\sqrt{f_c'}$, but the value of $f_{ct}/6.7$ shall not exceed $\sqrt{f_c'}$.

11.2.1.2 — When f_{ct} is not specified, all values of $\sqrt{f_c'}$ affecting V_c and M_{cr} shall be multiplied by 0.75 for "all-lightweight" concrete, and 0.85 for "sand-lightweight" concrete. Linear interpolation shall be permitted when partial sand replacement is used.

11.3 — Shear strength provided by concrete for nonprestressed members

11.3.1 — Shear strength V_c shall be computed by provisions of 11.3.1.1 through 11.3.1.3, unless a more detailed calculation is made in accordance with 11.3.2.

11.3.1.1 — For members subject to shear and flexure only,

$$V_c = 2\sqrt{f_c'}\,b_w d \qquad (11\text{-}3)$$

11.3.1.2 — For members subject to axial compression,

$$V_c = 2\left(1 + \frac{N_u}{2000A_g}\right)\sqrt{f_c'}\,b_w d \qquad (11\text{-}4)$$

Quantity N_u/A_g shall be expressed in psi.

11.3.1.3 — For members subject to significant axial tension, shear reinforcement shall be designed to

COMMENTARY

R11.1.3.2 — Because d frequently varies in prestressed members the location of the critical section has arbitrarily been taken as $h/2$ from the face of the support.

R11.2 — Lightweight concrete

Two alternate procedures are provided to modify the provisions for shear when lightweight aggregate concrete is used. The lightweight concrete modification applies only to the terms containing $\sqrt{f_c'}$ in the equations of Chapter 11.

R11.2.1.1 — The first alternate bases the modification on laboratory tests to determine the relationship between splitting tensile strength f_{ct} and the compressive strength f_c' for the lightweight concrete being used. For normal weight concrete, the splitting tensile strength f_{ct} is approximately equal to $6.7\sqrt{f_c'}$.[11.9,11.10]

R11.2.1.2 — The modification may also be based on the assumption that the tensile strength of lightweight concrete is a fixed fraction of the tensile strength of normal weight concrete.[11.10] The multipliers are based on data from tests on many types of structural lightweight aggregate concrete.

R11.3 — Shear strength provided by concrete for nonprestressed members

R11.3.1.1 — See R11.3.2.1.

R11.3.1.2 and R11.3.1.3 — See R11.3.2.2.

CODE

carry total shear unless a more detailed analysis is made using 11.3.2.3.

11.3.2 — Shear strength V_c shall be permitted to be computed by the more detailed calculation of 11.3.2.1 through 11.3.2.3.

11.3.2.1 — For members subject to shear and flexure only,

$$V_c = \left(1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \qquad (11\text{-}5)$$

but not greater than $3.5\sqrt{f_c'}\, b_w d$. Quantity $V_u d/M_u$ shall not be taken greater than 1.0 in computing V_c by Eq. (11-5), where M_u is factored moment occurring simultaneously with V_u at section considered.

11.3.2.2 — For members subject to axial compression, it shall be permitted to compute V_c using Eq. (11-5) with M_m substituted for M_u and $V_u d/M_u$ not then limited to 1.0, where

$$M_m = M_u - N_u \frac{(4h - d)}{8} \qquad (11\text{-}6)$$

However, V_c shall not be taken greater than

$$V_c = 3.5\sqrt{f_c'}\, b_w d \sqrt{1 + \frac{N_u}{500A_g}} \qquad (11\text{-}7)$$

Quantity N_u/A_g shall be expressed in psi. When M_m as computed by Eq. (11-6) is negative, V_c shall be computed by Eq. (11-7).

11.3.2.3 — For members subject to significant axial tension, up to designer's judgement.

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\sqrt{f_c'}\, b_w d \qquad (11\text{-}8)$$

but not less than zero, where N_u is negative for tension. Quantity N_u/A_g shall be expressed in psi.

COMMENTARY

R11.3.2.1 — Eq. (11-5) is the basic expression for shear strength of members without shear reinforcement.[11.3] Designers should recognize that the three variables in Eq. (11-5), $\sqrt{f_c'}$ (as a measure of concrete tensile strength), ρ_w, and $V_u d/M_u$, are known to affect shear strength, although some research data[11.1,11.11] indicate that Eq. (11-5) overestimates the influence of f_c' and underestimates the influence of ρ_w and $V_u d/M_u$. Further information[11.12] has indicated that shear strength decreases as the overall depth of the member increases.

The minimum value of M_u equal to $V_u d$ in Eq. (11-5) is to limit V_c near points of inflection.

For most designs, it is convenient to assume that the second term of Eq. (11-5) equals $0.1\sqrt{f_c'}$ and use V_c equal to $2\sqrt{f_c'}\, b_w d$ as permitted in 11.3.1.1.

R11.3.2.2 — Eq. (11-6) and (11-7) for members subject to axial compression in addition to shear and flexure, are derived in the ACI-ASCE Committee 326 report.[11.3] As N_u is increased, the value of V_c computed from Eq. (11-5) and (11-6) will exceed the upper limit given by Eq. (11-7) before the value of M_m given by Eq. (11-6) becomes negative. The value of V_c obtained from Eq. (11-5) has no physical significance if a negative value of M_m is substituted. For this condition, Eq. (11-7) or Eq. (11-4) should be used to calculate V_c. Values of V_c for members subject to shear and axial load are illustrated in Fig. R11.3. The background for these equations is discussed and comparisons are made with test data in Reference 11.2.

Because of the complexity of Eq. (11-5) and (11-6), an alternative design provision, Eq. (11-4), is permitted.

R11.3.2.3 — Eq. (11-8) may be used to compute V_c for members subject to significant axial tension. Shear reinforcement may then be designed for $V_n - V_c$. The term "significant" is used to recognize that a designer must use judgment in deciding whether axial tension needs to be considered. Low levels of axial tension often occur due to volume changes, but are not important in structures with adequate expansion joints and minimum reinforcement. It may be desirable to design shear reinforcement to carry total shear if there is uncertainty about the magnitude of axial tension.

CODE

11.4 — Shear strength provided by concrete for prestressed members

11.4.1 — For members with effective prestress force not less than 40 percent of the tensile strength of flexural reinforcement, unless a more detailed calculation is made in accordance with 11.4.2,

$$V_c = \left(0.6\sqrt{f_c'} + 700\frac{V_u d}{M_u}\right) b_w d \qquad (11\text{-}9)$$

but V_c need not be taken less than $2\sqrt{f_c'}\,b_w d$ nor shall V_c be taken greater than $5\sqrt{f_c'}\,b_w d$ nor the value given in 11.4.3 or 11.4.4. The quantity $V_u d/M_u$ shall not be taken greater than 1.0, where M_u is factored moment occurring simultaneously with V_u at section considered. When applying Eq. (11-9), d in the term $V_u d/M_u$ shall be the distance from extreme compression fiber to centroid of prestressed reinforcement.

COMMENTARY

R11.4 — Shear strength provided by concrete for prestressed members

R11.4.1 — Eq. (11-9) offers a simple means of computing V_c for prestressed concrete beams.[11.2] It may be applied to beams having prestressed reinforcement only, or to members reinforced with a combination of prestressed reinforcement and nonprestressed deformed bars. Eq. (11-9) is most applicable to members subject to uniform loading and may give conservative results when applied to composite girders for bridges.

In applying Eq. (11-9) to simply supported members subject to uniform loads $V_u d/M_u$ can be expressed as

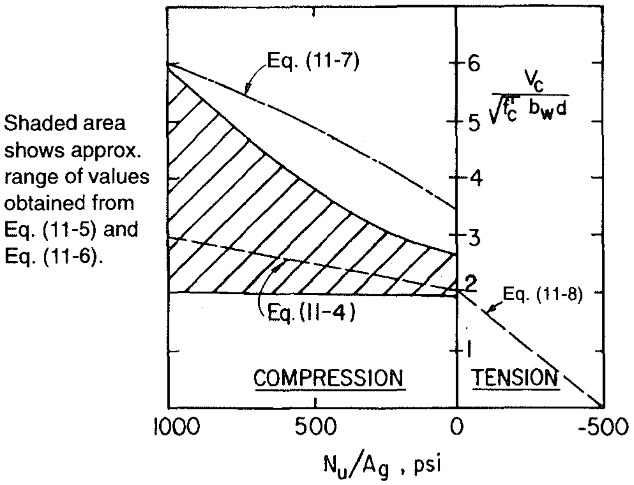


Fig. R11.3—Comparison of shear strength equations for members subject to axial load

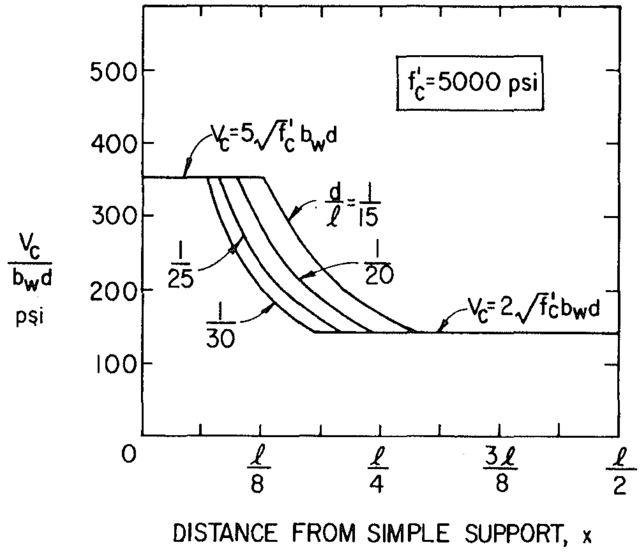


Fig. R11.4.1—Application of Eq. (11-9) to uniformly loaded prestressed members

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11.4.2 — Shear strength V_c shall be permitted to be computed in accordance with 11.4.2.1 and 11.4.2.2, where V_c shall be the lesser of V_{ci} or V_{cw}.

11.4.2.1—Shear strength V_{ci} shall be computed by

$$V_{ci} = 0.6\sqrt{f_c'}b_w d + V_d + \frac{V_i M_{cr}}{M_{max}} \qquad (11\text{-}10)$$

but V_{ci} need not be taken less than $1.7\sqrt{f_c'}\,b_w d$, where

$$M_{cr} = (I/y_t)(6\sqrt{f_c'} + f_{pe} - f_d) \qquad (11\text{-}11)$$

and values of M_{max} and V_i shall be computed from the load combination causing maximum moment to occur at the section.

11.4.2.2—Shear strength V_{cw} shall be computed by

$$V_{cw} = (3.5\sqrt{f_c'} + 0.3f_{pc})b_w d + V_p \qquad (11\text{-}12)$$

Alternatively, V_{cw} shall be computed as the shear force corresponding to dead load plus live load that results in a principal tensile stress of $4\sqrt{f_c'}$ at the centroidal axis of member, or at the intersection of flange and web when the centroidal axis is in the flange. In composite members, the principal tensile stress shall be computed using the cross section that resists live load.

11.4.2.3 — In Eq. (11-10) and (11-12), d shall be the distance from extreme compression fiber to centroid of prestressed reinforcement or $0.8h$, whichever is greater.

COMMENTARY

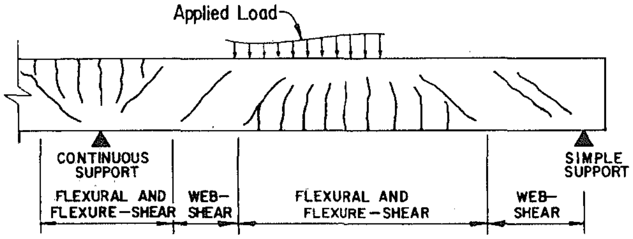

Fig. R11.4.2—Types of cracking in concrete beams

$$\frac{{}^{v}{}_{u}{}^{u}}{M_u} = \frac{d(\ell-2x)}{x(\ell-x)}$$

where ℓ is the span length and x is the distance from the section being investigated to the support. For concrete with f_c' equal to 5000 psi, V_c from 11.4.1 varies as shown in Fig. R11.4.1. Design aids based on this equation are given in Reference 11.13.

R11.4.2 — Two types of inclined cracking occur in concrete beams: web-shear cracking and flexure-shear cracking. These two types of inclined cracking are illustrated in Fig. R11.4.2.

Web-shear cracking begins from an interior point in a member when the principal tensile stresses exceed the tensile strength of the concrete. Flexure-shear cracking is initiated by flexural cracking. When flexural cracking occurs, the shear stresses in the concrete above the crack are increased. The flexure-shear crack develops when the combined shear and tensile stress exceeds the tensile strength of the concrete.

Eq. (11-10) and (11-12) may be used to determine the shear forces causing flexure-shear and web-shear cracking, respectively. The shear strength provided by the concrete V_c is assumed equal to the lesser of V_{ci} and V_{cw}. The derivations of Eq. (11-10) and (11-12) are summarized in Reference 11.14.

In deriving Eq. (11-10) it was assumed that V_{ci} is the sum of the shear required to cause a flexural crack at the point in question given by:

$$V = \frac{V_i M_{cr}}{M_{max}}$$

plus an additional increment of shear required to change the flexural crack to a flexure-shear crack. The externally applied factored loads, from which V_i and M_{max} are determined, include superimposed dead load, earth pressure, live load, etc. In computing M_{cr} for substitution into Eq. (11-10), I and y_t are the properties of the section resisting the externally applied loads.

CODE

COMMENTARY

For a composite member, where part of the dead load is resisted by only a part of the section, appropriate section properties should be used to compute f_d. The shear due to dead loads V_d and that due to other loads V_i are separated in this case. V_d is then the total shear force due to unfactored dead load acting on that part of the section carrying the dead loads acting prior to composite action plus the unfactored superimposed dead load acting on the composite member. The terms V_i and M_{max} may be taken as:

$$V_i = V_u - V_d$$

$$M_{max} = M_u - M_d$$

where V_u and M_u are the factored shear and moment due to the total factored loads, and M_d is the moment due to unfactored dead load (i.e., the moment corresponding to f_d).

For noncomposite uniformly loaded beams, the total cross section resists all the shear and the live and dead load shear force diagrams are similar. In this case Eq. (11-10) reduces to:

$$V_{ci} = 0.6\sqrt{f_c'}b_w d + \frac{V_u M_{ct}}{M_u}$$

where

$$M_{ct} = (I/y_t)\left(6\sqrt{f_c'} + f_{pe}\right)$$

The symbol M_{ct} in the two preceding equations represents the total moment, including dead load, required to cause cracking at the extreme fiber in tension. This is not the same as M_{cr} in code Eq. (11-10) where the cracking moment is that due to all loads except the dead load. In Eq. (11-10) the dead load shear is added as a separate term.

M_u is the factored moment on the beam at the section under consideration, and V_u is the factored shear force occurring simultaneously with M_u. Since the same section properties apply to both dead and live load stresses, there is no need to compute dead load stresses and shears separately, and the cracking moment M_{ct} reflects the total stress change from effective prestress to a tension of $6\sqrt{f_c'}$, assumed to cause flexural cracking.

Eq. (11-12) is based on the assumption that web-shear cracking occurs due to the shear causing a principal tensile stress of approximately $4\sqrt{f_c'}$ at the centroidal axis of the cross section. V_p is calculated from the effective prestress force without load factors.

CODE

11.4.3 — In a pretensioned member in which the section at a distance $h/2$ from face of support is closer to the end of member than the transfer length of the prestressing tendons, the reduced prestress shall be considered when computing V_{cw}. This value of V_{cw} shall also be taken as the maximum limit for Eq. (11-9). The prestress force shall be assumed to vary linearly from zero at end of tendon to a maximum at a distance from end of tendon equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

11.4.4—In a pretensioned member where bonding of some tendons does not extend to the end of member, a reduced prestress shall be considered when computing V_c in accordance with 11.4.1 or 11.4.2. The value of V_{cw} calculated using the reduced prestress shall also be taken as the maximum limit for Eq. (11-9). The prestress force due to tendons for which bonding does not extend to the end of member, shall be assumed to vary linearly from zero at the point at which bonding commences to a maximum at a distance from this point equal to the transfer length, assumed to be 50 diameters for strand and 100 diameters for single wire.

11.5 — Shear strength provided by shear reinforcement

11.5.1 — Types of shear reinforcement

11.5.1.1 — Shear reinforcement consisting of the following shall be permitted:

(a) Stirrups perpendicular to axis of member.

(b) Welded wire fabric with wires located perpendicular to axis of member.

11.5.1.2 — For nonprestressed members, shear reinforcement shall be permitted to also consist of:

(a) Stirrups making an angle of 45 deg or more with longitudinal tension reinforcement.

(b) Longitudinal reinforcement with bent portion making an angle of 30 deg or more with the longitudinal tension reinforcement.

(c) Combinations of stirrups and bent longitudinal reinforcement.

(d) Spirals.

COMMENTARY

R11.4.3 and R11.4.4 — The effect of the reduced prestress near the ends of pretensioned beams on the shear strength must be taken into account. Section 11.4.3 relates to the shear strength at sections within the transfer length of tendons when bonding of tendons extends to the end of the member.

Section 11.4.4 relates to the shear strength at sections within the length over which some tendons are not bonded to the concrete, or within the transfer length of those tendons for which bonding does not extend to the end of the beam.

R11.5 — Shear strength provided by shear reinforcement

CODE

11.5.2 — Design yield strength of shear reinforcement shall not exceed 60,000 psi, except that the design yield strength of welded deformed wire fabric shall not exceed 80,000 psi.

11.5.3 — Stirrups and other bars or wires used as shear reinforcement shall extend to a distance d from extreme compression fiber and shall be anchored at both ends according to 12.13 to develop the design yield strength of reinforcement.

11.5.4 — Spacing limits for shear reinforcement

11.5.4.1 — Spacing of shear reinforcement placed perpendicular to axis of member shall not exceed $d/2$ in nonprestressed members and $(^3/_4)h$ in prestressed members, nor 24 in.

11.5.4.2 — Inclined stirrups and bent longitudinal reinforcement shall be so spaced that every 45 deg line, extending toward the reaction from middepth of member $d/2$ to longitudinal tension reinforcement, shall be crossed by at least one line of shear reinforcement.

11.5.4.3 — When V_s exceeds $4\sqrt{f_c'}\,b_w d$, maximum spacings given in 11.5.4.1 and 11.5.4.2 shall be reduced by one-half.

11.5.5 — Minimum shear reinforcement

11.5.5.1 — A minimum area of shear reinforcement shall be provided in all reinforced concrete flexural members (prestressed and nonprestressed) where factored shear force V_u exceeds one-half the shear strength provided by concrete ϕV_c, except:

(a) Slabs and footings

(b) Concrete joist construction defined by 8.11

(c) Beams with total depth not greater than 10 in., $2^1/_2$ times thickness of flange, or $^1/_2$ the width of web, whichever is greatest.

COMMENTARY

R11.5.2 — Limiting the design yield strength of shear reinforcement to 60,000 psi provides a control on diagonal crack width. However, in the 1995 code, the limitation on design yield strength of 60,000 psi for shear reinforcement was raised to 80,000 psi for welded deformed wire fabric. Recent research[11.15,11.16,11.17] has indicated that the performance of higher strength steels as shear reinforcement has been satisfactory. In particular, full scale beam tests described in Reference 11.16 indicated that the widths of inclined shear cracks at service load levels were less for beams reinforced with smaller diameter deformed welded wire fabric cages designed on the basis of a yield strength of 75 ksi than beams reinforced with deformed Grade 60 stirrups.

R11.5.3 — It is essential that shear (and torsion) reinforcement be adequately anchored at both ends, to be fully effective on either side of any potential inclined crack. This generally requires a hook or bend at the end of the reinforcement as provided by 12.13.

R11.5. 5 — Minimum shear reinforcement

R11.5.5.1 — Shear reinforcement restrains the growth of inclined cracking. Ductility is increased and a warning of failure is provided. In an unreinforced web, the sudden formation of inclined cracking might lead directly to failure without warning. Such reinforcement is of great value if a member is subjected to an unexpected tensile force or an overload. Accordingly, a minimum area of shear reinforcement not less than that given by Eq. (11-13) or (11-14) is required wherever the total factored shear force V_u is greater than one-half the shear strength provided by concrete ϕV_c. Slabs, footings and joists are excluded from the minimum shear reinforcement requirement because there is a possibility of load sharing between weak and strong areas.

CODE

11.5.5.2 — Minimum shear reinforcement requirements of 11.5.5.1 shall be permitted to be waived if shown by test that required nominal flexural and shear strengths can be developed when shear reinforcement is omitted. Such tests shall simulate effects of differential settlement, creep, shrinkage, and temperature change, based on a realistic assessment of such effects occurring in service.

11.5.5.3 — Where shear reinforcement is required by 11.5.5.1 or for strength and where 11.6.1 allows torsion to be neglected, the minimum area of shear reinforcement for prestressed (except as provided in 11.5.5.4) and nonprestressed members shall be computed by

$$A_v = 50\frac{b_w s}{f_y} \qquad (11\text{-}13)$$

where b_w and s are in inches.

COMMENTARY

Even when the total factored shear strength V_u is less than one-half of the shear strength provided by the concrete ϕV_c, the use of some web reinforcement is recommended in all thin-web post-tensioned prestressed concrete members (joists, waffle slabs, beams, and T-beams) to reinforce against tensile forces in webs resulting from local deviations from the design tendon profile, and to provide a means of supporting the tendons in the design profile during construction. If sufficient support is not provided, lateral wobble and local deviations from the smooth parabolic tendon profile assumed in design may result during placement of the concrete. In such cases, the deviations in the tendons tend to straighten out when the tendons are stressed. This process may impose large tensile stresses in webs, and severe cracking may develop if no web reinforcement is provided. Unintended curvature of the tendons, and the resulting tensile stresses in webs, may be minimized by securely tying tendons to stirrups that are rigidly held in place by other elements of the reinforcing cage and held down in the forms. The maximum spacing of stirrups used for this purpose should not exceed the smaller of $1^1/_2 h$ or 4 ft. When applicable, the shear reinforcement provisions of 11.5.4 and 11.5.5 will require closer stirrup spacings.

For repeated loading of flexural members, the possibility of inclined diagonal tension cracks forming at stresses appreciably smaller than under static loading should be taken into account in the design. In these instances, it would be prudent to use at least the minimum shear reinforcement expressed by Eq. (11-13) or (11-14), even though tests or calculations based on static loads show that shear reinforcement is not required.

R11.5.5.2 — When a member is tested to demonstrate that its shear and flexural strengths are adequate, the actual member dimensions and material strengths are known. The strength used as a basis for comparison should therefore be that corresponding to a strength reduction factor of unity ($\phi = 1.0$), i.e. the required nominal strength V_n and M_n. This ensures that if the actual material strengths in the field were less than specified, or the member dimensions were in error such as to result in a reduced member strength, a satisfactory margin of safety will be retained.

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11.5.5.4 — For prestressed members with an effective prestress force not less than 40 percent of the tensile strength of the flexural reinforcement, the area of shear reinforcement shall not be less than the smaller $\boldsymbol{A_v}$ from Eq. (11-13) and (11-14).

$$A_v = \frac{A_{ps} f_{pu} s}{80 f_y d} \sqrt{\frac{d}{b_w}} \qquad (11\text{-}14)$$

11.5.6 — Design of shear reinforcement

11.5.6.1 — Where factored shear force $\boldsymbol{V_u}$ exceeds shear strength $\phi \boldsymbol{V_c}$, shear reinforcement shall be provided to satisfy Eq. (11-1) and (11-2), where shear strength $\boldsymbol{V_s}$ shall be computed in accordance with 11.5.6.2 through 11.5.6.8.

11.5.6.2 — When shear reinforcement perpendicular to axis of member is used,

$$V_s = \frac{A_v f_y d}{s} \qquad (11\text{-}15)$$

where $\boldsymbol{A_v}$ is the area of shear reinforcement within a distance *s*.

11.5.6.3 — When inclined stirrups are used as shear reinforcement,

$$V_s = \frac{A_v f_y (\sin\alpha + \cos\alpha)\, d}{s} \qquad (11\text{-}16)$$

11.5.6.4 — When shear reinforcement consists of a single bar or a single group of parallel bars, all bent up at the same distance from the support,

$$V_s = A_v f_y \sin\alpha \qquad (11\text{-}17)$$

but not greater than $\boldsymbol{3\sqrt{f_c'}\, b_w d}$.

11.5.6.5 — When shear reinforcement consists of a series of parallel bent-up bars or groups of parallel bent-up bars at different distances from the support, shear strength $\boldsymbol{V_s}$ shall be computed by Eq. (11-16).

11.5.6.6 — Only the center three-fourths of the inclined portion of any longitudinal bent bar shall be considered effective for shear reinforcement.

11.5.6.7 — Where more than one type of shear reinforcement is used to reinforce the same portion of a member, shear strength $\boldsymbol{V_s}$ shall be computed as the sum of the $\boldsymbol{V_s}$ values computed for the various types.

COMMENTARY

R11.5.5.4 — Tests of prestressed beams with minimum web reinforcement based on Eq. (11-13) and (11-14) indicated that the smaller A_v from these two equations was sufficient to develop ductile behavior.

Eq. (11-14) may be used only for prestressed members meeting the minimum prestress force requirements given in 11.5.5.4. This equation is discussed in Reference 11.18.

R11.5.6 — Design of shear reinforcement

Design of shear reinforcement is based on a modified truss analogy. The truss analogy assumes that the total shear is carried by shear reinforcement. However, considerable research on both nonprestressed and prestressed members has indicated that shear reinforcement need be designed to carry only the shear exceeding that which causes inclined cracking, provided the diagonal members in the truss are assumed to be inclined at 45 deg.

Eq. (11-15), (11-16), and (11-17) are presented in terms of shear strength V_s attributed to the shear reinforcement. When shear reinforcement perpendicular to axis of member is used, the required area of shear reinforcement A_v and its spacing *s* are computed by

$$\frac{A_v}{s} = \frac{(V_u - \phi V_c)}{\phi f_y d}$$

Research[11.19,11.20] has shown that shear behavior of wide beams with substantial flexural reinforcement is improved if the transverse spacing of stirrup legs across the section is reduced.

CODE

11.5.6.8 — Shear strength V_s shall not be taken greater than $8\sqrt{f_c'}\,b_w d$.

11.6 — Design for torsion

$T = 2A_o q \quad q = \frac{T}{2A_o}$

$q = \tau t$

$T = 2A_o \tau t$

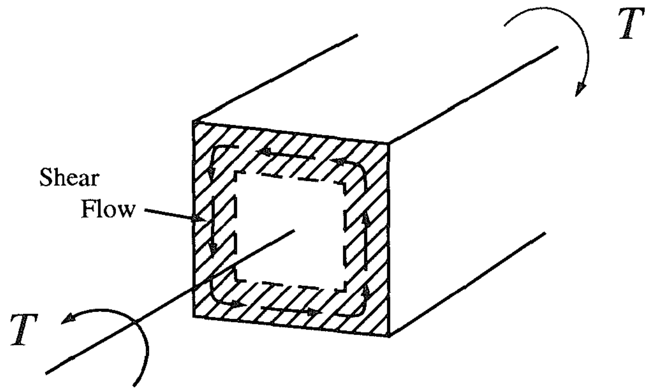
A_o = Area covered by shear flow
-either hollow or solid.

COMMENTARY

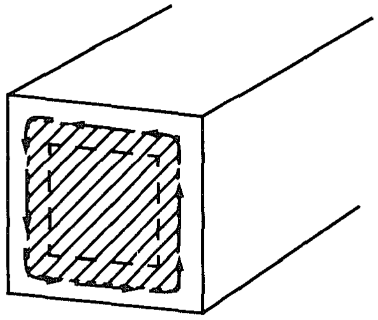
R11.6 — Design for torsion

In the 1995 code the design for torsion is based on a thin-walled tube, space truss analogy. A beam subjected to torsion is idealized as a thin-walled tube with the core concrete cross section in a solid beam neglected as shown in Fig. R11.6(a). Once a reinforced concrete beam has cracked in torsion its torsional resistance is provided primarily by closed stirrups and longitudinal bars located near the surface of the member. In the thin-walled tube analogy the resistance is assumed to be provided by the outer skin of the cross section roughly centered on the closed stirrups. Both hollow and solid sections are idealized as thin-walled tubes both before and after cracking.

In a closed thin-walled tube the shear stresses τ due to torsion act as shown in Fig. R11.6(a). The product of the shear stress τ and the wall thickness t at any point in the perimeter is known as the shear flow, $q = \tau t$. The shear flow q due to



(a) Thin-walled tube



(b) Area enclosed by shear flow path

Fig. R11.6—(a) Thin-walled tube; (b) area enclosed by shear flow path

CODE

11.6.1 — It shall be permitted to neglect torsion effects when the factored torsional moment T_u is less than:

(a) for nonprestressed members:

$$\phi\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

(b) for prestressed members:

$$\phi\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\sqrt{f_c'}}}$$

For members cast monolithically with a slab, the overhanging flange width used in computing A_{cp} and p_{cp} shall conform to 13.2.4.

COMMENTARY

torsion is constant at all points around the perimeter of the tube. The path along which it acts extends around the tube at midthickness of the walls of the tube. At any point along the perimeter of the tube the shear stress due to torsion is $\tau = T/(2A_o t)$ where A_o is the gross area enclosed by the shear flow path, shown shaded in Fig. R11.6(b), and t is the thickness of the wall at the point where τ is being computed. The shear flow path follows the midplane of the walls of the tube and A_o is the area enclosed by the midplane of the walls of the tube. For a hollow member with continuous walls, A_o includes the area of the hole.

In the 1995 code, the former elliptical interaction between the shear carried by the concrete, V_c, and the torsion carried by the concrete has been eliminated. V_c remains constant at the value it has when there is no torsion, and the torsion carried by the concrete is always taken as zero.

The design procedure is derived and compared to tests in Reference 11.21.

R11.6.1 — Torques that do not exceed approximately one-quarter of the cracking torque T_{cr} will not cause a structurally significant reduction in either the flexural or shear strength and hence can be ignored. The cracking torsion under pure torsion T_{cr} is derived by replacing the actual section with an equivalent thin-walled tube with a wall thickness t prior to cracking of $0.75A_{cp}/p_{cp}$ and an area enclosed by the wall centerline A_o equal to $2A_{cp}/3$. Cracking is assumed to occur when the principal tensile stress reaches $4\sqrt{f_c'}$. In a nonprestressed beam loaded with torsion alone, the principal tensile stress is equal to the torsional shear stress, $\tau = T/(2A_o t)$. Thus, cracking occurs when τ reaches $4\sqrt{f_c'}$, giving the cracking torque T_{cr} as:

$$T_{cr} = 4\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

The limit set in 11.6.1 is one-quarter of this value. The stress at cracking $4\sqrt{f_c'}$ has purposely been taken as a lower bound value.

For prestressed members the torsional cracking load is increased by the prestress. A Mohr's Circle analysis based on average stresses indicates the torque required to cause a principle tensile stress equal to $4\sqrt{f_c'}$ is $\sqrt{1+f_{pc}/(4\sqrt{f_c'})}$ times the corresponding torque in a nonprestressed beam.

For an isolated member with or without flanges, A_{cp} is the area of the entire cross section including the area of voids in hollow cross sections, and p_{cp} is the perimeter of the entire cross section. For a T-beam cast monolithically with a slab, A_{cp} and p_{cp} can include portions of the adjacent slabs conforming to 13.2.4.

CODE

11.6.2 — Calculation of factored torsional moment T_u

11.6.2.1 — If the factored torsional moment T_u in a member is required to maintain equilibrium and exceeds the minimum value given in 11.6.1, the member shall be designed to carry that torsional moment in accordance with 11.6.3 through 11.6.6.

11.6.2.2 — In a statically indeterminate structure where reduction of the torsional moment in a member can occur due to redistribution of internal forces upon cracking, the maximum factored torsional moment T_u shall be permitted to be reduced to

(a) for nonprestressed members, at the sections described in 11.6.2.4:

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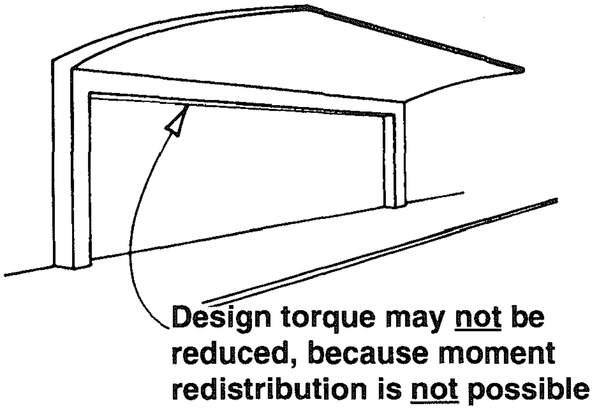
Design torque may not be reduced, because moment redistribution is not possible

Fig. R11.6.2.1—Design torque may not be reduced (11.6.2.1)

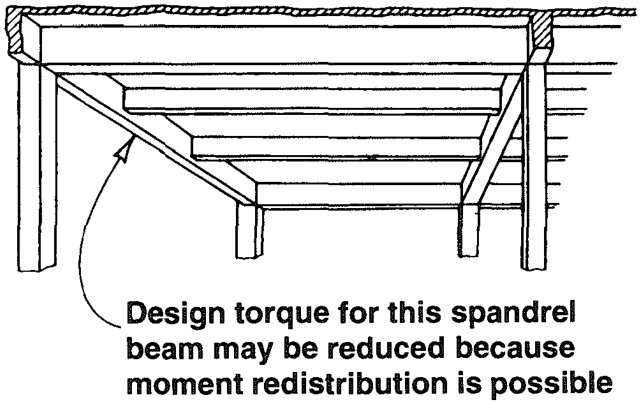
Design torque for this spandrel beam may be reduced because moment redistribution is possible

Fig. R11.6.2.2—Design torque may be reduced (11.6.2.2)

R11.6.2 — Calculation of factored torsional moment T_u

R11.6.2.1 and R11.6.2.2 — In designing for torsion in reinforced concrete structures, two conditions may be identified:[11.22,11.23]

(a) The torsional moment cannot be reduced by redistribution of internal forces (11.6.2.1). This is referred to as "equilibrium torsion," since the torsional moment is required for the structure to be in equilibrium.

For this condition, illustrated in Fig. R11.6.2.1, torsion reinforcement designed according to 11.6.3 through 11.6.6 must be provided to resist the total design torsional moments.

(b) The torsional moment can be reduced by redistribution of internal forces after cracking (11.6.2.2) if the tor-

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$$\phi 4\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

(b) for prestressed members, at the sections described in 11.6.2.5:

$$\phi 4\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\sqrt{f_c'}}}$$

In such a case, the correspondingly redistributed bending moments and shears in the adjoining members shall be used in the design of these members.

11.6.2.3 — Unless determined by a more exact analysis, it shall be permitted to take the torsional loading from a slab as uniformly distributed along the member.

11.6.2.4 — In nonprestressed members, sections located less than a distance ***d*** from the face of a support shall be designed for not less than the torsion T_u computed at a distance ***d***. If a concentrated torque occurs within this distance, the critical section for design shall be at the face of the support.

11.6.2.5 — In prestressed members, sections located less than a distance ***h/2*** from the face of a support shall be designed for not less than the torsion T_u computed at a distance ***h/2***. If a concentrated torque occurs within this distance, the critical section for design shall be at the face of the support.

11.6.3 — Torsional moment strength

11.6.3.1 — The cross-sectional dimensions shall be such that:

COMMENTARY

sion arises from the member twisting in order to maintain compatibility of deformations. This type of torsion is referred to as "compatibility torsion."

For this condition, illustrated in Fig. R11.6.2.2, the torsional stiffness before cracking corresponds to that of the uncracked section according to St. Venant's theory. At torsional cracking, however, a large twist occurs under an essentially constant torque, resulting in a large redistribution of forces in the structure.[11.22,11.23] The cracking torque under combined shear, flexure, and torsion corresponds to a principle tensile stress somewhat less than the $4\sqrt{f_c'}$ quoted in R11.6.1.

When the torsional moment exceeds the cracking torque, a maximum factored torsional moment equal to the cracking torque may be assumed to occur at the critical sections near the faces of the supports. This limit has been established to control the width of torsional cracks.

Section 11.6.2.2 applies to typical and regular framing conditions. With layouts that impose significant torsional rotations within a limited length of the member, such as a heavy torque loading located close to a stiff column, or a column that rotates in the reverse directions because of other loading, a more exact analysis is advisable.

When the factored torsional moment from an elastic analysis based on uncracked section properties is between the values in 11.6.1 and the values given in this section, torsion reinforcement should be designed to resist the computed torsional moments.

R11.6.2.4 and R11.6.2.5 — It is not uncommon for a beam to frame into one side of a girder near the support of the girder. In such a case a concentrated shear and torque are applied to the girder.

R11.6.3 — Torsional moment strength

R11.6.3.1 —The size of a cross section is limited for two reasons, first to reduce unsightly cracking and second to

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(a) for solid sections:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \qquad (11\text{-}18)$$

(b) for hollow sections:

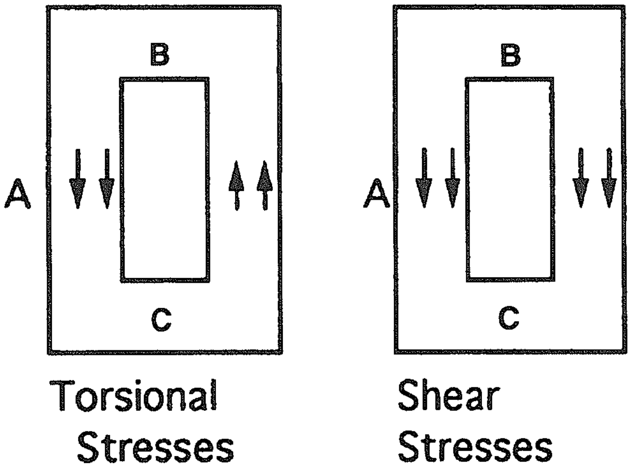
$$\left(\frac{V_u}{b_w d}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f_c'}\right) \qquad (11\text{-}19)$$

Shear caused by SHEAR + SHEAR CAUSED BY TORSION $\le \phi$

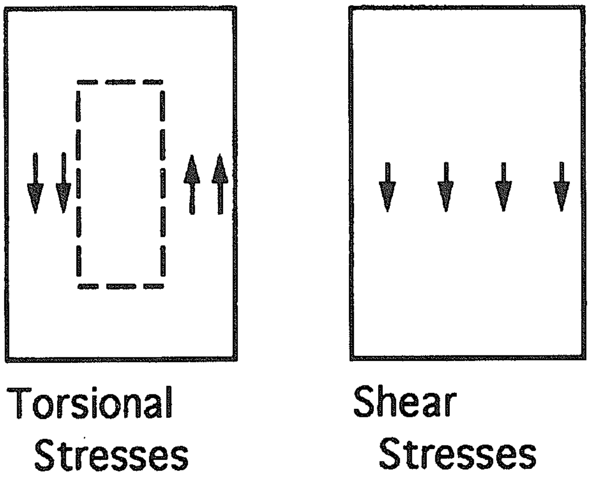
COMMENTARY

prevent crushing of the surface concrete due to inclined compressive stresses due to shear and torsion. In Eq. (11-18) and (11-19), the two terms on the left hand side are the shear stresses due to shear and torsion. The sum of these stresses may not exceed the stress causing shear cracking plus $8\sqrt{f_c'}$, similar to the limiting strength given in 11.5.6.8 for shear without torsion. The limit is expressed in terms of V_c to allow its use for nonprestressed or prestressed concrete. It was originally derived on the basis of crack control. It is not necessary to check against crushing of the web since this happens at higher shear stresses.

In a hollow section, the shear stresses due to shear and torsion both occur in the walls of the box as shown in Fig. 11.6.3.1(a) and hence are directly additive at point A as given in Eq. (11-19). In a solid section the shear stresses due to torsion act in the "tubular" outside section while the shear stresses due to V_u are spread across the width of the section



(a) Hollow Section



(b) Solid Section

Fig. R11.6.3.1—Addition of torsional and shear stresses

11.6.3.2—If the wall thickness varies around the perimeter of a hollow section, Eq. (11-19) shall be evaluated at the location where the left-hand side of Eq. (11-19) is a maximum.

11.6.3.3 — If the wall thickness is less than A_{oh}/p_h, the second term in Eq. (11-19) shall be taken as:

$$\left(\frac{T_u}{1.7A_{oh}t}\right)$$

where t is the thickness of the wall of the hollow section at the location where the stresses are being checked.

11.6.3.4 — Design yield strength of nonprestressed torsion reinforcement shall not exceed 60,000 psi.

11.6.3.5 — The reinforcement required for torsion shall be determined from:

$$\phi T_n \geq T_u \qquad (11\text{-}20)$$

as shown in Fig. R11.6.3.1(b). For this reason stresses are combined in Eq. (11-18) using the square root of the sum of the squares rather than by direct addition.

R11.6.3.2 — If the wall thickness varies around the perimeter of a hollow section, 11.6.3.2 requires that Eq. (11-19) be evaluated at the point in the cross section where the left side of Eq. (11-19) is a maximum. Generally, this will be on the wall where the torsional and shearing stresses are additive [Point A in Fig. R11.6.3.1(a)]. If the top or bottom flanges are thinner than the vertical webs, it may be necessary to evaluate Eq. (11-19) at points B and C in Fig. R11.6.3.1(a). At these points the stresses due to the shear force are usually negligible.

R11.6.3.4 — Limiting the design yield strength of torsion reinforcement to 60,000 psi provides a control on diagonal crack width.

R11.6.3.5 — The factored torsional resistance ϕT_n must equal or exceed the torsion T_u due to the factored loads. In the calculation of T_n, all the torque is assumed to be resisted by stirrups and longitudinal steel with $T_c = 0$. At the same time, the shear resisted by concrete V_c is assumed to be unchanged by the presence of torsion. For beams with V_u greater than about $0.8\phi V_c$ the resulting amount of combined shear and torsional reinforcement is essentially the same as required by the 1989 Code. For smaller values of V_u, more shear and torsion reinforcement will be required.

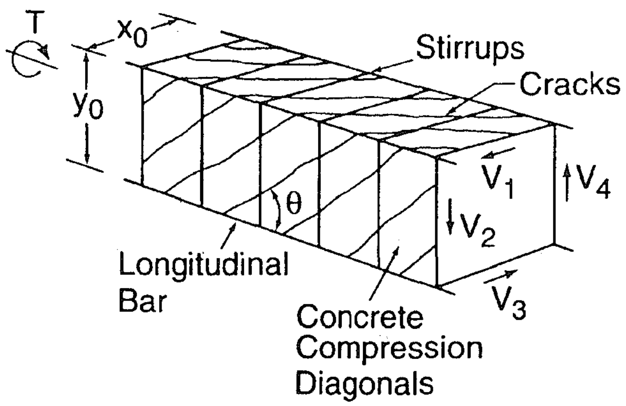


Fig. R11.6.3.6(a)—Space truss analogy

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11.6.3.6 — The transverse reinforcement for torsion shall be designed using:

$$T_n = \frac{2A_oA_tf_{yv}}{s}\cot\theta \qquad (11\text{-}21)$$

where A_o shall be determined by analysis except that it shall be permitted to take A_o equal to $0.85A_{oh}$; θ shall not be taken smaller than 30 deg nor larger than 60 deg. It shall be permitted to take θ equal to:

(a) 45 deg for nonprestressed members or members with less prestress than in (b),

(b) 37.5 deg for prestressed members with an effective prestress force not less than 40 percent of the tensile strength of the longitudinal reinforcement.

11.6.3.7 — The additional longitudinal reinforcement required for torsion shall not be less than:

$$A_\ell = \frac{A_t}{s}p_h\left(\frac{f_{yv}}{f_{y\ell}}\right)\cot^2\theta \qquad (11\text{-}22)$$

where θ shall be the same value used in Eq. (11-21) and A_t/s shall be taken as the amount computed from Eq. (11-21) not modified in accordance with 11.6.5.2 or 11.6.5.3.

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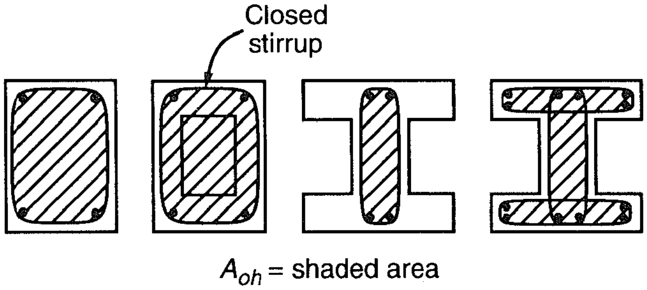

Fig. R11.6.3.6(b)—Definition of A_{oh}

R11.6.3.6 — Eq. (11-21) is based on the space truss analogy shown in Fig. R11.6.3.6(a) with compression diagonals at an angle θ, assuming the concrete carries no tension and the reinforcement yields. After torsional cracking develops, the torsional resistance is provided mainly by closed stirrups, longitudinal bars, and compression diagonals. The concrete outside these stirrups is relatively ineffective. For this reason A_o, the area enclosed by the shear flow path around the perimeter of the tube, is defined after cracking in terms of A_{oh}, the area enclosed by the centerline of the outermost closed hoops. The area A_{oh} is shown in Fig. R11.6.3.6(b) for various cross sections. In an I-, T-, or L-shaped section A_{oh} is taken as that area enclosed by the outermost legs of interlocking stirrups as shown in Fig. R11.6.3.6(b). The expression for A_o given by Hsu[11.24] may be used if greater accuracy is desired.

The shear flow q in the walls of the tube, discussed in R11.6, can be resolved into the shear forces V_1 to V_4 acting in the individual sides of the tube or space truss, as shown in Fig. R11.6.3.6(a).

The angle θ can be obtained by analysis[11.24] or may be taken to be equal to the values given in subsections (a) and (b). The same value of θ must be used in both Eq. (11-21) and (11-22). As θ gets smaller, the amount of stirrups required by Eq. (11-21) decreases. At the same time the amount of longitudinal steel required by Eq. (11-22) increases.

R11.6.3.7 — Fig. R11.6.3.7 shows one side of the equivalent tube assumed to resist torsion. The torsional cracks

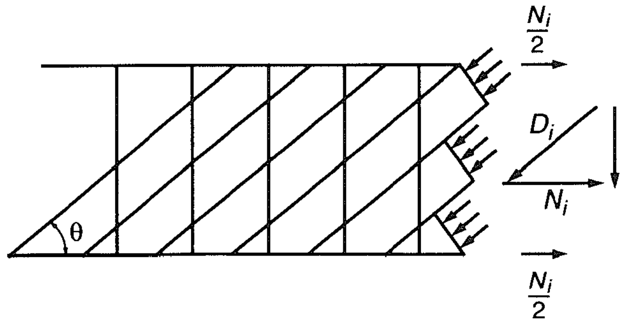

Fig. R11.6.3.7—resolution of shear force into diagonal compression and axial tension

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11.6.3.8 — Reinforcement required for torsion shall be added to that required for the shear, moment and axial force that act in combination with the torsion. The most restrictive requirements for reinforcement spacing and placement must be met.

COMMENTARY

have formed a series of inclined concrete struts crossed by stirrups. The component of the shear flow resisting the torque and acting on this side of the tube is V_i. This can be resolved into a diagonal compressive force D_i parallel to the concrete diagonals, and an axial tension force N_i. The force D_i causes diagonal compressive stresses in the walls of the tube. Longitudinal reinforcement with a capacity of $A_\ell f_{y\ell}$ must be provided to resist the sum ΣN_i of the axial forces N_i acting on all of the walls of the tube.

Frequently, the maximum allowable stirrup spacing governs the amount of stirrups provided. Furthermore, when combined shear and torsion act, the total stirrup area is the sum of the amounts provided for shear and torsion. To avoid the need to provide excessive amounts of longitudinal reinforcement, 11.6.3.7 states that the A_t/s used in calculating A_ℓ at any given section shall be taken as the A_t/s calculated at that section using Eq. (11-21).

R11.6.3.8 — The stirrup requirements for torsion and shear are added and stirrups are provided to supply at least the total amount required. Since the stirrup area A_v for shear is defined in terms of all the legs of a given stirrup while the stirrup area A_t for torsion is defined in terms of one leg only, the addition of stirrups is carried out as follows:

$$\text{Total}\left(\frac{A_{v+t}}{s}\right) = \frac{A_v}{s} + 2\frac{A_t}{s}$$

If a stirrup group had four legs for shear, only the legs adjacent to the sides of the beam would be included in this summation since the inner legs would be ineffective for torsion.

The longitudinal reinforcement required for torsion is added at each section to the longitudinal reinforcement required for bending moment that acts at the same time as the torsion. The longitudinal reinforcement is then chosen for this sum, but should not be less than the amount required for the maximum bending moment at that section if this exceeds the moment acting at the same time as the torsion. If the maximum bending moment occurs at one section, say midspan, while the maximum torsional moment occurs at another, such as the support, the total longitudinal steel required may be less than that obtained by adding the maximum flexural steel plus the maximum torsional steel. In such a case the required longitudinal steel is evaluated at several locations.

The most restrictive requirements for spacing, cut-off points, and placement for flexural, shear, and torsional steel must be satisfied. The flexural steel must be extended a distance d, but not less than $12d_b$, past where it is no longer needed for flexure as required in 12.10.3.

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11.6.3.9 — It shall be permitted to reduce the area of longitudinal torsion reinforcement in the flexural compression zone by an amount equal to $M_u/(0.9df_{y\ell})$, where M_u is the factored moment acting at the section in combination with T_u, except that the reinforcement provided shall not be less than that required by 11.6.5.3 or 11.6.6.2.

11.6.3.10 — In prestressed beams:

(a) the total longitudinal reinforcement including tendons at each section shall resist the factored bending moment at that section plus an additional concentric longitudinal tensile force equal to $A_\ell f_{y\ell}$, based on the factored torsion at that section, and

(b) the spacing of the longitudinal reinforcement including tendons shall satisfy the requirements in 11.6.6.2.

11.6.3.11 — In prestressed beams, it shall be permitted to reduce the area of longitudinal torsional reinforcement on the side of the member in compression due to flexure below that required by 11.6.3.10 in accordance with 11.6.3.9.

11.6.4 — Details of torsional reinforcement

11.6.4.1 — Torsion reinforcement shall consist of longitudinal bars or tendons and one or more of the following:

(a) closed stirrups or closed ties, perpendicular to the axis of the member, or

(b) a closed cage of welded wire fabric with transverse wires perpendicular to the axis of the member, or

(c) in nonprestressed beams, spiral reinforcement.

COMMENTARY

R11.6.3.9 — The longitudinal tension due to torsion is offset in part by the compression in the flexural compression zone, allowing a reduction in the longitudinal torsion steel required in the compression zone.

R11.6.3.10 — As explained in R11.6.3.7, torsion causes an axial tension force. In a nonprestressed beam this force is resisted by longitudinal reinforcement having an axial tensile capacity of $A_\ell f_{y\ell}$. This steel is in addition to the flexural reinforcement and is distributed uniformly around the sides of the perimeter so that the resultant of $A_\ell f_{y\ell}$ acts along the axis of the member.

In a prestressed beam the same technique (providing additional reinforcing bars with capacity $A_\ell f_{y\ell}$) can be followed, or the designer can use any overcapacity of the tendons to resist some of the axial force $A_\ell f_{y\ell}$ as outlined in the next paragraph.

In a prestressed beam the tendon stress at ultimate at the section of the maximum moment is f_{ps}. At other sections the tendon stress at ultimate will be between f_{se} and f_{ps}. A portion of the $A_\ell f_{y\ell}$ force acting on the sides of the perimeter where the tendons are located can be resisted by a force $A_{ps}\Delta f_p$ in the tendons where Δf_p is f_{ps} minus the tendon stress due to flexure at the ultimate load at the section in question. This can be taken as M_u at the section, divided by $(\phi 0.9 d_p A_{ps})$, but Δf_p should not be more than 60 ksi. Longitudinal reinforcing bars will be required on the other sides of the member to provide the remainder of the $A_\ell f_{y\ell}$ force, or to satisfy the spacing requirements given in 11.6.6.2, or both.

R11.6.4 — Details of torsional reinforcement

R11.6.4.1 — Both longitudinal and closed transverse reinforcement are required to resist the diagonal tension stresses due to torsion. The stirrups must be closed, since inclined cracking due to torsion may occur on all faces of a member.

In the case of sections subjected primarily to torsion, the concrete side cover over the stirrups spalls off at high torques.[11.25] This renders lapped-spliced stirrups ineffective, leading to a premature torsional failure.[11.26] In such cases, closed stirrups should not be made up of pairs of U-stirrups lapping one another.

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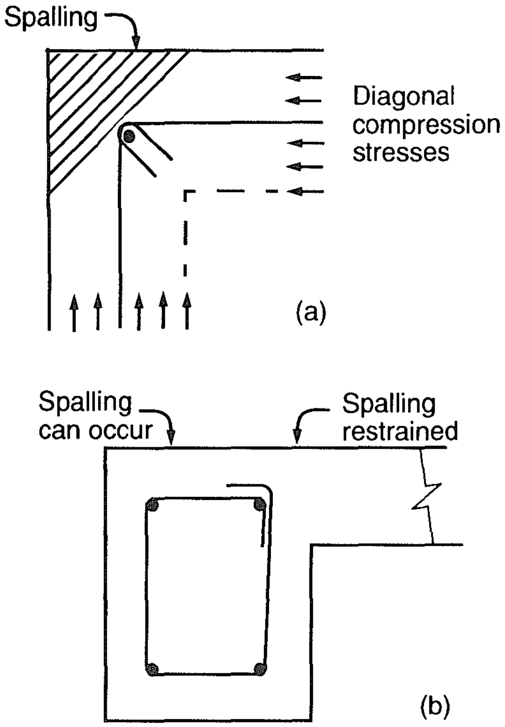

Fig. R11.6.4.2—Spalling of corners of beams loaded in torsion

11.6.4.2 — Transverse torsional reinforcement shall be anchored by one of the following:

(a) a 135 deg standard hook around a longitudinal bar, or

(b) according to 12.13.2.1, 12.13.2.2, or 12.13.2.3 in regions where the concrete surrounding the anchorage is restrained against spalling by a flange or slab or similar member.

11.6.4.3 — Longitudinal torsion reinforcement shall be developed at both ends.

11.6.4.4 — For hollow sections in torsion, the distance from the centerline of the transverse torsional reinforcement to the inside face of the wall of the hollow section shall not be less than $0.5A_{oh}/p_h$.

11.6.5 — Minimum torsion reinforcement

11.6.5.1 — A minimum area of torsion reinforcement shall be provided in all regions where the factored tor-

R11.6.4.2 — When a rectangular beam fails in torsion, the corners of the beam tend to spall off due to the inclined compressive stresses in the concrete diagonals of the space truss changing direction at the corner as shown in Fig. 11.6.4.2(a). In tests, closed stirrups anchored by 90 deg hooks failed when this occurred.[11.25] For this reason, 135 deg hooks are preferable for torsional stirrups in all cases. In regions where this spalling is prevented by an adjacent slab or flange, 11.6.4.2(b) relaxes this and allows 90 deg hooks.

R.11.6.4.3 — If high torsion acts near the end of a beam, the longitudinal torsion reinforcement must be adequately anchored. Sufficient development length must be provided outside the inner face of the support to develop the needed tension force in the bars or tendons. In the case of bars this may require hooks or horizontal U-shaped bars lapped with the longitudinal torsion reinforcement.

R11.6.4.4 — The closed stirrups provided for torsion in a hollow section should be located in the outer half of the wall thickness effective for torsion where the wall thickness can be taken as A_{oh}/p_h.

R11.6.5 — Minimum torsion reinforcement

R11.6.5.1 and R11.6.5.2 — If a member is subject to a factored torsional moment T_u greater than the values speci-

CODE

sional moment T_u exceeds the values specified in 11.6.1.

11.6.5.2 — Where torsional reinforcement is required by 11.6.5.1, the minimum area of transverse closed stirrups shall be computed by:

$$(A_v + 2A_t) = \frac{50 b_w s}{f_{yv}} \tag{11-23}$$

11.6.5.3 — Where torsional reinforcement is required by 11.6.5.1, the minimum total area of longitudinal torsional reinforcement shall be computed by:

$$A_{\ell,min} = \frac{5\sqrt{f_c'}A_{cp}}{f_{y\ell}} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yv}}{f_{y\ell}} \tag{11-24}$$

where A_t/s shall not be taken less than $25b_w/f_{yv}$.

11.6.6 — Spacing of torsion reinforcement

11.6.6.1 — The spacing of transverse torsion reinforcement shall not exceed the smaller of $p_h/8$ or 12 in.

11.6.6.2 — The longitudinal reinforcement required for torsion shall be distributed around the perimeter of the closed stirrups with a maximum spacing of 12 in. The longitudinal bars or tendons shall be inside the stirrups. There shall be at least one longitudinal bar or tendon in each corner of the stirrups. Bars shall have a diameter at least 1/24 of the stirrup spacing, but not less than a No. 3 bar.

11.6.6.3 — Torsion reinforcement shall be provided for a distance of at least $(b_t + d)$ beyond the point theoretically required.

11.7 — Shear-friction

11.7.1 — Provisions of 11.7 are to be applied where it is appropriate to consider shear transfer across a given plane, such as: an existing or potential crack, an

COMMENTARY

fied in 11.6.1, the minimum amount of transverse web reinforcement for combined shear and torsion is $50b_w s/f_{yv}$. The differences in the definition of A_v and the symbol A_t should be noted; A_v is the area of two legs of a closed stirrup while A_t is the area of only one leg of a closed stirrup.

R11.6.5.3 — Reinforced concrete beam specimens with less than 1 percent torsional reinforcement by volume have failed in pure torsion at torsional cracking.[11.21] In the 1989 and prior codes, a relationship was presented which required about 1 percent torsional reinforcement in beams loaded in pure torsion and less in beams with combined shear and torsion, as a function of the ratio of shear stresses due to torsion and shear. Eq. (11-24) was simplified by assuming a single value of this reduction factor and results in a volumetric ratio of about 0.5 percent.

R11.6.6 — Spacing of torsion reinforcement

R11.6.6.1 — The spacing of the stirrups is limited to ensure the development of the ultimate torsional strength of the beam, to prevent excessive loss of torsional stiffness after cracking, and to control crack widths. For a square cross section the $p_h/8$ limitation requires stirrups at $d/2$, which corresponds to 11.5.4.1.

R11.6.6.2 — In R11.6.3.7 it was shown that longitudinal reinforcement is needed to resist the sum of the longitudinal tensile forces due to torsion in the walls of the thin-walled tube. Since the force acts along the centroidal axis of the section, the centroid of the additional longitudinal reinforcement for torsion should approximately coincide with the centroid of the section. The code accomplishes this by requiring the longitudinal torsional reinforcement to be distributed around the perimeter of the closed stirrups. Longitudinal bars or tendons are required in each corner of the stirrups to provide anchorage for the legs of the stirrups. Corner bars have also been found to be very effective in developing torsional strength and in controlling cracks.

R11.6.6.3 — The distance $(b_t + d)$ beyond the point theoretically required for torsional reinforcement is larger than that used for shear and flexural reinforcement because torsional diagonal tension cracks develop in a helical form.

R11.7 — Shear-friction

R11.7.1 — With the exception of 11.7, virtually all provisions regarding shear are intended to prevent diagonal tension failures rather than direct shear transfer failures. The

CODE

interface between dissimilar materials, or an interface between two concretes cast at different times.

11.7.2 — Design of cross sections subject to shear transfer as described in 11.7.1 shall be based on Eq. (11-1), where V_n is calculated in accordance with provisions of 11.7.3 or 11.7.4.

11.7.3 — A crack shall be assumed to occur along the shear plane considered. The required area of shear-friction reinforcement A_{vf} across the shear plane shall be designed using either 11.7.4 or any other shear transfer design methods that result in prediction of strength in substantial agreement with results of comprehensive tests.

11.7.3.1 — Provisions of 11.7.5 through 11.7.10 shall apply for all calculations of shear transfer strength.

COMMENTARY

purpose of 11.7 is to provide design methods for conditions where shear transfer should be considered: an interface between concretes cast at different times, an interface between concrete and steel, reinforcement details for precast concrete structures, and other situations where it is considered appropriate to investigate shear transfer across a plane in structural concrete. (See References 11.27 and 11.28).

R11.7.3 — Although uncracked concrete is relatively strong in direct shear there is always the possibility that a crack will form in an unfavorable location. The shear-friction concept assumes that such a crack will form, and that reinforcement must be provided across the crack to resist relative displacement along it. When shear acts along a crack, one crack face slips relative to the other. If the crack faces are rough and irregular, this slip is accompanied by separation of the crack faces. At ultimate, the separation is sufficient to stress the reinforcement crossing the crack to its yield point. The reinforcement provides a clamping force $A_{vf}f_y$ across the crack faces. The applied shear is then resisted by friction between the crack faces, by resistance to the shearing off of protrusions on the crack faces, and by dowel action of the reinforcement crossing the crack. Successful application of 11.7 depends on proper selection of the location of an assumed crack.[11.13,11.27]

The relationship between shear-transfer strength and the reinforcement crossing the shear plane can be expressed in various ways. Eq. (11-25) and (11-26) of 11.7.4 are based on the shear-friction model. This gives a conservative prediction of shear-transfer strength. Other relationships which give a closer estimate of shear-transfer strength[11.13,11.29,11.30] can be used under the provisions of 11.7.3. For example, when the shear-friction reinforcement is perpendicular to the shear plane, the shear strength V_n is given by[11.29,11.30]

$$V_n = 0.8A_{vf}f_y + A_cK_1$$

where A_c is the area of concrete section resisting shear transfer (sq in.) and K_1 = **400 psi** for normal weight concrete, 200 psi for "all-lightweight" concrete, and 250 psi for "sand-lightweight" concrete. These values of K_1 apply to both monolithically cast concrete and to concrete cast against hardened concrete with a rough surface, as defined in 11.7.9.

In this equation, the first term represents the contribution of friction to shear-transfer resistance (0.8 representing the coefficient of friction). The second term represents the sum

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11.7.4 — Shear-friction design method

11.7.4.1 — When shear-friction reinforcement is perpendicular to shear plane, shear strength V_n shall be computed by

$$V_n = A_{vf} f_y \mu \qquad (11\text{-}25)$$

where μ is coefficient of friction in accordance with 11.7.4.3.

11.7.4.2 — When shear-friction reinforcement is inclined to shear plane, such that the shear force produces tension in shear-friction reinforcement, shear strength V_n shall be computed by

$$V_n = A_{vf} f_y (\mu \sin \alpha_f + \cos \alpha_f) \qquad (11\text{-}26)$$

where α_f is angle between shear-friction reinforcement and shear plane.

COMMENTARY

of: (1) the resistance to shearing of protrusions on the crack faces, and (2) the dowel action of the reinforcement.

When the shear-friction reinforcement is inclined to the shear plane, such that the shear force produces tension in that reinforcement, the shear strength V_n is given by

$$V_n = A_{vf} f_y\,(0.8 \sin\alpha_f + \cos\alpha_f) + A_c K_1 \sin^2\alpha_f$$

where α_f is the angle between the shear-friction reinforcement and the shear plane, (i.e. $\mathbf{0 < \alpha_f < 90}$ **deg**).

When using the modified shear-friction method, the terms $(A_{vf} f_y / A_c)$ or $(A_{vf} f_y \sin \alpha_f / A_c)$ must not be less than 200 psi for the design equations to be valid.

R11.7.4 — Shear-friction design method

R11.7.4.1 — The required area of shear-transfer reinforcement A_{vf} is computed using

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

The specified upper limit on shear strength must also be observed.

R11.7.4.2 — When the shear-friction reinforcement is inclined to the shear plane, such that the component of the shear force parallel to the reinforcement tends to produce tension in the reinforcement, as shown in Fig. R11.7.4, part of the shear is resisted by the component parallel to the shear plane of the tension force in the reinforcement.[11.30] Eq. (11-26) must be used only when the shear force component parallel to the reinforcement produces tension in the reinforcement, as shown in Fig. R11.7.4. When α_f is greater than 90 deg, the relative movement of the surfaces tends to compress the bar and Eq. (11-26) is not valid.

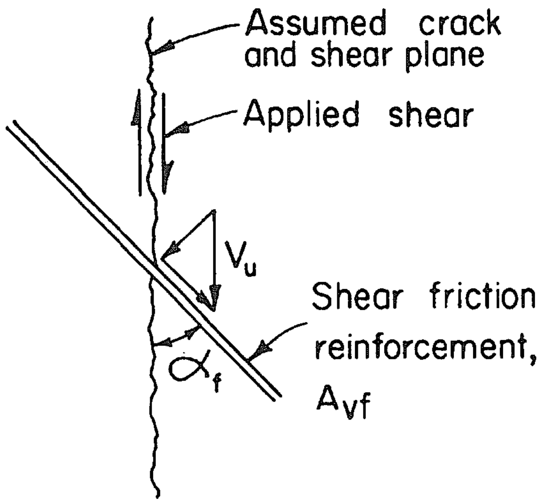


Fig. R11.7.4—Shear-friction reinforcement at an angle to assumed crack

11.7.4.3 — The coefficient of friction μ in Eq. (11-25) and Eq. (11-26) shall be

Concrete placed monolithically	1.4λ
Concrete placed against hardened concrete with surface intentionally roughened as specified in 11.7.9	1.0λ
Concrete placed against hardened concrete not intentionally roughened	0.6λ
Concrete anchored to as-rolled structural steel by headed studs or by reinforcing bars (see 11.7.10)	0.7λ

where λ = 1.0 for normal weight concrete, 0.85 for "sand-lightweight" concrete and 0.75 for "all lightweight" concrete. Linear interpolation shall be permitted when partial sand replacement is used.

11.7.5 — Shear strength V_n shall not be taken greater than $0.2f_c'A_c$ nor 800 A_c in pounds, where A_c is area of concrete section resisting shear transfer.

11.7.6 — Design yield strength of shear-friction reinforcement shall not exceed 60,000 psi.

11.7.7 — Net tension across shear plane shall be resisted by additional reinforcement. Permanent net compression across shear plane shall be permited to be taken as additive to the force in the shear-friction reinforcement $A_{vf}f_y$ when calculating required A_{vf}.

R11.7.4.3 — In the shear-friction method of calculation, it is assumed that all the shear resistance is due to the friction between the crack faces. It is, therefore, necessary to use artificially high values of the coefficient of friction in the shear-friction equations, so that the calculated shear strength will be in reasonable agreement with test results. For the case of concrete cast against hardened concrete not roughened in accordance with 11.7.9, shear resistance is primarily due to dowel action of the reinforcement and tests[11.31] indicate that reduced value of $\mu = 0.6\lambda$ specified for this case is appropriate.

The value of μ specified for concrete placed against as-rolled structural steel relates to the design of connections between precast concrete members, or between structural steel members and structural concrete members. The shear-transfer reinforcement may be either reinforcing bars or headed stud shear connectors; also, field welding to steel plates after casting of concrete is common. The design of shear connectors for composite action of concrete slabs and steel beams is not covered by these provisions, but should be in accordance with Reference 11.32.

R11.7.5 — This upper limit on shear strength is specified because Eq. (11-25) and (11-26) become unconservative if V_n has a greater value.

R11.7.7 — If a resultant tensile force acts across a shear plane, reinforcement to carry that tension must be provided in addition to that provided for shear transfer. Tension may be caused by restraint of deformations due to temperature change, creep, and shrinkage. Such tensile forces have caused failures, particularly in beam bearings.

When moment acts on a shear plane, the flexural tension stresses and flexural compression stresses are in equilibrium. There is no change in the resultant compression $A_{vf}f_y$ acting across the shear plane and the shear-transfer strength is not changed. It is therefore not necessary to provide additional reinforcement to resist the flexural tension stresses, unless the required flexural tension reinforcement exceeds the amount of shear-transfer reinforcement provided in the flexural tension zone. This has been demonstrated experimentally.[11.33]

It has also been demonstrated experimentally[11.28] that if a resultant compressive force acts across a shear plane, the shear-transfer strength is a function of the sum of the resultant compressive force and the force $A_{vf}f_y$ in the shear-friction reinforcement. In design, advantage should be taken of the existence of a compressive force across the shear plane to reduce the amount of shear-friction reinforcement

CODE

11.7.8 — Shear-friction reinforcement shall be appropriately placed along the shear plane and shall be anchored to develop the specified yield strength on both sides by embedment, hooks, or welding to special devices.

11.7.9 — For the purpose of 11.7, when concrete is placed against previously hardened concrete, the interface for shear transfer shall be clean and free of laitance. If μ is assumed equal to 1.0λ, interface shall be roughened to a full amplitude of approximately $^1/_4$ in.

11.7.10 — When shear is transferred between as-rolled steel and concrete using headed studs or welded reinforcing bars, steel shall be clean and free of paint.

11.8 — Special provisions for deep flexural members

11.8.1 — The provisions of 11.8 shall apply to members with ℓ_n/d less than 5 that are loaded on one face and supported on the opposite face so that compression struts can develop between the loads and the supports. See also 12.10.6.

11.8.2 — The design of simply supported deep flexural members for shear shall be based on Eq. (11-1) and (11-2), where the shear strength V_c shall be in accordance with 11.8.6 or 11.8.7 and the shear strength V_s shall be in accordance with 11.8.8.

COMMENTARY

required, only if it is absolutely certain that the compressive force is permanent.

R11.7.8 — If no moment acts across the shear plane, reinforcement should be uniformly distributed along the shear plane to minimize crack widths. If a moment acts across the shear plane, it is desirable to distribute the shear-transfer reinforcement primarily in the flexural tension zone.

Since the shear-friction reinforcement acts in tension, it must have full tensile anchorage on both sides of the shear plane. Further, the shear-friction reinforcement anchorage must engage the primary reinforcement, otherwise a potential crack may pass between the shear-friction reinforcement and the body of the concrete. This requirement applies particularly to welded headed studs used with steel inserts for connections in precast and cast-in-place concrete. Anchorage may be developed by bond, by a welded mechanical anchorage, or by threaded dowels and screw inserts. Space limitations often require a welded mechanical anchorage. For anchorage of headed studs in concrete see Reference 11.13.

R11.8 — Special provisions for deep flexural members

R11.8.1—The behavior of a deep beam is discussed in References 11.5 and 11.34. For a normal deep beam supporting gravity loads, this section applies if the loads are applied on the top of the beam and the beam is supported on its bottom face. If the loads are applied through the sides or bottom of such a member, the design for shear should be the same as for ordinary beams.

The longitudinal tension reinforcement in deep flexural members should be extended to the supports and adequately anchored by embedment, hooks, or welding to special devices. Truss bars are not recommended.

CODE

11.8.3 — The design of continuous deep flexural members for shear shall be based on 11.1 through 11.5 with 11.8.5 substituted for 11.1.3, or on methods satisfying equilibrium and strength requirements. In either case the design shall also satisfy 11.8.4, 11.8.9, and 11.8.10.

11.8.4 — Shear strength V_n for deep flexural members shall not be taken greater than $8\sqrt{f_c'}\, b_w d$ when ℓ_n/d is less than 2. When ℓ_n/d is between 2 and 5,

$$V_n = \frac{2}{3}\left(10 + \frac{\ell_n}{d}\right)\sqrt{f_c'}\, b_w d \qquad (11\text{-}27)$$

11.8.5 — Critical section for shear measured from face of support shall be taken at a distance $0.15\ell_n$ for uniformly loaded beams and $0.50a$ for beams with concentrated loads, but not greater than d.

11.8.6 — Unless a more detailed calculation is made in accordance with 11.8.7,

$$V_c = 2\sqrt{f_c'}\, b_w d \qquad (11\text{-}28)$$

11.8.7 — Shear strength V_c shall be permitted to be computed by

$$V_c = \left(3.5 - 2.5\frac{M_u}{V_u d}\right)\left(1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \quad (11\text{-}29)$$

except that the term

$$\left(3.5 - 2.5\frac{M_u}{V_u d}\right)$$

shall not exceed 2.5, and V_c shall not be taken greater than $6\sqrt{f_c'}\, b_w d$. M_u is factored moment occurring simultaneously with V_u at the critical section defined in 11.8.5.

11.8.8 — Where factored shear force V_u exceeds shear strength ϕV_c, shear reinforcement shall be provided to satisfy Eq. (11-1) and (11-2), where shear strength V_s shall be computed by

COMMENTARY

R11.8.3 — In a continuous beam, the critical section for shear defined in 11.8.5 occurs at a point where M_u approaches zero. As a result, the second term in Eq. (11-29) becomes large. For this reason, 11.8.3 requires continuous deep beams to be designed for shear according to the regular beam design procedures except that 11.8.5 is used to define the critical section for shear rather than 11.1.3. For a uniformly loaded beam, 11.1.3 allows one to design for the shear at d away from the support. This will frequently approach zero in a deep beam.

As an alternative to the regular beam design procedures, design methods satisfying equilibrium and strength requirements are allowed. Such methods are presented in References 11.34 and 11.35.

R11.8.7 — As the span-depth ratio of a member without web reinforcement decreases, its shear strength increases above the shear causing diagonal tension cracking. In Eq. (11-29) it is assumed that diagonal cracking occurs at the same shear strength as for ordinary beams, but that the shear strength carried by the concrete will be greater than the shear strength causing diagonal cracking.

Designers should note that shear in excess of the shear causing diagonal cracking may result in unsightly cracking unless shear reinforcement is provided.

R11.8.8 — The inclination of diagonal cracking may be greater than 45 deg, therefore, both horizontal and vertical shear reinforcement is required in deep flexural members.[11.36] The relative amounts of horizontal and vertical

CODE

$$V_s = \left[\frac{A_v}{s}\left(\frac{1+\frac{\ell_n}{d}}{12}\right)+\frac{A_{vh}}{s_2}\left(\frac{11-\frac{\ell_n}{d}}{12}\right)\right]f_y d \qquad (11\text{-}30)$$

where A_v is area of shear reinforcement perpendicular to flexural tension reinforcement within a distance s, and A_{vh} is area of shear reinforcement parallel to flexural reinforcement within a distance s_2.

11.8.9 — Area of shear reinforcement A_v shall not be less than $0.0015 b_w s$, and s shall not exceed $d/5$, nor 18 in.

11.8.10 — The area of horizontal shear reinforcement A_{vh} shall not be less than $0.0025 b_w s_2$ and s_2 shall not exceed $d/3$ nor 18 in.

11.8.11 — Shear reinforcement required at the critical section defined in 11.8.5 shall be used throughout the span.

11.9 — Special provisions for brackets and corbels

11.9.1 — Provisions of 11.9 shall apply to brackets and corbels with a shear span-to-depth ratio a/d not greater than unity, and subject to a horizontal tensile force N_{uc} not larger than V_u. Distance d shall be measured at face of support.

11.9.2 — Depth at outside edge of bearing area shall not be less than $0.5d$.

COMMENTARY

shear reinforcement that are selected from Eq. (11-30) may vary, as long as limits on the minimum amount and spacing are observed.

Special attention is directed to the importance of adequate anchorage for the shear reinforcement. Horizontal web reinforcement should be extended to the supports and anchored in the same manner as the tension reinforcement.

R11.8.11 — Based on the analysis carried out at the critical sections specified in 11.8.5, it may be determined that the member either does not need shear reinforcement, or that shear reinforcement is required, in which case it must be used throughout the span.

R11.9 — Special provisions for brackets and corbels

Brackets and corbels are cantilevers having shear span-to-depth ratios not greater than unity, which tend to act as simple trusses or deep beams rather than flexural members designed for shear according to 11.3.

The corbel shown in Fig. R11.9.1 may fail by shearing along the interface between the column and the corbel, by yielding of the tension tie, by crushing or splitting of the compression strut, or by localized bearing or shearing failure under the loading plate. These failure modes are illustrated and are discussed more fully in Reference 11.1. The notation used in 11.9 is illustrated in Fig. R11.9.2.

R11.9.1 — An upper limit of unity for a/d is specified for two reasons. First, for shear span-to-depth ratios exceeding unity, the diagonal tension cracks are less steeply inclined and the use of horizontal stirrups alone as specified in 11.9.4 is not appropriate. Second, the specified method of design has only been validated experimentally for a/d of unity or less. An upper limit is specified for N_{uc} because this method of design has only been validated experimentally for N_{uc} less than or equal to V_u, including N_{uc} equal to zero.

R11.9.2 — A minimum depth is specified at the outside edge of the bearing area so that a premature failure will not occur due to a major diagonal tension crack propagating

11.9.3 — Section at face of support shall be designed to resist simultaneously a shear V_u, a moment $[V_u a + N_{uc}(h - d)]$, and a horizontal tensile force N_{uc}.

11.9.3.1 — In all design calculations in accordance with 11.9, strength reduction factor ϕ shall be taken equal to 0.85.

11.9.3.2 — Design of shear-friction reinforcement A_{vf} to resist shear V_u shall be in accordance with 11.7.

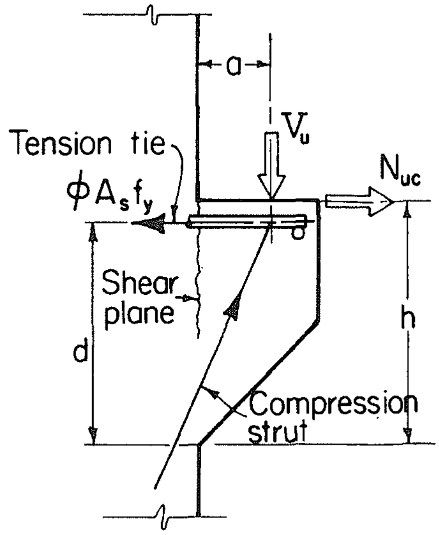

a
Tension tie
$\phi A_s f_y$
V_u
N_{uc}
Shear plane
d
h
Compression strut

Fig. R11.9.1—Structural action of a corbel

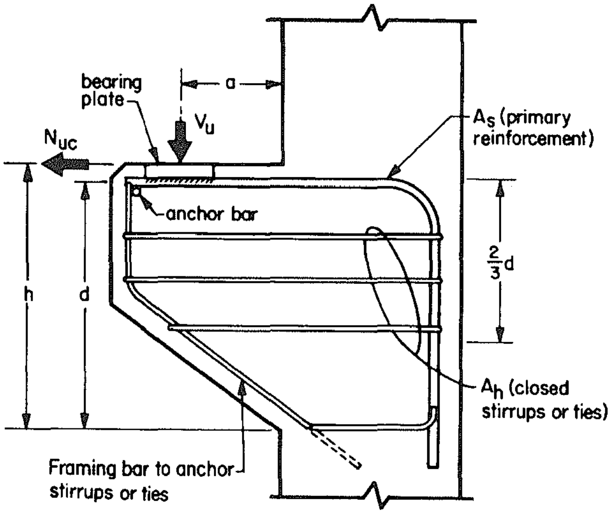

bearing plate
a
V_u
N_{uc}
A_s (primary reinforcement)
anchor bar
h
d
$\frac{2}{3}d$
A_h (closed stirrups or ties)
Framing bar to anchor stirrups or ties

Fig. R11.9.2—Notation used in Section 11.9

from below the bearing area to the outer sloping face of the corbel or bracket. Failures of this type have been observed[11.37] in corbels having depths at the outside edge of the bearing area less than specified in this section of the code.

R11.9.3.1 — Corbel and bracket behavior is predominantly controlled by shear; therefore, a single value of ϕ = **0.85** is specified for all design conditions.

CODE

11.9.3.2.1 — For normal weight concrete, shear strength V_n shall not be taken greater than $0.2f_c'b_wd$ nor $800b_wd$ in pounds.

11.9.3.2.2 — For "all-lightweight" or "sand-lightweight" concrete, shear strength V_n shall not be taken greater than $(0.2 - 0.07a/d)\ f_c'b_wd$ nor $(800 - 280a/d)\ b_wd$ in pounds.

11.9.3.3 — Reinforcement A_f to resist moment $[V_ua + N_{uc}(h - d)]$ shall be computed in accordance with 10.2 and 10.3.

11.9.3.4 — Reinforcement A_n to resist tensile force N_{uc} shall be determined from $N_{uc} \leq \phi A_nf_y$. Tensile force N_{uc} shall not be taken less than $0.2V_u$ unless special provisions are made to avoid tensile forces. Tensile force N_{uc} shall be regarded as a live load even when tension results from creep, shrinkage, or temperature change.

11.9.3.5 — Area of primary tension reinforcement A_s shall be made equal to the greater of $(A_f + A_n)$ or $(2A_{vf}/3 + A_n)$.

11.9.4 — Closed stirrups or ties parallel to A_s, with a total area A_h not less than $0.5\ (A_s - A_n)$, shall be uniformly distributed within two-thirds of the effective depth adjacent to A_s.

COMMENTARY

R11.9.3.2.2 — Tests[11.38] have shown that the maximum shear strength of lightweight concrete corbels or brackets is a function of both f_c' and a/d. No data are available for corbels or brackets made of sand-lightweight concrete. As a result, the same limitations have been placed on both all-lightweight and sand-lightweight brackets and corbels.

R11.9.3.3 — Reinforcement required to resist moment can be calculated using ordinary flexural theory. The factored moment is calculated by summing moments about the flexural reinforcement at the face of support.

R11.9.3.4 — Because the magnitude of horizontal forces acting on corbels or brackets cannot usually be determined with great accuracy, it is specified that N_{uc} be regarded as a live load.

R11.9.3.5 — Tests[11.38] suggest that the total amount of reinforcement $(A_s + A_h)$ required to cross the face of support must be the greater of:

(a) The sum of A_{vf} calculated according to 11.9.3.2 and A_n calculated according to 11.9.3.4, or

(b) The sum of $^3/_2$ times A_f calculated according to 11.9.3.3 and A_n calculated according to 11.9.3.4.

If (a) controls, $A_s = (2A_{vf}/3 + A_n)$ is required as primary tensile reinforcement, and the remaining $A_{vf}/3$ must be provided as closed stirrups parallel to A_s and distributed within $(^2/_3)d$, adjacent to A_s. Section 11.9.4 satisfies this by requiring $A_h = 0.5(2A_{vf}/3)$.

If (b) controls, $A_s = (A_f + A_n)$ is required as primary tension reinforcement, and the remaining $A_f/2$ must be provided as closed stirrups parallel to A_s and distributed within $(^2/_3)d$, adjacent to A_s. Again 11.9.4 satisfies this requirement.

R11.9.4 — Closed stirrups parallel to the primary tension reinforcement are necessary to prevent a premature diagonal tension failure of the corbel or bracket. The required area of closed stirrups $A_h = 0.5\ (A_s - A_n)$ automatically yields the appropriate amounts, as discussed in R11.9.3.5 above.

CODE

11.9.5 — Ratio $\rho = A_s/bd$ shall not be less than **0.04** (f_c'/f_y).

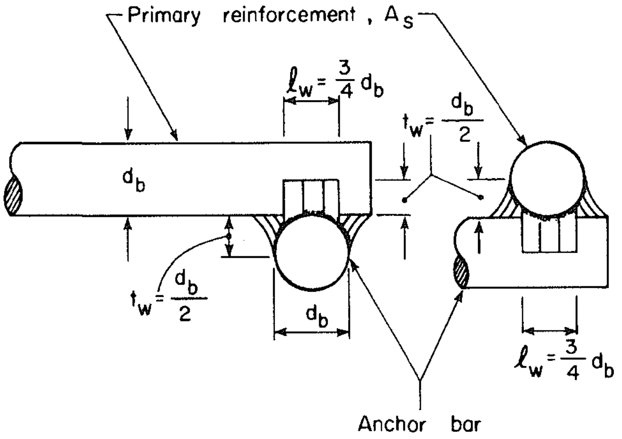
11.9.6 — At front face of bracket or corbel, primary tension reinforcement A_s shall be anchored by one of the following: (a) by a structural weld to a transverse bar of at least equal size; weld to be designed to develop specified yield strength f_y of A_s bars; (b) by bending primary tension bars A_s back to form a horizontal loop; or (c) by some other means of positive anchorage.

11.9.7 — Bearing area of load on bracket or corbel shall not project beyond straight portion of primary tension bars A_s, nor project beyond interior face of transverse anchor bar (if one is provided).

11.10 — Special provisions for walls

11.10.1 — Design for shear forces perpendicular to face of wall shall be in accordance with provisions for slabs in 11.12. Design for horizontal shear forces in plane of wall shall be in accordance with 11.10.2 through 11.10.8.

COMMENTARY



Primary reinforcement , A_s
$\ell_w = \frac{3}{4} d_b$
$t_w = \frac{d_b}{2}$
d_b
$t_w = \frac{d_b}{2}$
d_b
$\ell_w = \frac{3}{4} d_b$
Anchor bar

Fig. R11.9.6—Weld details used in tests of Reference 11.38

R11.9.5 — A minimum amount of reinforcement is specified to prevent the possibility of sudden failure should the bracket or corbel concrete crack under the action of flexural moment and outward tensile force N_{uc}.

R11.9.6 — Because the horizontal component of the inclined concrete compression strut (see Fig. R11.9.1) is transferred to the primary tension reinforcement at the location of the vertical load, the reinforcement A_s is essentially uniformly stressed from the face of the support to the point where the vertical load is applied. It must, therefore, be anchored at its outer end and in the supporting column, so as to be able to develop its yield strength from the face of support to the vertical load. Satisfactory anchorage at the outer end can be obtained by bending the A_s bars in a horizontal loop as specified in (b), or by welding a bar of equal diameter or a suitably sized angle across the ends of the A_s bars. The welds must be designed to develop the yield strength of the reinforcement A_s. The weld detail used successfully in the corbel tests reported in Reference 11.38 is shown in Fig. R11.9.6. The reinforcement A_s must be anchored within the supporting column in accordance with the requirements of Chapter 12. See additional discussion on end anchorage in R12.10.6.

R11.9.7 — The restriction on the location of the bearing area is necessary to ensure development of the yield strength of the reinforcement A_s near the load. When corbels are designed to resist horizontal forces, the bearing plate should be welded to the tension reinforcement A_s.

R11.10 — Special provisions for walls

R11.10.1 — Shear in the plane of the wall is primarily of importance for shearwalls with a small height-to-length ratio. The design of higher walls, particularly walls with uniformly distributed reinforcement, will probably be controlled by flexural considerations.

CODE

11.10.2 — Design of horizontal section for shear in plane of wall shall be based on Eq. (11-1) and (11-2), where shear strength V_c shall be in accordance with 11.10.5 or 11.10.6 and shear strength V_s shall be in accordance with 11.10.9.

11.10.3 — Shear strength V_n at any horizontal section for shear in plane of wall shall not be taken greater than $10\sqrt{f_c'}hd$.

11.10.4 — For design for horizontal shear forces in plane of wall, d shall be taken equal to $0.8\,\ell_w$. A larger value of d, equal to the distance from extreme compression fiber to center of force of all reinforcement in tension, shall be permitted to be used when determined by a strain compatibility analysis.

11.10.5 — Unless a more detailed calculation is made in accordance with 11.10.6, shear strength V_c shall not be taken greater than $2\sqrt{f_c'}hd$ for walls subject to N_u in compression, or V_c shall not be taken greater than the value given in 11.3.2.3 for walls subject to N_u in tension.

11.10.6 — Shear strength V_c shall be permitted to be computed by Eq. (11-31) and (11-32), where V_c shall be the lesser of Eq. (11-31) or (11-32).

$$V_c = 3.3\sqrt{f_c'}hd + \frac{N_u d}{4\ell_w} \qquad (11\text{-}31)$$

or

$$V_c = \left[0.6\sqrt{f_c'} + \frac{\ell_w\left(1.25\sqrt{f_c'} + 0.2\dfrac{N_u}{\ell_w h}\right)}{\dfrac{M_u}{V_u} - \dfrac{\ell_w}{2}}\right] hd \qquad (11\text{-}32)$$

where N_u is negative for tension. When $(M_u/V_u - \ell_w/2)$ is negative, Eq. (11-32) shall not apply.

11.10.7 — Sections located closer to wall base than a distance $\ell_w/2$ or one-half the wall height, whichever is less, shall be permitted to be designed for the same V_c as that computed at a distance $\ell_w/2$ or one-half the height.

11.10.8 — When factored shear force V_u is less than $\phi V_c/2$, reinforcement shall be provided in accordance with 11.10.9 or in accordance with Chapter 14. When V_u exceeds $\phi V_c/2$, wall reinforcement for resisting shear shall be provided in accordance with 11.10.9.

COMMENTARY

R11.10.3 — Although the width-to-depth ratio of shearwalls is less than that for ordinary beams, tests[11.39] on shearwalls with a thickness equal to $\ell_w/25$ have indicated that ultimate shear stresses in excess of $10\sqrt{f_c'}$ can be obtained.

R11.10.5 and R11.10.6 — Eq. (11-31) and (11-32) may be used to determine the inclined cracking strength at any section through a shearwall. Eq. (11-31) corresponds to the occurrence of a principal tensile stress of approximately $4\sqrt{f_c'}$ at the centroid of the shearwall cross section. Eq. (11-32) corresponds approximately to the occurrence of a flexural tensile stress of $6\sqrt{f_c'}$ at a section $\ell_w/2$ above the section being investigated. As the term

$$\left(\frac{M_u}{V_u} - \frac{\ell_w}{2}\right)$$

decreases, Eq. (11-31) will control before this term becomes negative. When this term becomes negative Eq. (11-31) should be used.

R11.10.7 — The values of V_c computed from Eq. (11-31) and (11-32) at a section located a distance $\ell_w/2$ or $h_w/2$ (whichever is less) above the base apply to that and all sections between this section and the base. However, the maximum factored shear force V_u at any section, including the base of the wall, is limited to ϕV_n in accordance with 11.10.3.

CODE

11.10.9 — Design of shear reinforcement for walls

11.10.9.1 — Where factored shear force V_u exceeds shear strength ϕV_c, horizontal shear reinforcement shall be provided to satisfy Eq. (11-1) and (11-2), where shear strength V_s shall be computed by

$$V_s = \frac{A_v f_y d}{s_2} \qquad (11\text{-}33)$$

where A_v is area of horizontal shear reinforcement within a distance s_2 and distance d is in accordance with 11.10.4. Vertical shear reinforcement shall be provided in accordance with 11.10.9.4.

11.10.9.2 — Ratio ρ_h of horizontal shear reinforcement area to gross concrete area of vertical section shall not be less than 0.0025.

11.10.9.3 — Spacing of horizontal shear reinforcement s_2 shall not exceed $\ell_w/5$, $3h$, nor 18 in.

11.10.9.4 — Ratio ρ_n of vertical shear reinforcement area to gross concrete area of horizontal section shall not be less than

$$\rho_n = 0.0025 + 0.5\left(2.5 - \frac{h_w}{\ell_w}\right)(\rho_h - 0.0025) \qquad (11\text{-}34)$$

nor 0.0025, but need not be greater than the required horizontal shear reinforcement.

11.10.9.5 — Spacing of vertical shear reinforcement s_1 shall not exceed $\ell_w/3$, $3h$, nor 18 in.

11.11 — Transfer of moments to columns

11.11.1 — When gravity load, wind, earthquake, or other lateral forces cause transfer of moment at connections of framing elements to columns, the shear resulting from moment transfer shall be considered in the design of lateral reinforcement in the columns.

11.11.2 — Except for connections not part of a primary seismic load-resisting system that are restrained on four sides by beams or slabs of approximately equal depth, connections shall have lateral reinforcement

COMMENTARY

R11.10.9 — Design of shear reinforcement for walls

Both horizontal and vertical shear reinforcement are required for all walls. For low walls, test data[11.40] indicate that horizontal shear reinforcement becomes less effective with vertical reinforcement becoming more effective. This change in effectiveness of the horizontal versus vertical reinforcement is recognized in Eq. (11-34); when h_w/ℓ_w is less than 0.5, the amount of vertical reinforcement is equal to the amount of horizontal reinforcement. When h_w/ℓ_w is greater than 2.5, only a minimum amount of vertical reinforcement is required ($0.0025 s_1 h$).

Eq. (11-33) is presented in terms of shear strength V_s provided by the horizontal shear reinforcement for direct application in Eq. (11-1) and (11-2).

Vertical shear reinforcement also must be provided in accordance with 11.10.9.4 within the spacing limitation of 11.10.9.5.

R11.11 — Transfer of moments to columns

R11.11.1 — Tests[11.41] have shown that the joint region of a beam to column connection in the interior of a building does not require shear reinforcement if the joint is confined on four sides by beams of approximately equal depth. However, joints without lateral confinement, such as at the exterior of a building, need shear reinforcement to prevent deterioration due to shear cracking.[11.42]

For regions where strong earthquakes may occur, joints may be required to withstand several reversals of loading that develop the flexural capacity of the adjoining beams. See Chapter 21 for special provisions for seismic design.

CODE

not less than that required by Eq. (11-13) within the column for a depth not less than that of the deepest connection of framing elements to the columns. See also 7.9.

11.12 — Special provisions for slabs and footings

11.12.1 — The shear strength of slabs and footings in the vicinity of columns, concentrated loads, or reactions is governed by the more severe of two conditions:

11.12.1.1 — Beam action where each critical section to be investigated extends in a plane across the entire width. For beam action the slab or footing shall be designed in accordance with 11.1 through 11.5.

11.12.1.2 — Two-way action where each of the critical sections to be investigated shall be located so that its perimeter b_o is a minimum but need not approach closer than $d/2$ to

(a) edges or corners of columns, concentrated loads, or reaction areas, or

(b) changes in slab thickness such as edges of capitals or drop panels.

For two-way action the slab or footing shall be designed in accordance with 11.12.2 through 11.12.6.

11.12.1.3 — For square or rectangular columns, concentrated loads, or reaction areas, the critical sections with four straight sides shall be permitted.

11.12.2 — The design of a slab or footing for two-way action is based on Eq. (11-1) and (11-2). V_c shall be computed in accordance with 11.12.2.1, 11.12.2.2, or 11.12.3.1. V_s shall be computed in accordance with 11.12.3. For slabs with shearheads, V_n shall be in accordance with 11.12.4. When moment is transferred between a slab and a column, 11.12.6 shall apply.

COMMENTARY

R11.12 — Special provisions for slabs and footings

R11.12.1 — Differentiation must be made between a long and narrow slab or footing acting as a beam, and a slab or footing subject to two-way action where failure may occur by "punching" along a truncated cone or pyramid around a concentrated load or reaction area.

R11.12.1.2 — The critical section for shear in slabs subjected to bending in two directions follows the perimeter at the edge of the loaded area.[11.3] The shear stress acting on this section at factored loads is a function of $\sqrt{f_c'}$ and the ratio of the side dimension of the column to the effective slab depth. A much simpler design equation results by assuming a pseudocritical section located at a distance $d/2$ from the periphery of the concentrated load. When this is done, the shear strength is almost independent of the ratio of column size to slab depth. For rectangular columns, this critical section was originally defined by straight lines drawn parallel to and at a distance $d/2$ from the edges of the loaded area. Section 11.12.1.3 allows the use of a rectangular critical section.

For slabs of uniform thickness it is sufficient to check shear on one section. For slabs with changes in thickness as happens, for example at the edge of drop panels, it is necessary to check shear at several sections.

For edge columns at points where the slab cantilevers beyond the column, the critical perimeter will either be three-sided or four-sided.

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11.12.2.1 — For nonprestressed slabs and footings, V_c shall be the smallest of:

(a) $$V_c = \left(2+\frac{4}{\beta_c}\right)\sqrt{f_c'}b_od \qquad (11\text{-}35)$$

where β_c is the ratio of long side to short side of the column, concentrated load or reaction area

(b) $$V_c = \left(\frac{\alpha_s d}{b_o}+2\right)\sqrt{f_c'}b_od \qquad (11\text{-}36)$$

where α_s is 40 for interior columns, 30 for edge columns, 20 for corner columns, and

(c) $$V_c = 4\sqrt{f_c'}\,b_od \qquad (11\text{-}37)$$

11.12.2.2 — At columns of two-way prestressed slabs and footings that meet the requirements of 18.9.3

$$V_c = \left(\beta_p\sqrt{f_c'}+0.3f_{pc}\right)b_od+V_p \qquad (11\text{-}38)$$

where β_p is the smaller of 3.5 or **($\alpha_s d/b_o$ + 1.5)**, α_s is 40 for interior columns, 30 for edge columns, and 20

COMMENTARY

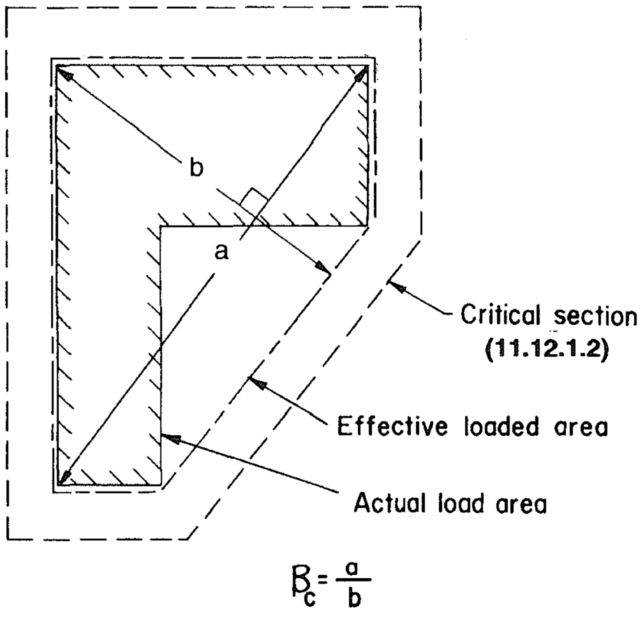

Fig. R11.12.2—Value of β_c *for a nonrectangular loaded area*

R11.12.2.1 — For square columns, the shear stress due to ultimate loads in slabs subjected to bending in two directions is limited to $4\sqrt{f_c'}$. However, tests[11.43] have indicated that the value of $4\sqrt{f_c'}$ is unconservative when the ratio β_c of the lengths of the long and short sides of a rectangular column or loaded area is larger than 2.0. In such cases, the actual shear stress on the critical section at punching shear failure varies from a maximum of about $4\sqrt{f_c'}$ around the corners of the column or loaded area, down to $2\sqrt{f_c'}$ or less along the long sides between the two end sections. Other tests[11.44] indicate that v_c decreases as the ratio b_o/d increases. Eq. (11-35) and (11-36) were developed to account for these two effects. The words "interior, edge, and corner columns" in 11.12.2.1(b) refer to critical sections with 4, 3, or 2 sides, respectively.

For shapes other than rectangular, β_c is taken to be the ratio of the longest overall dimension of the effective loaded area to the largest overall dimension of the effective loaded area measured perpendicular thereto, as illustrated for an L-shaped reaction area in Fig. R11.12.2. The effective loaded area is that area totally enclosing the actual loaded area, for which the perimeter is a minimum.

R11.12.2.2 — For prestressed slabs and footings, a modified form of code Eq. (11-35) and (11-36) is specified for two-way action shear strength. Research[11.45,11.46] indicates that the shear strength of two-way prestressed slabs around interior columns is conservatively predicted by Eq. (11-38). V_c from Eq. (11-36) corresponds to a diagonal tension failure of the concrete initiating at the critical section defined in 11.12.1.2. The mode of failure differs from a punching shear failure of the concrete compression zone around the perimeter of the loaded area predicted by Eq.

CODE

for corner columns, b_o is perimeter of critical section defined in 11.12.1.2, f_{pc} is the average value of f_{pc} for the two directions, and V_p is the vertical component of all effective prestress forces crossing the critical section. V_c shall be permitted to be computed by Eq. (11-38) if the following are satisfied; otherwise, 11.12.2.1 shall apply:

(a) no portion of the column cross section shall be closer to a discontinuous edge than 4 times the slab thickness, and

(b) f_c' in Eq. (11-38) shall not be taken greater than 5000 psi, and

(c) f_{pc} in each direction shall not be less than 125 psi, nor be taken greater than 500 psi.

11.12.3 — Shear reinforcement consisting of bars or wires shall be permitted in slabs and footings in accordance with the following:

11.12.3.1 — V_n shall be computed by Eq. (11-2), where V_c shall not be taken greater than $2\sqrt{f_c'}b_od$, and the required area of shear reinforcement A_v and V_s shall be calculated in accordance with 11.5 and anchored in accordance with 12.13.

11.12.3.2 — V_n shall not be taken greater than $6\sqrt{f_c'}b_od$.

COMMENTARY

(11-35). Consequently, the term β_c does not enter into Eq. (11-38). Design values for f_c' and f_{pc} are restricted due to limited test data available for higher values. When computing f_{pc}, loss of prestress due to restraint of the slab by shearwalls and other structural elements must be taken into account.

In a prestressed slab with distributed tendons, the V_p term in Eq. (11-38) contributes only a small amount to the shear strength; therefore, it may be conservatively taken as zero. If V_p is to be included, the tendon profile assumed in the calculations must be specified.

For an exterior column support where the distance from the outside of the column to the edge of the slab is less than four times the slab thickness, the prestress is not fully effective around the total perimeter b_o of the critical section. Shear strength in this case is therefore conservatively taken the same as for a nonprestressed slab.

R11.12.3 — Research has shown that shear reinforcement consisting of bars or wires can be used in slabs provided that it is well anchored. The anchorage detail used in the tests is shown in Fig. R11.12.3(a). Anchorage of stirrups according to the requirements of 12.13 may be difficult in slabs thinner than 10 in. For such thin slabs, stirrups should only be used if they are closed and enclose a longitudinal bar at each corner. Shear reinforcement consisting of vertical bars mechanically anchored at each end by a plate or head capable of developing the yield strength of the bars have been used successfully.

In a slab-column joint in which the moment transfer is negligible, the shear reinforcement should be symmetrical about the centroid of the critical section in location, number and spacing of stirrups as shown in Fig. R11.12.3(b). At edge columns or in the case of interior columns transferring moment, the shear reinforcement should be as symmetrical as possible. Although the average shear stresses on faces *AD* and *BC* of the exterior column in Fig. R11.12.3(c) are

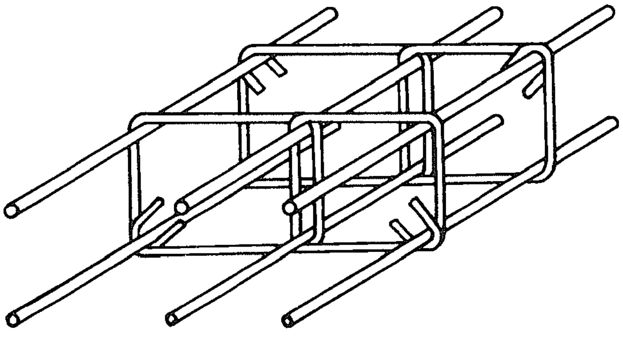

Fig. R11.12.3(a)—Slab stirrups

11.12.4—Shear reinforcement consisting of steel I- or channel-shaped sections (shearheads) shall be permitted in slabs. The provisions of 11.12.4.1 through 11.12.4.9 shall apply where shear due to gravity load is transferred at interior column supports. Where moment is transferred to columns, 11.12.6.3 shall apply.

11.12.4.1 — Each shearhead shall consist of steel shapes fabricated by welding with a full penetration weld into identical arms at right angles. Shearhead arms shall not be interrupted within the column section.

11.12.4.2 — A shearhead shall not be deeper than 70 times the web thickness of the steel shape.

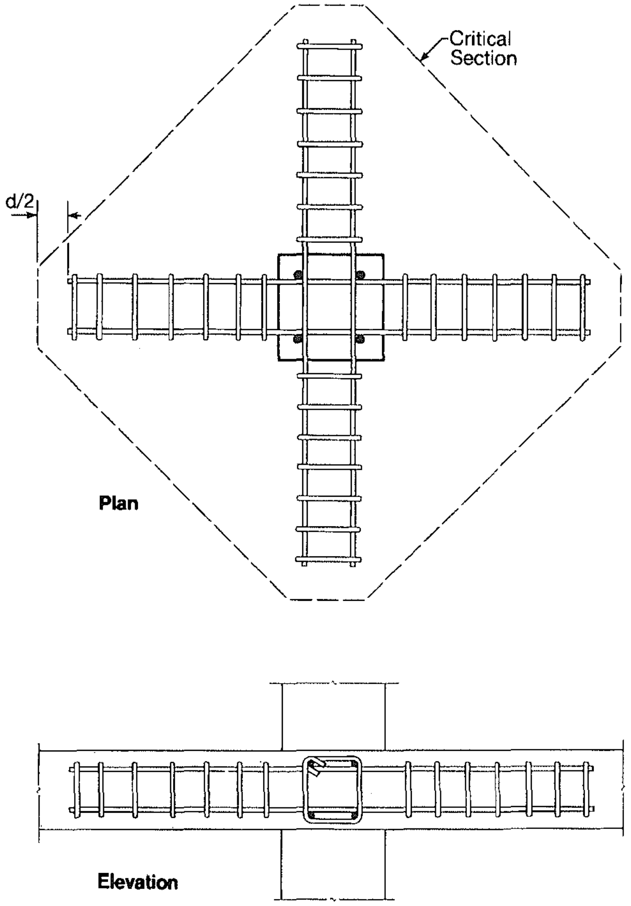

Fig. R11.12.3(b)—Arrangement of stirrup shear reinforcement, interior column

lower than on face *AB*, the stirrups extending from faces *AD* and *BC* reinforce against torsional stresses in the strip of slab along the edge.

When bars or wires are provided as shear reinforcement, the shear strength may be increased to a maximum shear stress of $6\sqrt{f_c'}$. However, shear reinforcement must be designed to carry all shear in excess of a stress of $2\sqrt{f_c'}$.[11.47]

R11.12.4—Based on reported test data,[11.48] design procedures are presented for shearhead reinforcement consisting of structural steel shapes. For a column connection transferring moment, the design of shearheads is given in 11.12.6.3.

Three basic criteria must be considered in the design of shearhead reinforcement for connections transferring shear due to gravity load. First, a minimum flexural strength must be provided to assure that the required shear strength of the slab is reached before the flexural strength of the shearhead is exceeded. Second, the shear stress in the slab at the end of the shearhead reinforcement must be limited. Third, after these two requirements are satisfied, the designer can reduce the negative slab reinforcement in proportion to the moment contribution of the shearhead at the design section.

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11.12.4.3 — The ends of each shearhead arm shall be permitted to be cut at angles not less than 30 deg with the horizontal, provided the plastic moment strength of the remaining tapered section is adequate to resist the shear force attributed to that arm of the shearhead.

11.12.4.4 — All compression flanges of steel shapes shall be located within **0.3*d*** of compression surface of slab.

11.12.4.5 — The ratio α_v between the stiffness of each shearhead arm and that of the surrounding composite cracked slab section of width $(c_2 + d)$ shall not be less than 0.15.

11.12.4.6 — The plastic moment strength M_p required for each arm of the shearhead shall be computed by

$$\phi M_p = \frac{V_u}{2\eta}\left[h_v + \alpha_v\left(\ell_v - \frac{c_1}{2}\right)\right] \qquad (11\text{-}39)$$

where ϕ is the strength reduction factor for flexure, η is the number of arms, and ℓ_v is the minimum length of each shearhead arm required to comply with requirements of 11.12.4.7 and 11.12.4.8.

COMMENTARY

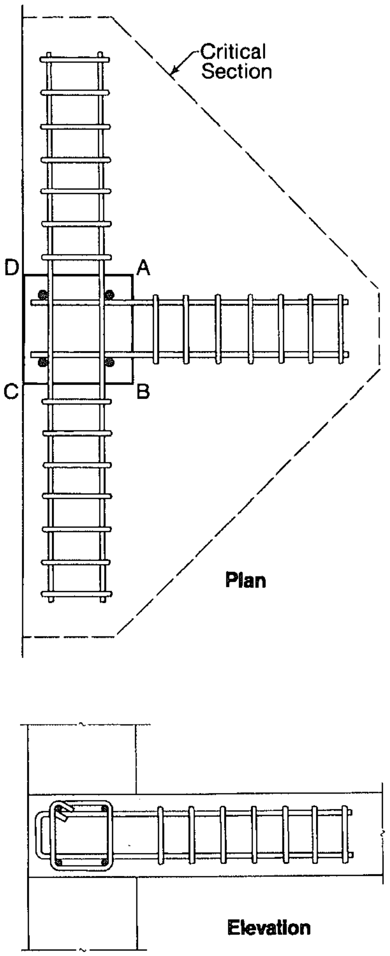

Fig. R11.12.3(c)—Arrangement of stirrup shear reinforcement, edge column

R11.12.4.5 and R11.12.4.6 — The assumed idealized shear distribution along an arm of a shearhead at an interior column is shown in Fig. R11.12.4.5. The shear along each of the arms is taken as $\alpha_v V_c/\eta$, where α_v and η are defined in 11.12.4.5 and 11.12.4.6, and V_c is defined in 11.12.2.1. However, the peak shear at the face of the column is taken as the total shear considered per arm $V_u/\phi\eta$ minus the shear considered carried to the column by the concrete compression zone of the slab. The latter term is expressed as

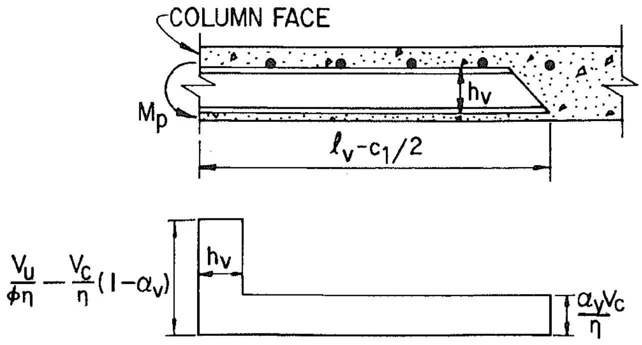
COLUMN FACE
M_p
h_v
$\ell_v - c_1/2$
h_v
$\frac{V_u}{\phi\eta} - \frac{V_c}{\eta}(1-\alpha_v)$
$\frac{\alpha_v V_c}{\eta}$

Fig. R11.12.4.5—Idealized shear acting on shearhead

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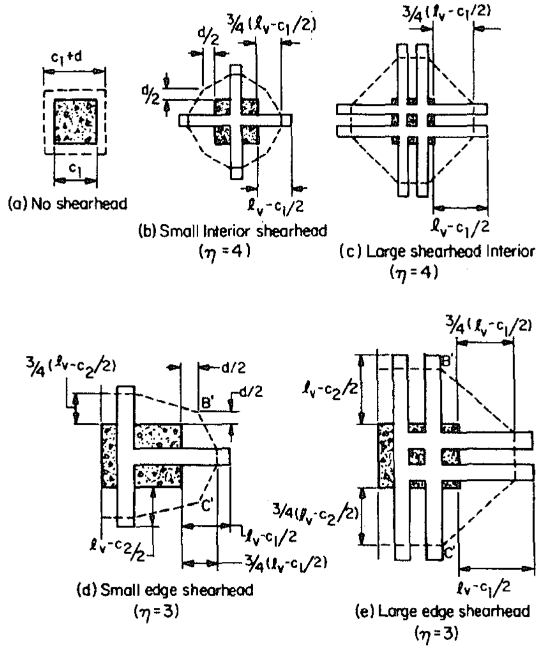
11.12.4.7 — The critical slab section for shear shall be perpendicular to the plane of the slab and shall cross each shearhead arm at three-quarters the distance **[ℓ_v - (c_1/2)]** from the column face to the end of the shearhead arm. The critical section shall be located so that its perimeter b_o is a minimum, but need not be closer than the perimeter defined in 11.12.1.2(a).

11.12.4.8 — V_n shall not be taken greater than $4\sqrt{f_c'}\,b_o d$ on the critical section defined in 11.12.4.7. When shearhead reinforcement is provided, V_n shall not be

COMMENTARY



c1+d
c1
(a) No shearhead
d/2
3/4(ℓv-c1/2)
d/2
ℓv-c1/2
(b) Small Interior shearhead
(η=4)
3/4(ℓv-c1/2)
ℓv-c1/2
(c) Large shearhead Interior
(η=4)
3/4(ℓv-c2/2)
d/2
d/2
B'
C'
ℓv-c2/2
3/4(ℓv-c2/2)
ℓv-c1/2
3/4(ℓv-c1/2)
(d) Small edge shearhead
(η=3)
3/4(ℓv-c1/2)
ℓv-c2/2
B'
C'
3/4(ℓv-c2/2)
ℓv-c1/2
(e) Large edge shearhead
(η=3)

Fig. R11.12.4.7—Location of critical section defined in 11.12.4.7

(V_c/η)(1 - α_v), so that it approaches zero for a heavy shearhead and approaches V_u/φη when a light shearhead is used. Eq. (11-39) then follows from the assumption that the inclined cracking shear force V_c is about one-half the shear force V_u. In this equation, M_p is the required plastic moment strength of each shearhead arm necessary to assure that ultimate shear is attained as the moment strength of the shearhead is reached. The quantity ℓ_v is the length from the center of the column to the point at which the shearhead is no longer required, and the distance c_1/2 is one-half the dimension of the column in the direction considered.

R11.12.4.7 — The test results indicated that slabs containing "underreinforcing" shearheads failed at a shear stress on a critical section at the end of the shearhead reinforcement less than $4\sqrt{f_c'}$. Although the use of "overreinforcing" shearheads brought the shear strength back to about the equivalent of $4\sqrt{f_c'}$, the limited test data suggest that a conservative design is desirable. Therefore, the shear strength is calculated as $4\sqrt{f_c'}$ on an assumed critical section located inside the end of the shearhead reinforcement.

The critical section is taken through the shearhead arms three-fourths of the distance [ℓ_v - (c_1/2)] from the face of the column to the end of the shearhead. However, this assumed critical section need not be taken closer than **d/2** to the column. See Fig. R11.12.4.7.

CODE

taken greater than $7\sqrt{f_c'}\,b_o d$ on the critical section defined in 11.12.1.2(a).

11.12.4.9 — The moment resistance M_v contributed to each slab column strip by a shearhead shall not be taken greater than

$$M_v = \frac{\phi\alpha_v V_u}{2\eta}\left(\ell_v - \frac{c_1}{2}\right) \qquad (11\text{-}40)$$

where ϕ is the strength reduction factor for flexure, η is the number of arms, and ℓ_v is the length of each shearhead arm actually provided. However, M_v shall not be taken larger than the smaller of:

(a) 30 percent of the total factored moment required for each slab column strip,

(b) the change in column strip moment over the length ℓ_v,

(c) the value of M_p computed by Eq. (11-39)

11.12.4.10 — When unbalanced moments are considered, the shearhead must have adequate anchorage to transmit M_p to column.

11.12.5 — Openings in slabs

When openings in slabs are located at a distance less than 10 times the slab thickness from a concentrated load or reaction area, or when openings in flat slabs are located within column strips as defined in Chapter 13, the critical slab sections for shear defined in 11.12.1.2 and 11.12.4.7 shall be modified as follows:

11.12.5.1 — For slabs without shearheads, that part of the perimeter of the critical section that is enclosed by straight lines projecting from the centroid of the column, concentrated load, or reaction area and tangent to the boundaries of the openings shall be considered ineffective.

11.12.5.2 — For slabs with shearheads, the ineffective portion of the perimeter shall be one-half of that defined in 11.12.5.1.

COMMENTARY

R11.12.4.9 — If the peak shear at the face of the column is neglected, and the cracking load V_c is again assumed to be about one-half of V_u, the moment contribution of the shearhead M_v can be conservatively computed from Eq. (11-40), in which ϕ is the factor for flexure (0.9).

R11.12.4.10 — See R11.12.6.3.

R11.12.5 — Openings in slabs

Provisions for design of openings in slabs (and footings) were developed in Reference 11.3. The locations of the

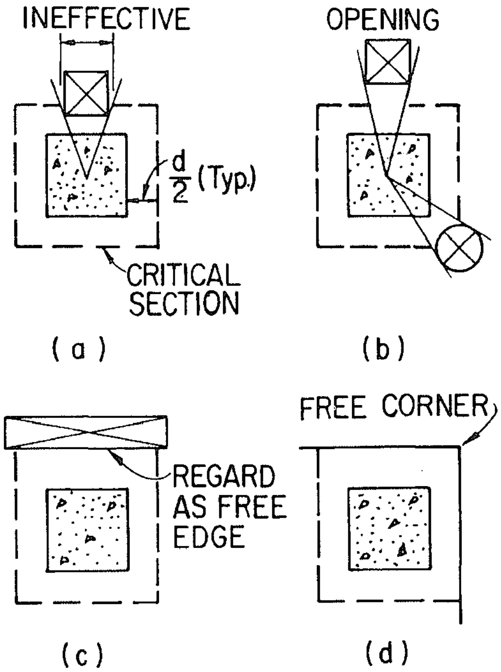


Fig. R11.12.5—Effect of openings and free edges (effective perimeter shown with dashed lines)

11.12.6 — Transfer of moment in slab-column connections

11.12.6.1 — When gravity load, wind, earthquake, or other lateral forces cause transfer of unbalanced moment $\boldsymbol{M_u}$ between a slab and a column, a fraction $\gamma_f \boldsymbol{M_u}$ of the unbalanced moment shall be transferred by flexure in accordance with 13.5.3. The remainder of the unbalanced moment given by $\gamma_v \boldsymbol{M_u}$ shall be considered to be transferred by eccentricity of shear about the centroid of the critical section defined in 11.12.1.2 where

$$\gamma_v = (1 - \gamma_f) \qquad (11\text{-}41)$$

11.12.6.2 — The shear stress resulting from moment transfer by eccentricity of shear shall be assumed to vary linearly about the centroid of the critical sections defined in 11.12.1.2. The maximum shear stress due to the factored shear force and moment shall not exceed $\phi \boldsymbol{v_n}$:

For members without shear reinforcement

$$\phi v_n = \phi V_c / (b_o d) \qquad (11\text{-}42)$$

where $\boldsymbol{V_c}$ is as defined in 11.12.2.1 or 11.12.2.2.

For members with shear reinforcement other than shearheads:

$$\phi v_n = \phi (V_c + V_s) / (b_o d) \qquad (11\text{-}43)$$

where $\boldsymbol{V_c}$ and $\boldsymbol{V_s}$ are defined in 11.12.3. If shear reinforcement is provided, the design shall take into account the variation of shear stress around the column.

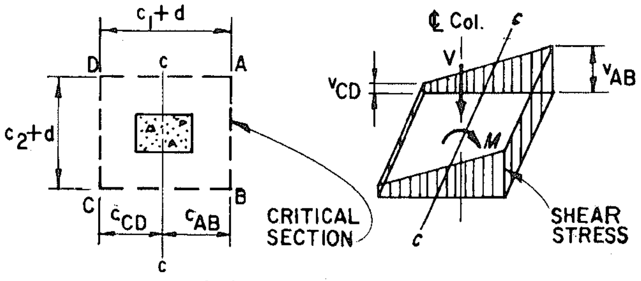
effective portions of the critical section near typical openings and free edges are shown by the dashed lines in Fig. R11.12.5. Additional research[11.43] has confirmed that these provisions are conservative.

R11.12.6 — Transfer of moment in slab-column connections

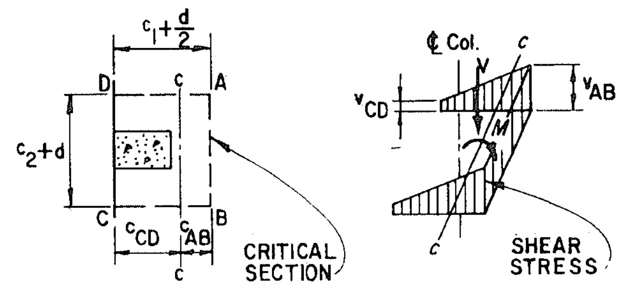
R11.12.6.1 — In Reference 11.49 it was found that where moment is transferred between a column and a slab, 60 percent of the moment should be considered transferred by flexure across the perimeter of the critical section defined in 11.12.1.2, and 40 percent by eccentricity of the shear about the centroid of the critical section. For rectangular columns, it has been assumed that the portion of the moment transferred by flexure increases as the width of the face of the critical section resisting the moment increases as given by Eq. (13-1).

Most of the data in Reference 11.49 were obtained from tests of square columns, and little information is available for round columns. These can be approximated as square columns. Fig. R13.6.2.5 shows square supports having the same area as some nonrectangular members.

R11.12.6.2 — The stress distribution is assumed as illustrated in Fig. R11.12.6.2 for an interior or exterior column. The perimeter of the critical section, *ABCD*, is determined in accordance with 11.12.1.2. The factored shear force $\boldsymbol{V_u}$ and unbalanced moment $\boldsymbol{M_u}$ are determined at the centroi-



(a) INTERIOR COLUMN



(b) EDGE COLUMN

Fig. R11.12.6.2—Assumed distribution of shear stress

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11.12.6.3 — When shear reinforcement consisting of steel I- or channel-shaped sections (shearheads) is provided, the sum of the shear stresses due to vertical load acting on the critical section defined by 11.12.4.7 and the shear stresses resulting from moment transferred by eccentricity of shear about the centroid of the critical section defined in 11.12.1.2 shall not exceed $\phi 4\sqrt{f_c'}$.

COMMENTARY

dal axis *c-c* of the critical section. The maximum factored shear stress may be calculated from:

$$v_{u(AB)} = \frac{V_u}{A_c} + \frac{\gamma_v M_u c_{AB}}{J_c}$$

or

$$v_{u(CD)} = \frac{V_u}{A_c} - \frac{\gamma_v M_u c_{CD}}{J_c}$$

where γ_v is given by Eq. (11-41). For an interior column, A_c and J_c may be calculated by

A_c = area of concrete of assumed critical section
= $2d(c_1 + c_2 + 2d)$

J_c = property of assumed critical section analogous to polar moment of inertia

$$= \frac{d(c_1+d)^3}{6} + \frac{(c_1+d)d^3}{6} + \frac{d(c_2+d)(c_1+d)^2}{2}$$

Similar equations may be developed for A_c and J_c for columns located at the edge or corner of a slab.

The fraction of the unbalanced moment between slab and column not transferred by eccentricity of the shear must be transferred by flexure in accordance with 13.5.3. A conservative method assigns the fraction transferred by flexure over an effective slab width defined in 13.5.3.2. Often designers concentrate column strip reinforcement near the column to accommodate this unbalanced moment. Available test data seem to indicate that this practice does not increase shear strength but may be desirable to increase the stiffness of the slab-column junction.

Test data[11.50] indicate that the moment transfer capacity of a prestressed slab to column connection can be calculated using the procedures of 11.12.6 and 13.5.3.

R11.12.6.3 — Tests[11.51] indicate that the critical section defined in 11.12.1.2 is appropriate for calculations of shear stresses caused by transfer of moments even when shearheads are used. Then, even though the critical sections for direct shear and shear due to moment transfer differ, they coincide or are in close proximity at the column corners where the failures initiate. Because a shearhead attracts most of the shear as it funnels toward the column, it is conservative to take the maximum shear stress as the sum of the two components.

Section 11.12.4.10 requires the moment M_p transferred to the column in shearhead connections transferring unbalanced moments. This may be done by bearing within the column or positive mechanical anchorage.

CHAPTER 12 — DEVELOPMENT AND SPLICES OF REINFORCEMENT

CODE

12.0 — Notation

a = depth of equivalent rectangular stress block as defined in 10.2.7.1

A_b = area of an individual bar, in.^{2}

A_s = area of nonprestressed tension reinforcement, in.^{2}

A_{tr} = total cross-sectional area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed, in.^{2}

A_v = area of shear reinforcement within a distance s, in.^{2}

A_w = area of an individual wire to be developed or spliced, in.^{2}

b_w = web width, or diameter of circular section, in.

c = spacing or cover dimension, in. See 12.2.4

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

d_b = nominal diameter of bar, wire, or prestressing strand, in.

f_c' = specified compressive strength of concrete, psi

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi

f_{ps} = stress in prestressed reinforcement at nominal strength, ksi

f_{se} = effective stress in prestressed reinforcement (after allowance for all prestress losses), ksi

f_y = specified yield strength of nonprestressed reinforcement, psi

f_{yt} = specified yield strength of transverse reinforcement, psi

h = overall thickness of member, in.

K_{tr} = transverse reinforcement index

= $\frac{A_{tr}f_{yt}}{1500sn}$ (constant 1500 carries the unit lb/in.^{2})

ℓ_a = additional embedment length at support or at point of inflection, in.

ℓ_d = development length, in.

= ℓ_{db} x applicable modification factors

ℓ_{db} = basic development length, in.

ℓ_{dh} = development length of standard hook in tension, measured from critical section to outside end of hook (straight embedment length between critical section and start of hook

COMMENTARY

The development length concept for anchorage of reinforcement was first introduced in the 1971 ACI Building Code, to replace the dual requirements for flexural bond and anchorage bond contained in earlier editions of the ACI Building Code. It is no longer necessary to consider the flexural bond concept which placed emphasis on the computation of nominal peak bond stresses. Consideration of an average bond resistance over a full development length of the reinforcement is more meaningful, partially because all bond tests consider an average bond resistance over a length of embedment of the reinforcement, and partially because uncalculated extreme variations in local bond stresses exist near flexural cracks.[12.1]

The development length concept is based on the attainable average bond stress over the length of embedment of the reinforcement. The specified development lengths are required because of the tendency of highly stressed bars to split relatively thin sections of restraining concrete. A single bar embedded in a mass of concrete should not require as great a development length; although a row of bars, even in mass concrete, can create a weakened plane, with longitudinal splitting along the plane of the bars.

In application, the development length concept requires the specified minimum lengths or extensions of reinforcement beyond all points of peak stress in the reinforcement. Such peak stresses generally occur at the points specified in 12.10.2.

The strength reduction factor ϕ *is not used in this chapter. The basic development lengths* ℓ_{db} *already include an allowance for understrength. The required lengths are the same for either the strength design method or the alternate design method of Appendix A, since* ℓ_{db} *is based on* f_y *in either case.*

CODE

[point of tangency] plus radius of bend and one bar diameter), in.

= ℓ_{hb} x applicable modification factors

ℓ_{hb} = basic development length of standard hook in tension, in.

M_n = nominal moment strength at section, in.-lb

= $A_s f_y(d - a/2)$

n = number of bars or wires being spliced or developed along the plane of splitting

s = maximum spacing of transverse reinforcement within ℓ_d center-to-center, in.

s_w = spacing of wire to be developed or spliced, in.

V_u = factored shear force at section

α = reinforcement location factor. See 12.2.4

β = coating factor. See 12.2.4

β_b = ratio of area of reinforcement cut off to total area of tension reinforcement at section

γ = reinforcement size factor. See 12.2.4

λ = lightweight aggregate concrete factor. See 12.2.4

12.1 — Development of reinforcement — General

12.1.1 — Calculated tension or compression in reinforcement at each section of structural concrete members shall be developed on each side of that section by embedment length, hook or mechanical device, or a combination thereof. Hooks shall not be used to develop bars in compression.

12.1.2 — The values of $\sqrt{f_c'}$ used in this chapter shall not exceed 100 psi.

12.2 — Development of deformed bars and deformed wire in tension

12.2.1 — Development length ℓ_d, in terms of diameter d_b for deformed bars and deformed wire in tension shall be determined from either 12.2.2 or 12.2.3, but ℓ_d shall not be less than 12 in.

12.2.2 — For deformed bars or deformed wire, ℓ_d/d_b shall be as follows:

COMMENTARY

R12.1 — Development of reinforcement — General

From a point of peak stress in reinforcement, some length of reinforcement or anchorage is necessary through which to develop the stress. This development length or anchorage is necessary on both sides of such peak stress points. Often the reinforcement continues for a considerable distance on one side of a critical stress point so that calculations need involve only the other side, e.g., the negative moment reinforcement continuing through a support to the middle of the next span.

R12.2 — Development of deformed bars and deformed wire in tension

In the 1989 Building Code, major changes were made in the procedures for calculating development lengths for deformed bars and deformed wires in tension. While the 1989 revisions were based on extensive research and professional judgment, many of those applying the 1989 provisions in design, detailing, and fabrication found them to be overly complex in application. Also, in some circumstances, the provisions required longer development lengths than prior experience indicated necessary. Committee 318 reexamined the basic tension development length procedures with a view of formulating a more "user friendly" format while maintaining general agreement with research results and professional judgment. In the 1995 code, the format for determining the development lengths for deformed bars and deformed wires in tension has been extensively revised. The revision, however, is still based on the same general

CODE

	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the code minimum or Clear spacing of bars being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\frac{\ell_d}{d_b} = \frac{f_y\alpha\beta\lambda}{25\sqrt{f_c'}}$	$\frac{\ell_d}{d_b} = \frac{f_y\alpha\beta\lambda}{20\sqrt{f_c'}}$
Other cases	$\frac{\ell_d}{d_b} = \frac{3f_y\alpha\beta\lambda}{50\sqrt{f_c'}}$	$\frac{\ell_d}{d_b} = \frac{3f_y\alpha\beta\lambda}{40\sqrt{f_c'}}$

12.2.3 — For deformed bars or deformed wire, ℓ_d/d_b shall be:

$$\frac{\ell_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f_c'}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+K_{tr}}{d_b}\right)} \qquad (12\text{-}1)$$

in which the term $(c + K_{tr})/d_b$ shall not be taken greater than 2.5.

COMMENTARY

equation[12.9] for development length previously endorsed by Committee 408.[12.2,12.3]

After extensive discussion, the committee decided to show as many of the previous multipliers as possible in the basic equation, as well as to rearrange terms and to eliminate compounding ϕ-factors. This results in the development length equation (expressed in terms of bar or wire diameter) given in 12.2.3:

$$\frac{\ell_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f_c'}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+K_{tr}}{d_b}\right)}$$

c is a factor which represents the smallest of the side cover, cover over the bar or wire (in both cases measured to the center of the bar or wire) or one-half the center-to-center spacing of the bars or wires. K_{tr} is a factor which represents the contribution of confining reinforcement across potential splitting planes. α is the traditional reinforcement location factor to reflect the adverse effects of the top reinforcement casting position. β is a coating factor reflecting the effects of epoxy coating for some applications. These factors have been revised to reflect recent research findings and there is a limit on the product $\alpha\beta$. γ is a reinforcement size factor which reflects the more favorable performance of smaller diameter reinforcement. λ is a lightweight concrete factor which reflects the generally lower tensile strength of lightweight concrete and the resulting reduction of splitting resistance which is important in the development of deformed reinforcement. A limit on the term $(c + K_{tr})/d_b$ of 2.5 is included to safeguard against pullout type failures. Provision of this limit eliminated the need for the check of $0.03\, d_b f_y/\sqrt{f_c'}$ previously required under ACI 318-89, Section 12.2.3.6.

The general Eq. (12-1) allows the designer to see the effect of all variables controlling the development length. The designer is permitted to disregard terms when such omission results in longer and hence more conservative development lengths. Evaluation of Eq. (12-1) for certain design conditions, and for given concrete strengths and reinforcing steel grades gives the basic development length in bar diameter multiples. This format was judged by designers and reinforcing bar suppliers to be a much more practical formulation.

However, practical implementation requires that either the user calculate ℓ_d based on the actual $(c + K_{tr})/d_b$ for each case or that a range of $(c + K_{tr})/d_b$ values be preselected for common cases. Committee 318 chose a final format which allows the user to choose between either of two approaches:

(1) Section 12.2.2 presents a "simpler" approach which recognizes that many current practical construction cases

utilize spacing and cover values along with confining reinforcement such as stirrups or ties which result in a value of **$(c + K_{tr})/d_b$** of at least 1.5. Typical examples would be minimum clear cover of **$1.0d_b$** along with either minimum clear spacing of **$2d_b$** or a combination of minimum clear spacing of **$1.0d_b$** and minimum ties or stirrups. For these frequently occurring cases, the development length for larger bars can be taken as $\ell_d/d_b = {}^1/_{20}(f_y \alpha\beta\lambda/\sqrt{f_c'})$. Comparison with past provisions and a check of massive data bank of experimental results maintained by Committee 408 indicated that for No. 6 deformed bars and smaller, as well as for deformed wire, these values could be reduced 20 percent using $\gamma = 0.80$. This became the basis for the first row of the table in 12.2.2. With lesser cover and in the absence of minimum ties or stirrups, the minimum clear spacing limits of 7.6.1 and the minimum concrete cover requirements of 7.7 result in minimum values of **c** of **$1.0d_b$**. Thus, for "other cases," the values are multiplied by 1.5 to restore them to equivalence with Eq. (12.1).

While the equations in the table may initially look complex, they are readily evaluated and for the generally occurring conditions, the user may easily construct very simple, quite useful expressions. For example, in all structures with normal weight concrete (**$\lambda = 1.0$**), uncoated reinforcement (**$\beta = 1.0$**), No. 6 or smaller bottom bars (**$\alpha = 1.0$**) with **$f_c' = 4$** ksi and Grade 60 reinforcement, the equations reduce to

$$\frac{\ell_d}{d_b} = \frac{(60,000)\,(1.0)\,(1.0)\,(1.0)}{25\sqrt{4000}} = 38$$

or

$$\frac{\ell_d}{d_b} = \frac{3\,(60,000)\,(1.0)\,(1.0)\,(1.0)}{50\sqrt{4000}} = 57$$

Thus, a designer or detailer knows that for these widely occurring cases as long as minimum cover of **d_b**, and either minimum clear spacing of **$2d_b$** or minimum clear spacing of **d_b** along with minimum ties or stirrups are provided, **$\ell_d = 38d_b$**. The penalty for spacing bars closer or providing less cover is the requirement that **$\ell_d = 57d_b$**.

(2) A "more general" approach, which is basically quite similar in many respects to the original 408 proposal,[12.2,12.3] is included in 12.2.3. This allows the user to evaluate **$(c + K_{tr})/d_b$** for each particular combination of cover, spacing, and transverse reinforcement. This allows one to more rigorously calculate development lengths where critical or in special investigations. A limit on **$(c + K_{tr})/d_b$** of 2.5 is imposed to maintain the 1989 Section 12.2.3.6 limit of $\ell_{db} \geq \mathbf{0.03}\,d_b f_y/\sqrt{f_c'}$ based on the pullout failure mode controlling.

CODE

12.2.4 — The factors for use in the expressions for development of deformed bars and deformed wires in tension in Chapter 12 are as follows:

α = reinforcement location factor

Horizontal reinforcement so placed that more than 12 in. of fresh concrete is cast in the member below the development length or splice 1.3

Other reinforcement 1.0

β = coating factor

Epoxy-coated bars or wires with cover less than $3d_b$, or clear spacing less than $6d_b$ 1.5

All other epoxy-coated bars or wires 1.2

Uncoated reinforcement 1.0

However, the product of $\alpha\beta$ need not be taken greater than 1.7.

γ = reinforcement size factor

No. 6 and smaller bars and deformed wires 0.8

No. 7 and larger bars 1.0

λ = lightweight aggregate concrete factor

When lightweight aggregate concrete is used 1.3

However, when f_{ct} is specified, λ shall be permitted to be taken as $6.7\sqrt{f_c'}/f_{ct}$ but not less than 1.0

When normal weight concrete is used 1.0

c = spacing or cover dimension, in.

Use the smaller of either the distance from the center of the bar or wire to the nearest concrete surface or one-half the center-to-center spacing of the bars or wires being developed.

COMMENTARY

There are many practical combinations of side cover, clear cover, and confining reinforcement which can be used with 12.2.3 to produce significantly shorter development lengths than allowed by 12.2.2. For example: Bars or wires with minimum clear cover not less than $2d_b$ and minimum clear spacing not less than $4d_b$ and without any confining reinforcement would have a $(c + K_{tr})/d_b$ value of 2.5 and hence would require only 0.6 times the values of 12.2.2.

The new provisions of 12.2.2 and 12.2.3 give a two-tier approach as provided in many other places in the code. They should result in simpler computations where approximations are acceptable while retaining the more general ACI 408 approach where special cases or many repetitions make the greater efficiency desirable.

The basis for determining tension development length in the 1995 code is the same as that in the 1989 code. Thus, design aids and computer programs based on Section 12.2 of ACI 318-89 can be used for complying with the 1995 ACI Building Code.

R12.2.4 — The reinforcement location factor α accounts for position of the reinforcement in freshly placed concrete. The factor had been reduced from 1.4 in the 1983 code to 1.3 in the 1989 code to reflect recent research.[12.4,12.5]

The factor λ for lightweight aggregate concrete was made the same for all types of aggregates in 1989. Research on hooked bar anchorages did not support the variations specified in previous codes for "all-lightweight and sand-lightweight" concrete and a single value, 1.3, was selected. Section 12.2.4 allows a lower factor to be used when the splitting tensile strength of the lightweight concrete is specified. See 5.1.4.

Studies[12.6,12.7,12.8] of the anchorage of epoxy-coated bars show that bond strength is reduced because the coating prevents adhesion and friction between the bar and the concrete. The various factors reflect the type of anchorage failure likely to occur. When the cover or spacing is small, a splitting failure can occur and the anchorage or bond strength is substantially reduced. If the cover and spacing between bars is large, a splitting failure is precluded and the effect of the epoxy coating on anchorage strength is not as large. Studies[12.9] have shown that although the cover or spacing may be small, the anchorage strength may be increased by adding transverse steel crossing the plane of splitting, and restraining the splitting crack.

Although no studies on the effect of coated transverse steel have been reported to date, the addition of transverse steel should improve the anchorage strength of epoxy-coated bars. Since the bond of epoxy-coated bars is already reduced due to the loss of adhesion between the bar and the concrete, an upper limit of 1.7 is established for the product

CODE

K_{tr} = transverse reinforcement index

$$= \frac{A_{tr} f_{yt}}{1500 s n}$$

where

A_{tr} = total cross-sectional area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed, in.^{2}

f_{yt} = specified yield strength of transverse reinforcement, psi

s = maximum spacing of transverse reinforcement within ℓ_d, center-to-center, in.

n = number of bars or wires being developed along the plane of splitting

It shall be permitted to use $K_{tr} = 0$ as a design simplification even if transverse reinforcement is present.

12.2.5 — Excess reinforcement

Reduction in development length shall be permitted where reinforcement in a flexural member is in excess of that required by analysis except where anchorage or development for f_y is specifically required or the reinforcement is designed under provisions of 21.2.1.4 .... (A_s required)/(A_s provided)

12.3 — Development of deformed bars in compression

12.3.1 — Development length ℓ_d, in inches, for deformed bars in compression shall be computed as the product of the basic development length ℓ_{db} of 12.3.2 and applicable modification factors of 12.3.3, but ℓ_d shall be not less than 8 in.

12.3.2 — Basic development length

ℓ_{db} shall be .... $0.02 d_b f_y / \sqrt{f_c'}$

but not less than.................................. $0.0003 d_b f_y$*

12.3.3 — Basic development length ℓ_{db} shall be permitted to be multiplied by applicable factors for:

12.3.3.1 — Excess reinforcement

Reinforcement in excess of that required by analysis........................... (A_s required)/(A_s provided)

COMMENTARY

of the top reinforcement and epoxy-coated reinforcement factors.

R12.2.5 — Excess reinforcement

The reduction factor based on area is not to be used in those cases where anchorage development for full f_y is required. For example, the excess reinforcement factor does not apply for development of positive moment reinforcement at supports according to 12.11.2, for development of shrinkage and temperature reinforcement according to 7.12.2.3, or for development of reinforcement provided according to 7.13 and 13.3.8.5.

R12.3 — Development of deformed bars in compression

The weakening effect of flexural tension cracks is not present for bars in compression and usually end bearing of the bars on the concrete is beneficial. Therefore, shorter basic development lengths ℓ_{db} are specified for compression than for tension. The basic development length may be reduced 25 percent in 12.3.3.2 when the reinforcement is enclosed within a column type spiral or an individual spiral around each bar or group of bars.

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*The constant carries the unit of in.^{2}/lb.

CODE

12.3.3.2 — Spirals and ties
Reinforcement enclosed within spiral reinforcement not less than $^1/_4$ in. diameter and not more than 4 in. pitch or within No. 4 ties in conformance with 7.10.5 and spaced at not more than 4 in. on center 0.75

12.4 — Development of bundled bars

12.4.1 — Development length of individual bars within a bundle, in tension or compression, shall be that for the individual bar, increased 20 percent for three-bar bundle, and 33 percent for four-bar bundle.

12.4.2 — For determining the appropriate factors in 12.2, a unit of bundled bars shall be treated as a single bar of a diameter derived from the equivalent total area.

12.5 — Development of standard hooks in tension

12.5.1 — Development length ℓ_{dh}, in inches, for deformed bars in tension terminating in a standard hook (see 7.1) shall be computed as the product of the basic development length ℓ_{hb} of 12.5.2 and the applicable modification factor or factors of 12.5.3, but ℓ_{dh} shall not be less than $8d_b$ nor less than 6 in.

12.5.2 — Basic development length ℓ_{hb} for a hooked bar with f_y equal to 60,000 psi shall be .... $1200d_b/\sqrt{f_c'}$*

COMMENTARY

R12.4 — Development of bundled bars

R12.4.1 — An increased development length for individual bars is required when three or four bars are bundled together. The extra extension is needed because the grouping makes it more difficult to mobilize bond resistance from the "core" between the bars.

The designer should also note 7.6.6.4 relating to the cutoff points of individual bars within a bundle and 12.14.2.2 relating to splices of bundled bars. The increases in development length of 12.4 do apply when computing splice lengths of bundled bars in accordance with 12.14.2.2. The development of bundled bars by a standard hook of the bundle is not covered by the provisions of 12.5.

R12.4.2 — Although splice and development lengths of bundled bars are based on the diameter of individual bars increased by 20 percent or 33 percent as appropriate, it is necessary to use an equivalent diameter of the entire bundle derived from the equivalent total area of bars when determining factors in 12.2 which considers cover and clear spacing and represent the tendency of concrete to split.

R12.5 — Development of standard hooks in tension

The provisions for hooked bar anchorage were extensively revised in the 1983 code. Study of failures of hooked bars indicate that splitting of the concrete cover in the plane of the hook is the primary cause of failure and that splitting originates at the inside of the hook where the local stress concentrations are very high. Thus, hook development is a direct function of bar diameter d_b which governs the magnitude of compressive stresses on the inside of the hook. Only standard hooks (see 7.1) are considered and the influence of larger radius of bend cannot be evaluated by 12.5.

The hooked bar anchorage provisions give the total hooked bar embedment length as shown in Fig. R12.5.1. The development length ℓ_{dh} is measured from the critical section to the outside end (or edge) of the hook.

The development length ℓ_{dh} is the product of the basic development length ℓ_{hb} of 12.5.2 and the applicable modifi-

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* Constant carries unit of lb/in.^{2}

CODE

12.5.3 — Basic development length ℓ_{hb} shall be multiplied by applicable factor or factors for:

12.5.3.1 — Bar yield strength
Bars with f_y other than 60,000 psi .... f_y/60,000

12.5.3.2 — Concrete cover
For No. 11 bar and smaller, side cover (normal to plane of hook) not less than 2½ in., and for 90 deg hook, cover on bar extension beyond hook not less than 2 in. 0.7

12.5.3.3 — Ties or stirrups
For No. 11 bar and smaller, hook enclosed vertically or horizontally within ties or stirrup-ties spaced along the full development length ℓ_{dh} not greater than $3d_b$, where d_b is diameter of hooked bar 0.8

12.5.3.4 — Excess reinforcement
Where anchorage or development for f_y is not specifically required, reinforcement in excess of that required by analysis .... (A_s required)/(A_s provided)

12.5.3.5 — Lightweight aggregate concrete ....
.... 1.3

12.5.3.6 — Epoxy-coated reinforcement
Hooked bars with epoxy coating 1.2

12.5.4 — For bars being developed by a standard hook at discontinuous ends of members with both side cover and top (or bottom) cover over hook less than 2½ in., hooked bar shall be enclosed within ties or stir-

COMMENTARY

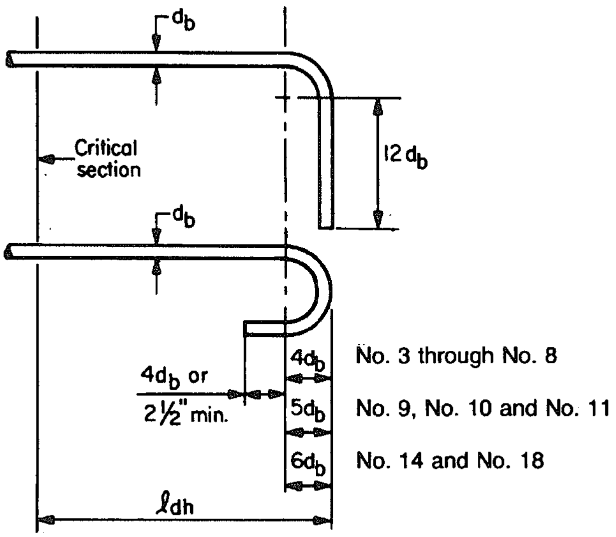

Fig. R12.5.1—Hooked bar details for development of standard hooks

cation factors of 12.5.3. If side cover is large so that splitting is effectively eliminated, and ties are provided, both factors of 12.5.3.2 and 12.5.3.3 may be applied: ($\ell_{dh} = \ell_{hb} \times 0.7 \times 0.8$). If, for the same case, anchorage is in lightweight concrete: ($\ell_{dh} = \ell_{hb} \times 0.7 \times 0.8 \times 1.3$).

Modification factors are provided for bar yield strength, excess reinforcement, lightweight concrete, and factors to reflect the resistance to splitting provided from confinement by concrete and transverse ties or stirrups. The factors are based on recommendations from References 12.2 and 12.3.

The factor for excess reinforcement applies only where anchorage or development for full f_y is not specifically required. The factor for lightweight concrete is a simplification over the procedure in 12.2.3.3 of ACI 318-83 in which the increase varies from 18 percent to 33 percent, depending on the amount of lightweight aggregate used. Unlike straight bar development, no distinction is made between top bars and other bars; such a distinction is difficult for hook bars in any case. A minimum value of ℓ_{dh} is specified to prevent failure by direct pullout in cases where a hook may be located very near the critical section. Hooks cannot be considered effective in compression.

Recent tests[12.10] indicate that the development length for hooked bars should be increased by 20 percent to account for reduced bond when reinforcement is epoxy coated.

R12.5.4 — Bar hooks are especially susceptible to a concrete splitting failure if both side cover (normal to plane of hook) and top or bottom cover (in plane of hook) are small. See Fig. R12.5.4. With minimum confinement provided by rup-ties spaced along the full development length ℓ_{dh} not greater than $3d_b$, where d_b is diameter of hooked bar. For this case, factor of 12.5.3.3 shall not apply.

12.5.5 — Hooks shall not be considered effective in developing bars in compression.

12.6 — Mechanical anchorage

12.6.1 — Any mechanical device capable of developing the strength of reinforcement without damage to concrete is allowed as anchorage.

12.6.2 — Test results showing adequacy of such mechanical devices shall be presented to the building official.

12.6.3 — Development of reinforcement shall be permitted to consist of a combination of mechanical anchorage plus additional embedment length of reinforcement between the point of maximum bar stress and the mechanical anchorage.

12.7 — Development of welded deformed wire fabric in tension

12.7.1 — Development length ℓ_d, in inches, of welded deformed wire fabric measured from the point of criti-

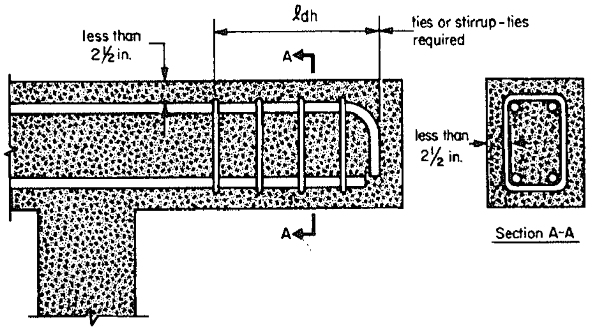

Fig. R12.5.4—Concrete cover per 12.5.4

concrete, confinement provided by ties or stirrups is essential, especially if full bar strength must be developed by a hooked bar with such small cover. Typical cases where hooks may require ties or stirrups for confinement are at ends of simply supported beams, at free end of cantilevers, and at ends of members framing into a joint where members do not extend beyond the joint. In contrast, if calculated bar stress is low so that the hook is not needed for bar anchorage, the ties or stirrups are not necessary. Also, for hooked bars at discontinuous ends of slabs with confinement provided by the slab continuous on both sides normal to the plane of the hook, provisions of 12.5.4 do not apply.

R12.5.5 — In compression, hooks are ineffective and cannot be used as anchorage.

R12.6 — Mechanical anchorage

R12.6.1 — Mechanical anchorage can be made adequate for strength both for prestressing tendons and for bar reinforcement.

R12.6.3 — Total development of a bar simply consists of the sum of all the parts that contribute to anchorage. When a mechanical anchorage is not capable of developing the required design strength of the reinforcement, additional embedment length of reinforcement must be provided between the mechanical anchorage and the critical section.

R12.7 — Development of welded deformed wire fabric in tension

Fig. R12.7 shows the development requirements for deformed wire fabric with one cross wire within the devel-

CODE

cal section to the end of wire shall be computed as the product of the development length ℓ_d, from 12.2.2 or 12.2.3 times a wire fabric factor from 12.7.2 or 12.7.3. It shall be permitted to reduce the development length in accordance with 12.2.5 when applicable, but ℓ_d shall not be less than 8 in. except in computation of lap splices by 12.18. When using the wire fabric factor from 12.7.2, it shall be permitted to use an epoxy-coating factor β of 1.0 for epoxy-coated welded wire fabric in 12.2.2 and 12.2.3.

12.7.2 — For welded deformed wire fabric with at least one cross wire within the development length and not less than 2 in. from the point of the critical section, the wire fabric factor shall be the greater of:

$$\left(\frac{f_y - 35{,}000}{f_y}\right)$$

or

$$\left(\frac{5d_b}{s_w}\right)$$

but need not be taken greater than 1.

12.7.3 — For welded deformed wire fabric with no cross wires within the development length or with a single cross wire less than 2 in. from the point of the critical section, the wire fabric factor shall be taken as 1, and the development length shall be determined as for deformed wire.

12.7.4—When any plain wires are present in the deformed wire fabric in the direction of the development length, the fabric shall be developed in accordance with 12.8.

12.8—Development of welded plain wire fabric in tension

Yield strength of welded plain wire fabric shall be considered developed by embedment of two cross wires with the closer cross wire not less than 2 in. from the point of the critical section. However, the development length ℓ_d, in inches, measured from the point of the critical section to the outermost cross wire shall not be less than

$$0.27\,\frac{A_w}{s_w}\left(\frac{f_y}{\sqrt{f'_c}}\right)\lambda$$

except that when reinforcement provided is in excess of that required, this length may be reduced in accordance with 12.2.5. ℓ_d shall not be less than 6 in. except in computation of lap splices by 12.19.

COMMENTARY

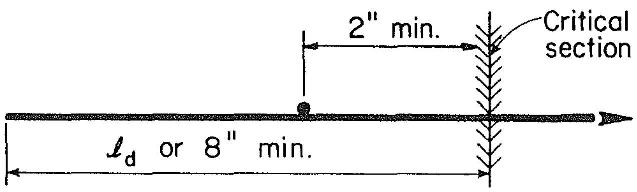

Fig. R12.7—Development of welded deformed wire fabric

opment length. ASTM A 497 for deformed wire fabric requires the same strength of the weld as required for plain wire fabric (ASTM A 185). Some of the development is assigned to welds and some assigned to the length of deformed wire. The development computations are simplified from earlier code provisions for wire development by assuming that only one cross wire is contained in the development length. The factors in 12.7.2 are applied to the deformed wire development length computed from 12.2 but with an absolute minimum of 8 in. The explicit statement that the mesh multiplier not be taken greater than 1 corrects an oversight in earlier codes. The multipliers were derived using the general relationships between deformed wire mesh and deformed wires in the ℓ_{db} values of ACI 318-83.

Tests[12.11] have indicated that epoxy-coated welded wire fabric has essentially the same development and splice strengths as uncoated fabric since the cross wires provide the primary anchorage for the wire. Therefore, an epoxy-coating factor of 1.0 is used for development and splice lengths of epoxy-coated welded wire fabric with cross wires within the splice or development length.

R12.8 — Development of welded smooth wire fabric in tension

Fig. R12.8 shows the development requirements for plain wire fabric with development primarily dependent on the location of cross wires. For fabrics made with the smaller wires, an embedment of at least two cross wires 2 in. or more beyond the point of critical section is adequate to develop the full yield strength of the anchored wires. However, for fabrics made with larger closely spaced wires a longer embedment is required and a minimum development length is provided for these fabrics.

12.9 — Development of prestressing strand

12.9.1 — Three- or seven-wire pretensioning strand shall be bonded beyond the critical section for a development length, in inches, not less than

$$\left(f_{ps} - \frac{2}{3} f_{se}\right) d_b{}^*$$

where d_b is strand diameter in inches, and f_{ps} and f_{se} are expressed in kips/in.^{2}

12.9.2 — Limiting the investigation to cross sections nearest each end of the member that are required to develop full design strength under specified factored loads shall be permitted.

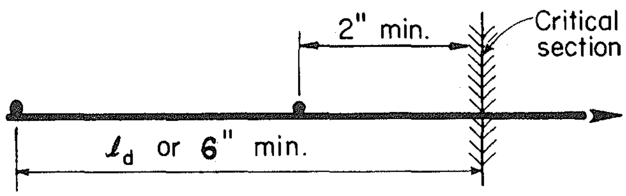

Fig. R12.8—Development of welded plain wire fabric

R12.9 — Development of prestressing strand

The development requirements for prestressing strand are intended to provide bond integrity for the strength of the member. The provisions are based on tests performed on normal weight concrete members with a minimum cover of 2 in. These tests may not represent the behavior of strand in low water-cementitious materials ratio, no-slump concrete. Fabrication methods should ensure consolidation of concrete around the strand with complete contact between the steel and concrete. Extra precautions should be exercised when low water-cementitious materials ratio, no-slump concrete is used. In general, this section will control only for the design of cantilever and short-span members.

The expression for development length ℓ_d may be rewritten as:

$$\ell_d = \frac{f_{se}}{3} d_b + (f_{ps} - f_{se}) d_b$$

where ℓ_d and d_b are in inches, and f_{ps} and f_{se} are in kips/in.^{2} The first term represents the transfer length of the strand, i.e., the distance over which the strand must be bonded to the concrete to develop the prestress f_{se} in the strand. The second term represents the additional length over which the strand must be bonded so that a stress f_{ps} may develop in the strand at nominal strength of the member.

The variation of strand stress along the development length of the strand is shown in Fig. R12.9. The expressions for transfer length, and for the additional bonded length necessary to develop an increase in stress of $(f_{ps} - f_{se})$ are based on tests of members prestressed with clean, $^1/_4$, $^3/_8$, and $^1/_2$ in. diameter strands for which the maximum value of f_{ps} was 275 kips/in.^{2} See References 12.12, 12.13, and 12.14.

The transfer length of strand is a function of the perimeter configuration area and surface condition of the steel, the stress in the steel, and the method used to transfer the steel force to the concrete. Strand with a slightly rusted surface can have an appreciably shorter transfer length than clean strand. Gentle release of the strand will permit a shorter

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[*] Expression in parenthesis used as a constant without units.

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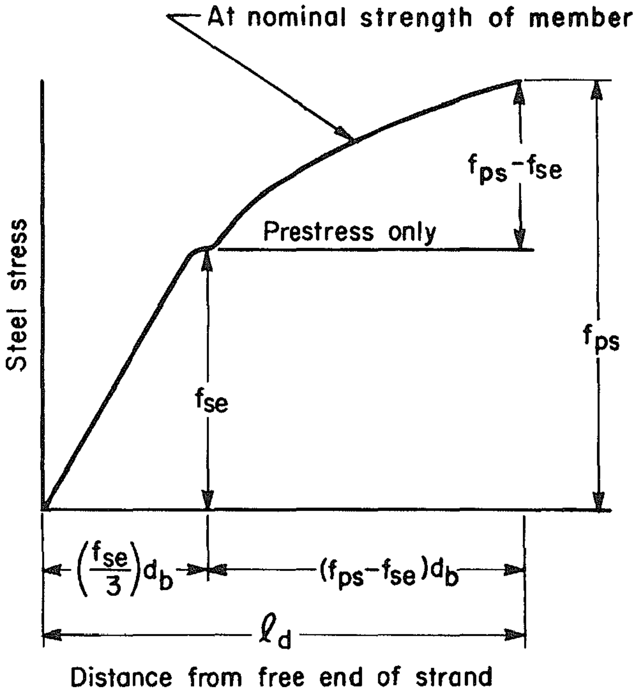

At nominal strength of member
Steel stress
Prestress only
$f_{ps}-f_{se}$
f_{se}
f_{ps}
$\left(\frac{f_{se}}{3}\right)d_b$
$(f_{ps}-f_{se})d_b$
ℓ_d
Distance from free end of strand

Fig. R12.9—Variation of steel stress with distance from free end of strand

transfer length than abruptly cutting the strands.

The provisions of 12.9 do not apply to plain wires nor to end anchored tendons. The length for smooth wire could be expected to be considerably greater due to the absence of mechanical interlock. Flexural bond failure would occur with plain wire when first slip occurred.

12.9.3 — Where bonding of a strand does not extend to end of member, and design includes tension at service load in precompressed tensile zone as permitted by 18.4.2, development length specified in 12.9.1 shall be doubled.

R12.9.3 — Exploratory tests conducted in 1965[12.12] to study the effect of debonded strand (bond not permitted to extend to the ends of members) on performance of pretensioned girders, indicated that the performance of these girders with embedment lengths twice those required by 12.9.1 closely matched the flexural performance of similar pretensioned girders with strand fully bonded to ends of girders. Accordingly, doubled development length is required for strand not bonded through to the end of a member. Subsequent tests[12.15] indicated that in pretensioned members designed for zero tension in the concrete under service load conditions (see 18.4.2), the development length for debonded strands need not be doubled.

12.10 — Development of flexural reinforcement — General

R12.10 — Development of flexural reinforcement — General

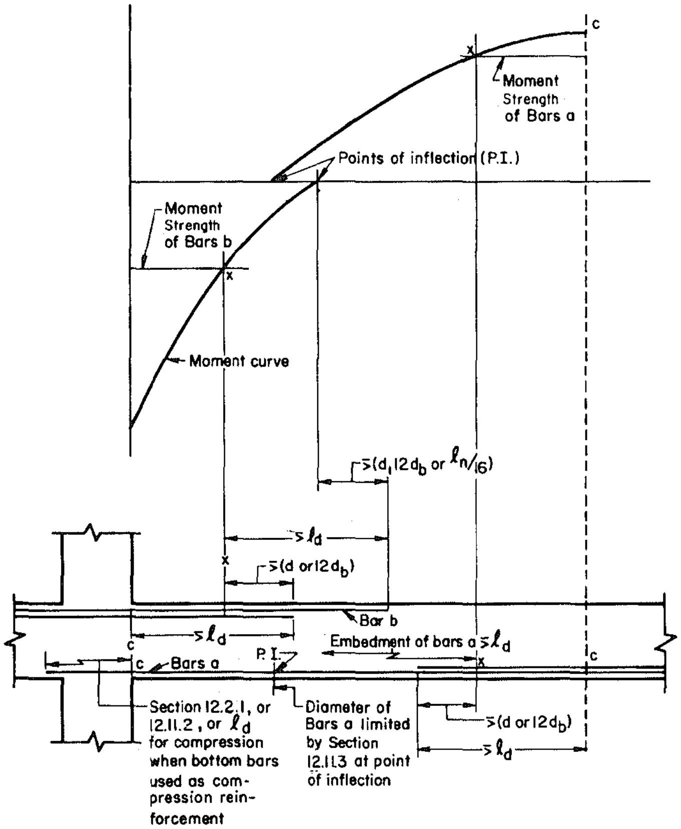
12.10.1 — Development of tension reinforcement by bending across the web to be anchored or made continuous with reinforcement on the opposite face of member shall be permitted.

CODE

12.10.2 — Critical sections for development of reinforcement in flexural members are at points of maximum stress and at points within the span where adjacent reinforcement terminates, or is bent. Provisions of 12.11.3 must be satisfied.

12.10.3 — Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the effective depth of member or $12d_b$, whichever is greater, except at supports of simple spans and at free end of cantilevers.

COMMENTARY



c
x
Moment Strength of Bars a
Points of inflection (P.I.)
Moment Strength of Bars b
x
Moment curve
$\geq (d, 12d_b \text{ or } \ell_n/16)$
$\geq \ell_d$
x
$\geq (d \text{ or } 12d_b)$
Bar b
$\geq \ell_d$
Embedment of bars a $\geq \ell_d$
c
c
Bars a
P.I.
x
c
Section 12.2.1, or 12.11.2, or ℓ_d for compression when bottom bars used as compression reinforcement
Diameter of Bars a limited by Section 12.11.3 at point of inflection
$\geq (d \text{ or } 12d_b)$
$\geq \ell_d$

Fig. R12.10.2—Development of flexural reinforcement in a typical continuous beam

R12.10.2 — Critical sections for a typical continuous beam are indicated with a "c" or an "x" in Fig. R12.10.2. For uniform loading, the positive reinforcement extending into the support is more apt to be governed by the requirements of 12.11.3 rather than by development length measured from a point of maximum moment or bar cutoff.

R12.10.3 — The moment diagrams customarily used in design are approximate; some shifting of the location of maximum moments may occur due to changes in loading, settlement of supports, lateral loads, or other causes. A diagonal tension crack in a flexural member without stirrups may shift the location of the calculated tensile stress approximately a distance d towards a point of zero moment. When stirrups are provided, this effect is less severe, although still present to some extent.

To provide for shifts in the location of maximum moments, the code requires the extension of reinforcement a distance d or $12d_b$ beyond the point at which it is theoretically no longer required to resist flexure, except as noted.

Cutoff points of bars to meet this requirement are illustrated in Fig. R12.10.2.

CODE

12.10.4 — Continuing reinforcement shall have an embedment length not less than the development length ℓ_d beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure.

12.10.5 — Flexural reinforcement shall not be terminated in a tension zone unless one of the following conditions is satisfied:

12.10.5.1 — Shear at the cutoff point does not exceed two-thirds that permitted, including shear strength of shear reinforcement provided.

12.10.5.2—Stirrup area in excess of that required for shear and torsion is provided along each terminated bar or wire over a distance from the termination point equal to three-fourths the effective depth of member. Excess stirrup area A_v shall be not less than $60b_w s/f_y$. Spacing s shall not exceed $d/8\beta_b$ where β_b is the ratio of area of reinforcement cut off to total area of tension reinforcement at the section.

12.10.5.3—For No. 11 bar and smaller, continuing reinforcement provides double the area required for flexure at the cutoff point and shear does not exceed three-fourths that permitted.

12.10.6 — Adequate anchorage shall be provided for tension reinforcement in flexural members where reinforcement stress is not directly proportional to moment, such as: sloped, stepped, or tapered footings; brackets; deep flexural members; or members in which tension reinforcement is not parallel to compression face. See 12.11.4 and 12.12.4 for deep flexural members.

COMMENTARY

When bars of different sizes are used, the extension should be in accordance with the diameter of bar being terminated. A bar bent to the far face of a beam and continued there may logically be considered effective, in satisfying this section, to the point where the bar crosses the middepth of the member.

R12.10.4 — Peak stresses exist in the remaining bars wherever adjacent bars are cut off, or bent, in tension regions. In Fig. R12.10.2 an "x" mark is used to indicate the peak stress points remaining in continuing bars after part of the bars have been cut off. If bars are cut off as short as the moment diagrams allow, these peak stresses become the full f_y, which requires a full ℓ_d extension as indicated. This extension may exceed the length required for flexure.

R12.10.5 — Reduced shear strength and loss of ductility when bars are cut off in a tension zone, as in Fig. R12.10.2, has been reported. The code does not permit flexural reinforcement to be terminated in a tension zone unless special conditions are satisfied. Flexure cracks tend to open early wherever any reinforcement is terminated in a tension zone. If the steel stress in the continuing reinforcement and the shear strength are each near their limiting values, diagonal tension cracking tends to develop prematurely from these flexure cracks. Diagonal cracks are less likely to form where shear stress is low (see 12.10.5.1). Diagonal cracks can be restrained by closely spaced stirrups (see 12.10.5.2). A lower steel stress reduces the probability of such diagonal cracking (see 12.10.5.3). These requirements are not intended to apply to tension splices which are covered by 12.15, 12.13.5, and the related 12.2.

R12.10.6 — Brackets, members of variable depth, and other members where steel stress f_s does not decrease linearly in proportion to a decreasing moment require special consideration for proper development of the flexural reinforcement. For the bracket shown in Fig. R12.10.6, the stress at ultimate in the reinforcement is almost constant at approxi-

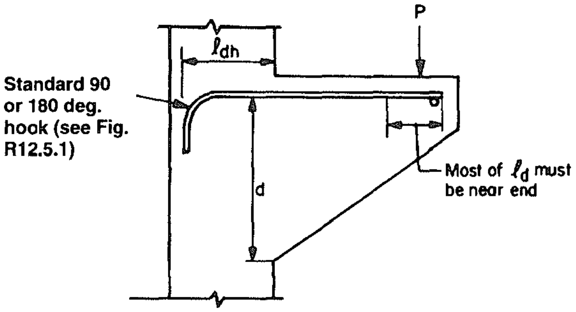

P
ℓ_{dh}
Standard 90 or 180 deg. hook (see Fig. R12.5.1)
d
Most of ℓ_d must be near end

Fig. R12.10.6—Special member largely dependent on end anchorage

CODE

COMMENTARY

mately f_y from the face of support to the load point. In such a case, development of the flexural reinforcement depends largely on the end anchorage provided at the loaded end. Reference 12.1 suggests a welded cross bar of equal diameter as a means of providing effective end anchorage. An end hook in the vertical plane, with the minimum diameter bend, is not totally effective because an essentially plain concrete corner will exist near loads applied close to the corner. For wide brackets (perpendicular to the plane of the figure) and loads not applied close to the corners, U-shaped bars in a horizontal plane provide effective end hooks.

12.11 — Development of positive moment reinforcement

12.11.1 — At least one-third the positive moment reinforcement in simple members and one-fourth the positive moment reinforcement in continuous members shall extend along the same face of member into the support. In beams, such reinforcement shall extend into the support at least 6 in.

12.11.2 — When a flexural member is part of a primary lateral load resisting system, positive moment reinforcement required to be extended into the support by 12.11.1 shall be anchored to develop the specified yield strength f_y in tension at the face of support.

12.11.3 — At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that ℓ_d computed for f_y by 12.2 satisfies Eq. (12-2); except, Eq. (12-2) need not be satisfied for reinforcement terminating beyond centerline of simple supports by a standard hook, or a mechanical anchorage at least equivalent to a standard hook.

$$\ell_d \le \frac{M_n}{V_u} + \ell_a \qquad (12\text{-}2)$$

where:

M_n is nominal moment strength assuming all reinforcement at the section to be stressed to the specified yield strength f_y.

V_u is factored shear force at the section.

ℓ_a at a support shall be the embedment length beyond center of support.

R12.11 — Development of positive moment reinforcement

R12.11.1 — Specified amounts of the positive moment reinforcement are required to be carried into the support to provide for some shifting of the moments due to changes in loading, settlement of supports, lateral loads, and other causes.

R12.11.2 — When a flexural member is part of a primary lateral load resisting system, loads greater than those anticipated in design may cause reversal of moment at supports; some positive reinforcement should be well anchored into the support. This anchorage is required to assure ductility of response in the event of serious overstress, such as from blast or earthquake. It is not sufficient to use more reinforcement at lower stresses.

R12.11.3 — At simple supports and points of inflection such as those marked "PI" in Fig. R12.10.2, the diameter of the positive reinforcement must be small enough so that computed development length of the bar ℓ_d does not exceed $M_n/V_u + \ell_a$, or under favorable support conditions, $1.3M_n/V_u + \ell_a$. Fig. R12.11.3(a) illustrates the use of the provision.

At the point of inflection the value of ℓ_a must not exceed the actual bar extension used beyond the point of zero moment. The M_n/V_u portion of the available length is a theoretical quantity not generally associated with an obvious maximum stress point. M_n is the nominal strength of the cross section without the ϕ-factor and is not the applied factored moment.

The length M_n/V_u corresponds to the development length for the maximum size bar obtained from the previously used flexural bond equation $\Sigma_o = V/ujd$, where u is bond stress, and the jd is moment arm. In the 1971 ACI Building Code, this anchorage requirement was relaxed from previous codes by crediting the available end anchorage length ℓ_a and by including a 30 percent increase for M_n/V_u when the ends of the reinforcement are confined by a compressive reaction.

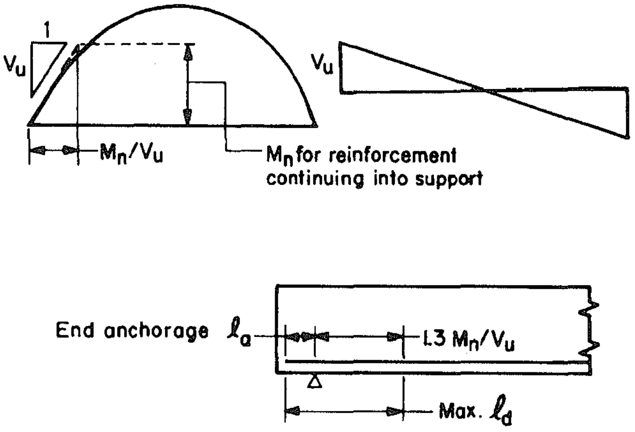
CODE

ℓ_a at a point of inflection shall be limited to the effective depth of member or $12d_b$, whichever is greater.

An increase of 30 percent in the value of M_n/V_u shall be permitted when the ends of reinforcement are confined by a compressive reaction.

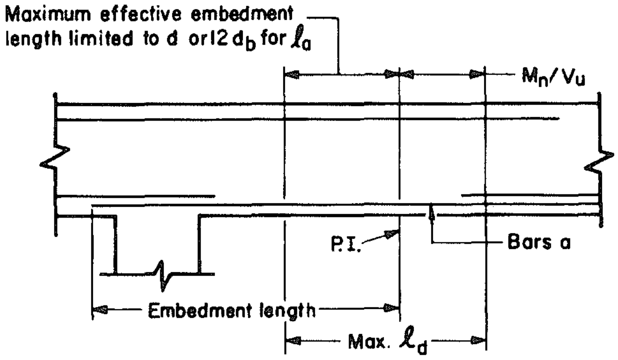
12.11.4 — At simple supports of deep flexural members, positive moment tension reinforcement shall be anchored to develop the specified yield strength f_y in tension at the face of support. At interior supports of

COMMENTARY


1
Vu
Vu
Mn/Vu
Mn for reinforcement continuing into support
End anchorage ℓ_a
1.3 Mn/Vu
Max. ℓ_d

Note: The 1.3 factor is usable only if the reaction confines the ends of the reinforcement.

(a) Maximum size of bar at simple support


Maximum effective embedment length limited to d or 12 d_b for ℓ_a
Mn/Vu
P.I.
Bars a
Embedment length
Max. ℓ_d

(b) Maximum size of Bar "a" at point of inflection

Fig. R12.11.3—Concept for determining maximum bar size per 12.11.3

As an example, consider a bar size is provided at a simple support such that ℓ_d as computed by 12.2 is equal to $0.04 A_b f_y/\sqrt{f_c'}$. The bar size provided is satisfactory only if $0.04 A_b f_y/\sqrt{f_c'}$ does not exceed $1.3M_n/V_u + \ell_a$.

The ℓ_a to be used at points of inflection is limited to the effective depth of the member d or 12 bar diameters ($12d_b$), whichever is greater. Fig. R12.11.3(b) illustrates this provision at points of inflection. The ℓ_a limitation is added since test data are not available to show that a long end anchorage length will be fully effective in developing a bar that has only a short length between a point of inflection and a point of maximum stress.

R12.11.4 — The use of the strut and tie model for the design of reinforced concrete deep flexural members clarifies that there is significant tension in the reinforcement at the face of the support. This requires the tension reinforce-deep flexural members, positive moment tension reinforcement shall be continuous or be spliced with that of the adjacent spans.

12.12 — Development of negative moment reinforcement

12.12.1 — Negative moment reinforcement in a continuous, restrained, or cantilever member, or in any member of a rigid frame, shall be anchored in or through the supporting member by embedment length, hooks, or mechanical anchorage.

12.12.2 — Negative moment reinforcement shall have an embedment length into the span as required by 12.1 and 12.10.3.

12.12.3 — At least one-third the total tension reinforcement provided for negative moment at a support shall have an embedment length beyond the point of inflection not less than effective depth of member, $12d_b$, or one-sixteenth the clear span, whichever is greater.

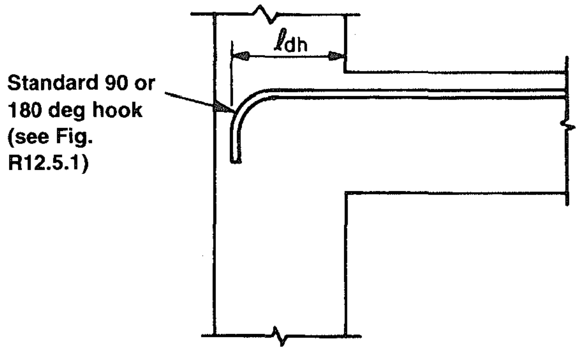
12.12.4 — At interior supports of deep flexural members, negative moment tension reinforcement shall be continuous with that of the adjacent spans.

ment to be continuous or be developed through and beyond the support.[12.16]

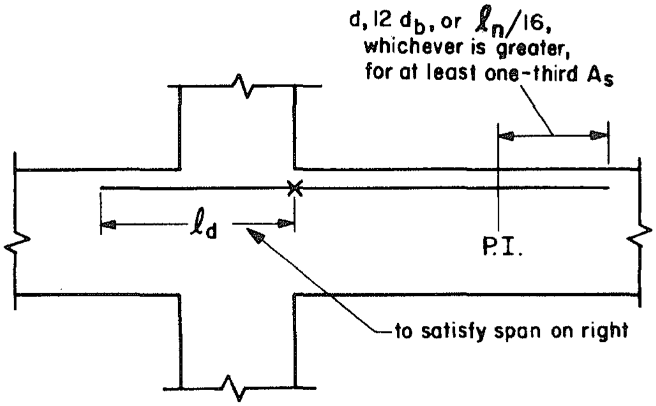
R12.12 — Development of negative moment reinforcement

Fig. R12.12 illustrates two methods of satisfying requirements for anchorage of tension reinforcement beyond the face of support. For anchorage of reinforcement with hooks, see R12.5.

Section 12.12.3 provides for possible shifting of the moment diagram at a point of inflection, as discussed under R12.10.3. This requirement may exceed that of 12.10.3, and the more restrictive of the two provisions governs.



(a) Anchorage into exterior column


d, 12 d_b, or ℓ_n/16, whichever is greater, for at least one-third A_s
ℓ_d
P.I.
to satisfy span on right

Note: Usually such anchorage becomes part of the adjacent beam reinforcement.

(b) Anchorage into adjacent beam

Fig. R12.12—Development of negative moment reinforcement

CODE

12.13 — Development of web reinforcement

12.13.1 — Web reinforcement shall be carried as close to compression and tension surfaces of member as cover requirements and proximity of other reinforcement will permit.

12.13.2 — Ends of single leg, simple U-, or multiple U-stirrups shall be anchored by one of the following means:

12.13.2.1 — For No. 5 bar and D31 wire, and smaller, and for No. 6, No. 7, and No. 8 bars with f_y of 40,000 psi or less, a standard hook around longitudinal reinforcement.

12.13.2.2 — For No. 6, No. 7, and No. 8 stirrups with f_y greater than 40,000 psi, a standard stirrup hook around a longitudinal bar plus an embedment between midheight of the member and the outside end of the hook equal to or greater than $0.014 d_b f_y / \sqrt{f_c'}$.

12.13.2.3—For each leg of welded plain wire fabric forming simple U-stirrups, either:

(a) Two longitudinal wires spaced at a 2 in. spacing along the member at the top of the U.

(b) One longitudinal wire located not more than $d/4$ from the compression face and a second wire closer to the compression face and spaced not less than 2 in. from the first wire. The second wire shall be per-

COMMENTARY

R12.13 — Development of web reinforcement

R12.13.1 — Stirrups must be carried as close to the compression face of the member as possible because near ultimate load the flexural tension cracks penetrate deeply.

R12.13.2 — The anchorage or development requirements for stirrups composed of bars or deformed wire were changed in the 1989 code to simplify the requirements. The straight anchorage was deleted as this stirrup is difficult to hold in place during concrete placement and the lack of a hook may make the stirrup ineffective as it crosses shear cracks near the end of the stirrup.

R12.13.2.1 — For a No. 5 bar or smaller, anchorage is provided by a standard stirrup hook, as defined in 7.1.3, hooked around a longitudinal bar. The 1989 code eliminated the need for a calculated straight embedment length in addition to the hook for these small bars, but 12.13.1 requires a full depth stirrup. Likewise, larger stirrups with f_y equal to or less than 40,000 are sufficiently anchored with a standard stirrup hook around the longitudinal reinforcement.

R12.13.2.2—Since it is not possible to bend a No. 6, No. 7, or No. 8 stirrup tightly around a longitudinal bar and due to the force in a bar with a design stress greater than 40,000 psi, stirrup anchorage depends on both the value of the hook and whatever development length is provided. A longitudinal bar within a stirrup hook limits the width of any flexural cracks, even in a tensile zone. Since such a stirrup hook cannot fail by splitting parallel to the plane of the hooked bar, the hook strength as utilized in 12.5.2 has been adjusted to reflect cover and confinement around the stirrup hook.

For stirrups with f_y of only 40,000 psi, a standard stirrup hook provides sufficient anchorage and these bars are covered in 12.13.2.1. For bars with higher strength, the embedment must be checked. A 135-deg or 180-deg hook is preferred, but a 90-deg hook may be used provided the free end of the 90-deg hook is extended the full 12 bar diameters as required in 7.1.3.

R12.13.2.3—The requirements for anchorage of welded plain wire fabric stirrups are illustrated in Fig. R12.13.2.3.

CODE

mitted to be located on the stirrup leg beyond a bend, or on a bend with an inside diameter of bend not less than $8d_b$.

12.13.2.4 — For each end of a single leg stirrup of welded plain or deformed wire fabric, two longitudinal wires at a minimum spacing of 2 in. and with the inner wire at least the greater of $d/4$ or 2 in. from middepth of member $d/2$. Outer longitudinal wire at tension face shall not be farther from the face than the portion of primary flexural reinforcement closest to the face.

COMMENTARY

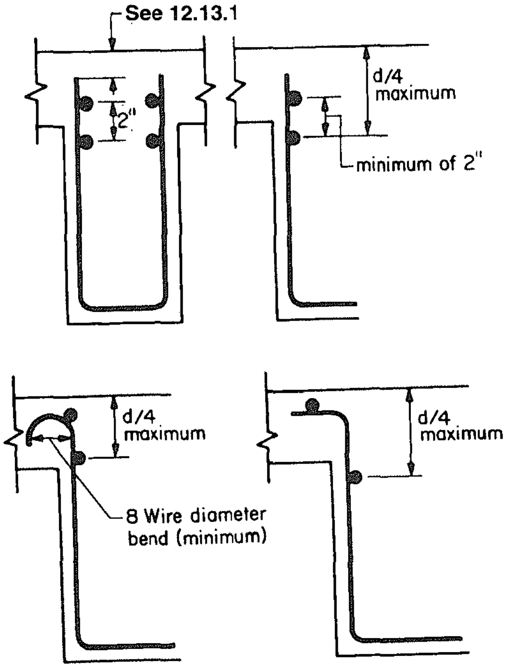

See 12.13.1
2"
d/4 maximum
minimum of 2"
d/4 maximum
8 Wire diameter bend (minimum)
d/4 maximum

Fig. R12.13.2.3—Anchorage in compression zone of welded plain wire fabric U-stirrups

R12.13.2.4 — Use of welded wire fabric for shear reinforcement has become commonplace in the precast, prestressed concrete industry. Rationale for acceptance of straight sheets of wire fabric as shear reinforcement is presented in a report by a joint PCI/WRI Ad Hoc Committee on Welded Wire Fabric for Shear Reinforcement.[12.17]

The provisions for anchorage of single leg welded wire fabric in the tension face emphasize the location of the longitudinal wire at the same depth as the primary flexural

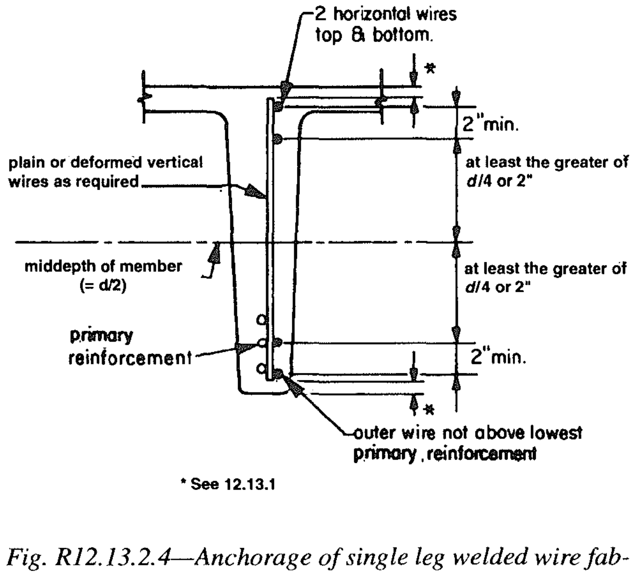

2 horizontal wires top & bottom.
*
2"min.
plain or deformed vertical wires as required
at least the greater of d/4 or 2"
middepth of member (= d/2)
at least the greater of d/4 or 2"
primary reinforcement
2"min.
*
outer wire not above lowest primary reinforcement
* See 12.13.1

Fig. R12.13.2.4—Anchorage of single leg welded wire fabric shear reinforcement

CODE

12.13.2.5 — In joist construction as defined in 8.11, for No. 4 bar and D20 wire and smaller, a standard hook.

12.13.3 — Between anchored ends, each bend in the continuous portion of a simple U-stirrup or multiple U-stirrup shall enclose a longitudinal bar.

12.13.4 — Longitudinal bars bent to act as shear reinforcement, if extended into a region of tension, shall be continuous with longitudinal reinforcement and, if extended into a region of compression, shall be anchored beyond middepth $d/2$ as specified for development length in 12.2 for that part of f_y required to satisfy Eq. (11-17).

12.13.5 — Pairs of U-stirrups or ties so placed as to form a closed unit shall be considered properly spliced when length of laps are $1.3\ell_d$. In members at least 18 in. deep, such splices with $A_b f_y$ not more than 9000 lb per leg shall be considered adequate if stirrup legs extend the full available depth of member.

12.14 — Splices of reinforcement — General

12.14.1 — Splices of reinforcement shall be made only as required or permitted on design drawings, or in specifications, or as authorized by the engineer.

12.14.2 — Lap splices

12.14.2.1 — Lap splices shall not be used for bars larger than No. 11 except as provided in 12.16.2 and 15.8.2.3.

12.14.2.2 — Lap splices of bars in a bundle shall be based on the lap splice length required for individual bars within the bundle, increased in accordance with 12.4. Individual bar splices within a bundle shall

COMMENTARY

reinforcement to avoid a splitting problem at the tension steel level. Fig. R12.13.2.4 illustrates the anchorage requirements for single leg welded wire fabric. For anchorage of single leg welded wire fabric, the code has permitted hooks and embedment length in the compression and tension faces of members (see 12.13.2.1 and 12.13.2.3), and embedment only in the compression face (see 12.13.2.2). Section 12.13.2.4 provides for anchorage of straight single leg welded wire fabric using longitudinal wire anchorage with adequate embedment length in compression and tension faces of members.

R12.13.2.5 — In joists, a small bar or wire can be anchored by a standard hook not engaging longitudinal reinforcement, allowing a continuously bent bar to form a series of single leg stirrups in the joist.

R12.13.5 — These requirements for lapping of double U-stirrups to form closed stirrups control over the provisions of 12.15.

R12.14 — Splices of reinforcement — General

Splices should, if possible, be located away from points of maximum tensile stress. The lap splice requirements of 12.15 encourage this practice.

R12.14.2 — Lap splices

R12.14.2.1 — Because of lack of adequate experimental data on lap splices of No. 14 and No. 18 bars in compression and in tension, lap splicing of these bar sizes is prohibited except as permitted in 12.16.2 and 15.8.2.3 for compression lap splices of No. 14 and No. 18 bars with smaller bars.

R12.14.2.2 — The increased length of lap required for bars in bundles is based on the reduction in the exposed perimeter of the bars. The ℓ_d to be used in computing splice length prescribed in 12.15.1 or 12.16.1 is for an individual

CODE

not overlap. Entire bundles shall not be lap spliced.

12.14.2.3 — Bars spliced by noncontact lap splices in flexural members shall not be spaced transversely farther apart than one-fifth the required lap splice length, nor 6 in.

12.14.3 — Welded splices and mechanical connections

12.14.3.1 — Welded splices and other mechanical connections are allowed.

12.14.3.2 — Except as provided in this code, all welding shall conform to "Structural Welding Code—Reinforcing Steel" (ANSI/AWS D1.4).

12.14.3.3—A full welded splice shall develop at least 125 percent of the specified yield strength f_y of the bar.

12.14.3.4—A full mechanical connection shall develop in tension or compression, as required, at least 125 percent of specified yield strength f_y of the bar.

12.14.3.5—Welded splices and mechanical connections not meeting requirements of 12.14.3.3 or 12.14.3.4 are allowed only for No. 5 bars and smaller and in accordance with 12.15.4.

COMMENTARY

bar. Bundled bars are lap spliced only by splicing individual bars along a length of the bundle. Two bundles should not be lap spliced as individual bars.

R12.14.2.3 — If individual bars in noncontact lap splices are too widely spaced, an unreinforced section is created. Forcing a potential crack to follow a zigzag line (5 to 1 slope) is considered a minimum precaution. The 6 in. maximum spacing is added because most research available on the lap splicing of deformed bars was conducted with reinforcement within this spacing.

R12.14.3 — Welded splices and mechanical connections

R12.14.3.2 — The code requires all welding of reinforcing steel to conform to the American Welding Society **"Structural Welding Code—Reinforcing Steel"** (ANSI/AWS D1.4). See R3.5.2 for discussion on welding.

R12.14.3.3—A full welded splice is primarily intended for large bars (No. 6 and larger) in main members. The tensile strength requirement of 125 percent of specified yield will ensure sound welding, adequate also for compression. The maximum reinforcement stress used in design under the code is the yield strength. To ensure sufficient strength in splices so that yielding can be achieved in a member and thus brittle failure avoided, the 25 percent increase above the specified yield strength was selected as both an adequate minimum for safety and a practicable maximum for economy. The 1995 edition eliminated a requirement that the bars be butted since indirect butt welds are permitted by ANSI/AWS D1.4, although ANSI/AWS D1.4 does indicate that wherever practical, direct butt splices are preferable for No. 7 bars and larger.

R12.14.3.4—Full mechanical connections are also required to develop 125 percent of the yield strength, in tension or compression as required, for the same reasons discussed for full welded splices in R12.14.3.3.

R12.14.3.5—The use of welded splices or mechanical connections of less strength than 125 percent of yield strength is permitted if the minimum design criteria of 12.15.4 are met. Therefore, lap welds of reinforcing bars, either with or without backup material, welds to plate connections, and end-bearing splices are allowed under certain conditions. The 1995 edition limited these lower strength welds and connections to No. 5 bars and smaller due to the potentially brittle nature of failure at these welds.

CODE

12.15 — Splices of deformed bars and deformed wire in tension

12.15.1 — Minimum length of lap for tension lap splices shall be as required for Class A or B splice, but not less than 12 in., where:

Class A splice .... 1.0 ℓ_d

Class B splice .... 1.3ℓ_d

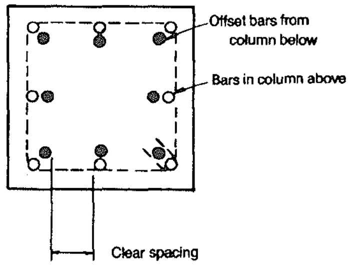
where ℓ_d is the tensile development length for the specified yield strength f_y in accordance with 12.2 without the modification factor of 12.2.5.

COMMENTARY

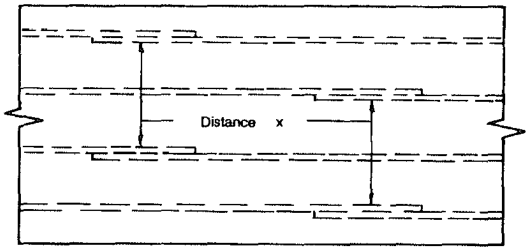
R12.15 — Splices of deformed bars and deformed wire in tension

R12.15.1 — Lap splices in tension are classified as Types A and B, with length of lap a multiple of the tensile development length ℓ_d. The development length $\ell_d = \ell_{db}$ times the applicable modification factors (see 12.2) used to obtain lap length must be based on full f_y because the splice classifications already reflect any excess reinforcement at the splice location; therefore, the factor from 12.2.5 for excess A_s must not be used. The increasing modification factors of 12.2.4 for top reinforcement, lightweight concrete, and epoxy-coated reinforcement must be applied where appropriate. A modifying factor must also be included to account for clear spacing, amount of cover, and transverse reinforcement. When multiple bars are spliced at the same section, the clear spacing is the minimum clear distance between the bars measured outside the splice length less one bar diameter. For splices in columns with offset bars, Fig. R12.15.1(a) illustrates the clear spacing to be used. For staggered splices, the clear spacing is the distance between adjacent spliced bars [distance x in Fig. R12.15.1(b)] less the diameters of any intermediate unspliced bars.

The 1989 code contained several changes in development length in tension which eliminated many of the concerns regarding tension splices due to closely spaced bars with minimal cover. Thus, the Class C splice was eliminated although development lengths, on which splice lengths are based, have in some cases increased. Committee 318 con-



a) Offset column bars



b) Staggered splices

Fig. R12.15.1—Clear spacing of spliced bars

CODE

12.15.2 — Lap splices of deformed bars and deformed wire in tension shall be Class B splices except that Class A splices are allowed when: (a) the area of reinforcement provided is at least twice that required by analysis over the entire length of the splice, and (b) one-half or less of the total reinforcement is spliced within the required lap length.

12.15.3 — Welded splices or mechanical connections used where area of reinforcement provided is less than twice that required by analysis shall meet requirements of 12.14.3.3 or 12.14.3.4.

12.15.4 — Welded splices and mechanical connections not meeting the requirements of 12.14.3.3 or 12.14.3.4 are allowed for No. 5 bars and smaller when the area of reinforcement provided is at least twice that required by analysis, and the following requirements are met:

12.15.4.1 — Splices shall be staggered at least 24 in. and in such manner as to develop at every section at least twice the calculated tensile force at that section but not less than 20,000 psi for total area of reinforcement provided.

12.15.4.2 — In computing tensile forces developed at each section, rate the spliced reinforcement at the specified splice strength. Unspliced reinforcement shall be rated at that fraction of f_y defined by the ratio of the shorter actual development length to ℓ_d required to develop the specified yield strength f_y.

COMMENTARY

sidered suggestions from many sources, including Committee 408, but has retained a two-level splice length primarily to encourage designers to splice bars at points of minimum stress and to stagger splices to improve behavior of critical details.

R12.15.2 — The tension lap splice requirements of 12.15.1 encourage the location of splices away from regions of high tensile stress, to locations where the area of steel provided is at least twice that required by analysis. Table R12.15.2 presents the splice requirements in tabular form as presented in earlier code editions.

TABLE R12.15.2—TENSION LAP SPLICES

$\frac{A_s \text{ provided}^*}{A_s \text{ required}}$	Maximum percent of A_s spliced within required lap length	
	50	100
Equal to or greater than 2	Class A	Class B
Less than 2	Class B	Class B

* Ratio of area of reinforcement provided to area of reinforcement required by analysis at splice locations.

R12.15.3 — A welded splice or mechanical connection must develop at least 125 percent of the specified yield strength when located in regions of high tensile stress in the reinforcement. Such splices or connections need not be staggered although such staggering is encouraged where the area of reinforcement provided is less than twice that required by the analysis.

R12.15.4 — See R12.14.3.5. This section describes the situation where welded splices or mechanical connections of less strength than 125 percent of the specified yield strength of the reinforcement may be used. It provides a relaxation in the splice requirements where the splices or connections are staggered and excess reinforcement area is available. The criterion of twice the computed tensile force is used to cover sections containing partial tensile splices with various percentages of total steel continuous. The usual partial tensile splice will be a flare groove weld between bars or bar and structural steel piece.

To detail such welding, the length of weld must be specified. Such welds are rated as the product of total weld length times effective size of groove weld (established by bar size) times allowable stress permitted by **"Structural Welding Code—Reinforcing Steel"** (ANSI/AWS D1.4).

A full welded splice or connection conforming to 12.14.3.3 or 12.14.3.4 can be used without the stagger requirement in lieu of the lower strength weld or connection.

CODE

12.15.5 — Splices in "tension tie members" shall be made with a full welded splice or full mechanical connection in accordance with 12.14.3.3 or 12.14.3.4 and splices in adjacent bars shall be staggered at least 30 in.

12.16 — Splices of deformed bars in compression

12.16.1 — Compression lap splice length shall be **$0.0005 f_y d_b$**, for f_y of 60,000 psi or less, or **$(0.0009 f_y - 24) d_b$** for f_y greater than 60,000 psi, but not less than 12 in. For f_c' less than 3000 psi, length of lap shall be increased by one-third.

12.16.2 — When bars of different size are lap spliced in compression, splice length shall be the larger of development length of larger bar, or splice length of smaller bar. Lap splices of No. 14 and No. 18 bars to No. 11 and smaller bars shall be permitted.

COMMENTARY

R12.15.5 — A tension tie member, as envisioned by ACI Committee 318, has the following characteristics: member having an axial tensile force sufficient to create tension over the cross section; a level of stress in the reinforcement such that every bar must be fully effective; and limited concrete cover on all sides. Examples of members which may be classified as tension ties are arch ties, hangers carrying load to an overhead supporting structure, and main tension elements in a truss.

In determining if a member should be classified as a tension tie, consideration must be given to the importance, function, proportions, and stress conditions of the member related to the above characteristics. For example, the usual large circular tank, with many bars and with splices well staggered and widely spaced should not be classified as a tension tie member, and Class B splices may be used.

R12.16 — Splices of deformed bars in compression

Bond research has been primarily related to bars in tension. Bond behavior of compression bars is not complicated by the problem of transverse tension cracking and thus compression splices do not require provisions as strict as those specified for tension splices. The minimum lengths specified for column splices contained originally in the 1956 ACI Building Code have been carried forward in the later code editions, and extended to compression bars in beams and to higher strength steels. No changes have been made in the provisions for compression splices since the 1971 code edition.

R12.16.1 — Essentially, lap requirements for compression splices have remained the same since the 1963 ACI Building Code.

The 1963 ACI Building Code values were modified in the 1971 code to recognize various degrees of confinement and to permit design with reinforcement up to 80,000 psi yield strength. Tests[12.1,12.18] have shown that splice strengths in compression depend considerably on end bearing and hence do not increase proportionally in strength when the splice length is doubled. Accordingly, for yield strengths above 60,000 psi, compression lap lengths are significantly increased, except where spiral enclosures are used (as in spiral columns) the increase is about 10 percent at 75,000 psi.

R12.16.2 — The lap splice length is to be computed based on the larger of: (1) the compression splice length of the smaller bar or (2) the compression development length of the larger bar. Lap splices are generally prohibited for No. 14 or No. 18 bars; however, for compression only, lap splices are permitted for No. 14 or No. 18 bars to No. 11 or smaller bars.

CODE

12.16.3 — Welded splices or mechanical connections used in compression shall meet requirements of 12.14.3.3 or 12.14.3.4.

12.16.4 — End-bearing splices

12.16.4.1 — In bars required for compression only, transmission of compressive stress by bearing of square cut ends held in concentric contact by a suitable device shall be permitted.

12.16.4.2 — Bar ends shall terminate in flat surfaces within $1^1/_2$ deg of a right angle to the axis of the bars and shall be fitted within 3 deg of full bearing after assembly.

12.16.4.3 — End-bearing splices shall be used only in members containing closed ties, closed stirrups, or spirals.

12.17 — Special splice requirements for columns

12.17.1 — Lap splices, butt welded splices, mechanical connections, or end-bearing splices shall be used with the limitations of 12.17.2 through 12.17.4. A splice shall satisfy requirements for all load combinations for the column.

COMMENTARY

R12.16.4 — End-bearing splices

R12.16.4.1 — Experience with end-bearing splices has been almost exclusively with vertical bars in columns. If bars are significantly inclined from the vertical, special attention is required to ensure that adequate end-bearing contact can be achieved and maintained.

R12.16.4.2 — These tolerances were added in the 1971 code, representing practice based on tests of full-size members containing No. 18 bars.

R12.16.4.3 — This limitation was added in the 1971 code to ensure a minimum shear resistance in sections containing end-bearing splices.

R12.17 — Special splice requirements for columns

In columns subject to flexure and axial loads, tension stresses may occur on one face of the column for moderate and large eccentricities as shown in Fig. R12.17. When such tensions occur, 12.17 requires tension splices to be used or an adequate tensile resistance provided. Furthermore, a minimum tension capacity is required in each face of all columns even where analysis indicates compression only.

The 1989 Code clarifies this section on the basis that a compressive lap splice has a tension capacity of at least one-

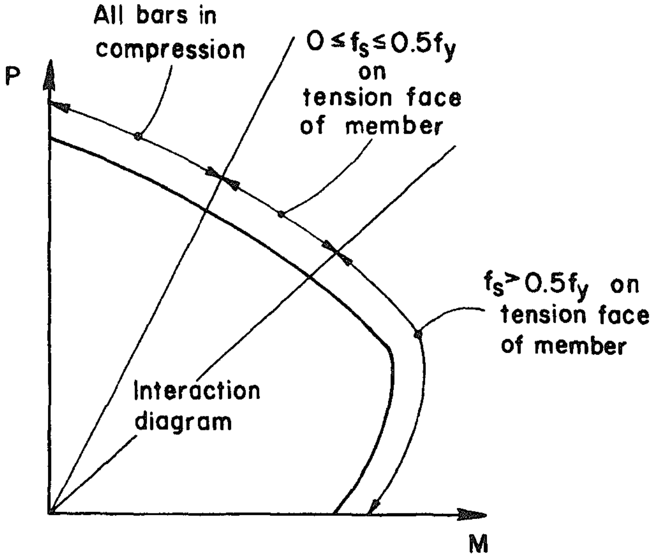

All bars in compression
P
$0 \le f_s \le 0.5f_y$ on tension face of member
$f_s > 0.5f_y$ on tension face of member
Interaction diagram
M

Fig. R12.17—Special splice requirements for columns

CODE

12.17.2 — Lap splices in columns

12.17.2.1 — Where the bar stress due to factored loads is compressive, lap splices shall conform to 12.16.1, 12.16.2, and, where applicable, to 12.17.2.4 or 12.17.2.5.

12.17.2.2 — Where the bar stress due to factored loads is tensile and does not exceed $0.5f_y$ in tension, lap splices shall be Class B tension lap splices if more than one-half of the bars are spliced at any section, or Class A tension lap splices if half or fewer of the bars are spliced at any section and alternate lap splices are staggered by ℓ_d.

12.17.2.3 — Where the bar stress due to factored loads is greater than $0.5f_y$ in tension, lap splices shall be Class B tension lap splices.

12.17.2.4 — In tied reinforced compression members, where ties throughout the lap splice length have

COMMENTARY

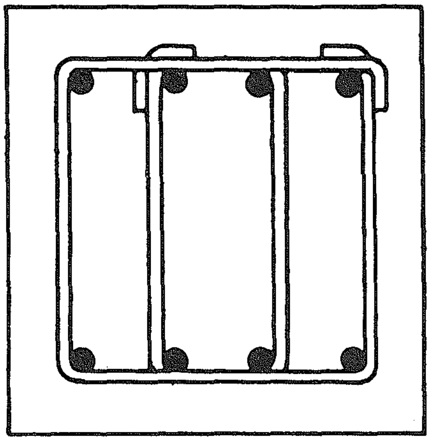


Fig. R.12.17.2—Tie legs which cross the axis of bending are used to compute effective area. In the case shown, four legs are effective

quarter f_y, which simplifies the calculation requirements suggested in the previous code editions.

Note that the column splice must satisfy requirements for all load combinations for the column. Frequently, the basic gravity load combination will govern the design of the column itself, but a load combination including wind or seismic loads may induce greater tension in some column bars, and the column splice must be designed for this tension.

Section 12.17 has been reorganized to define more clearly the requirements for different types of bar splices in columns.

R12.17.2 — Lap splices in columns

R12.17.2.1 — The 1989 code was simplified for column bars always in compression on the basis that a compressive lap splice is adequate for sufficient tension to preclude special requirements.

R12.17.2.4 — Reduced lap lengths are allowed when the splice is enclosed throughout its length by minimum ties.

CODE

an effective area not less than **0.0015*hs***, lap splice length shall be permitted to be multiplied by 0.83, but lap length shall not be less than 12 in. Tie legs perpendicular to dimension ***h*** shall be used in determining effective area.

12.17.2.5 — In spirally reinforced compression members, lap splice length of bars within a spiral shall be permitted to be multiplied by 0.75, but lap length shall not be less than 12 in.

12.17.3 — Welded splices or mechanical connectors in columns

Welded splices or mechanical connectors in columns shall meet the requirements of 12.14.3.3 or 12.14.3.4.

12.17.4 — End-bearing splices in columns

End-bearing splices complying with 12.16.4 shall be permitted to be used for column bars stressed in compression provided the splices are staggered or additional bars are provided at splice locations. The continuing bars in each face of the column shall have a tensile strength, based on the specified yield strength f_y, not less than $0.25f_y$ times the area of the vertical reinforcement in that face.

12.18 — Splices of welded deformed wire fabric in tension

12.18.1 — Minimum length of lap for lap splices of welded deformed wire fabric measured between the ends of each fabric sheet shall be not less than $1.3\ \ell_d$ nor 8 in., and the overlap measured between outermost cross wires of each fabric sheet shall be not less than 2 in. ℓ_d shall be the development length for the specified yield strength f_y in accordance with 12.7.

12.18.2 — Lap splices of welded deformed wire fabric, with no cross wires within the lap splice length, shall be determined as for deformed wire.

COMMENTARY

Compression splice lengths may be multiplied by 0.83 for tied compression members when the tie area throughout the lap length is at least **0.0015 *hs***, but the splice length may not be less than 12 in.

The tie legs perpendicular to each direction are computed separately and the requirement must be satisfied in each direction. This is illustrated in Fig. R12.17.2, where four legs are effective in one direction and two legs in the other direction. This calculation is critical in one direction which normally can be determined by inspection.

R12.17.2.5 — Compression lap lengths may be reduced when the lap splice is enclosed throughout its length by spirals because of increased splitting resistance. Spirals should meet requirements of 7.10.4 and 10.9.3.

R12.17.3 — Welded splices or mechanical connectors in columns

Welded splices or couplers are allowed for splices in columns but must be designed as a full welded splice or a full mechanical connection developing 125 percent f_y as required by 12.16.3 and 12.14.3.3, or 12.14.3.4. Splice capacity is traditionally tested in tension and full strength is required to reflect the high compression loads possible in column reinforcement due to creep effects. If a coupler developing less than a full mechanical connection is desired, then the splice must conform to all requirements of end bearing splices of 12.16.4 and 12.17.4.

R12.17.4 — End-bearing splices in columns

End-bearing splices used to splice column bars always in compression must have tension capacity of 25 percent of the yield strength of the steel area on each face of the column, either by staggering the end-bearing splices or by adding additional steel through the splice location. The end-bearing splice must conform to 12.16.4.

R12.18 — Splices of welded deformed wire fabric in tension

Splice provisions for deformed fabric are based on available tests.[12.19] The requirements were simplified (1976 code supplement) from provisions of the 1971 ACI Building Code by assuming that only one cross wire of each fabric sheet is overlapped and by computing the splice length as $1.3\ell_d$. The development length ℓ_d is that computed in accordance with the provisions of 12.7 without regard to the 8 in. minimum. The 8 in. applies to the overall splice length. See Fig. R12.18. If no cross wires are within the lap length, the provisions for deformed wire apply.

CODE

12.18.3—When any plain wires are present in the deformed wire fabric in the direction of the lap splice or when deformed wire fabric is lap spliced to plain wire fabric, the fabric shall be lap spliced in accordance with 12.19.

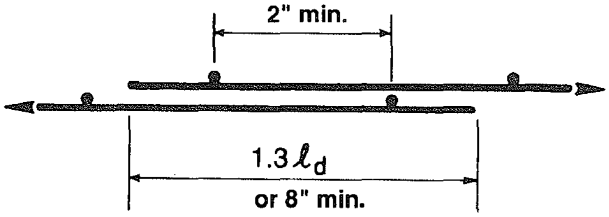
12.19 — Splices of welded plain wire fabric in tension

Minimum length of lap for lap splices of welded plain wire fabric shall be in accordance with the following.

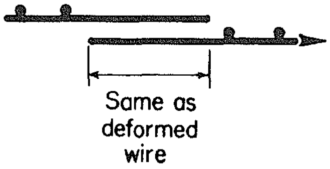
12.19.1 — When area of reinforcement provided is less than twice that required by analysis at splice location, length of overlap measured between outermost cross wires of each fabric sheet shall be not less than one spacing of cross wires plus 2 in., nor less than **1.5**ℓ_d, nor 6 in. ℓ_d shall be the development length for the specified yield strength f_y in accordance with 12.8.

12.19.2 — When area of reinforcement provided is at least twice that required by analysis at splice location, length of overlap measured between outermost cross wires of each fabric sheet shall be not less than **1.5** ℓ_d, nor 2 in. ℓ_d shall be the development length for the specified yield strength f_y in accordance with 12.8.

COMMENTARY



(a) Section 12.18.1

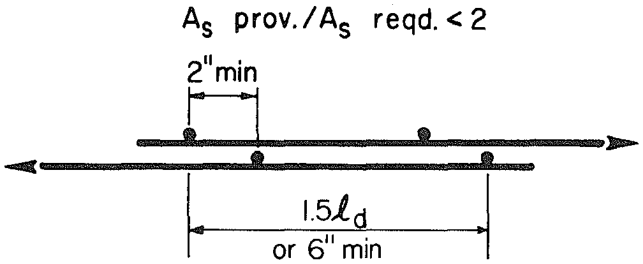


(b) Section 12.18.2

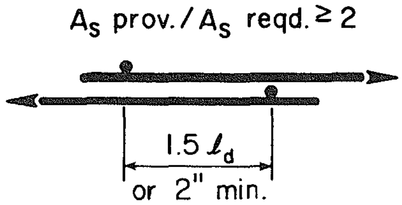
Fig. R12.18—Lap splices of deformed fabric

R12.19 — Splices of welded plain wire fabric in tension

The strength of lap splices of welded plain wire fabric is dependent primarily on the anchorage obtained from the cross wires rather than on the length of wire in the splice. For this reason, the lap is specified in terms of overlap of cross wires rather than in wire diameters or inches. The 2 in. additional lap required is to assure overlapping of the cross wires and to provide space for satisfactory consolidation of the concrete between cross wires. Research[12.20] has shown



(a) Section 12.19.1



(b) Section 12.19.2

Fig. R12.19—Lap splices of plain fabric

CODE

COMMENTARY

an increased splice length is required when fabric of large, closely spaced wires, is lapped and as a consequence additional splice length requirements are provided for these fabrics, in addition to an absolute minimum of 6 in. The development length ℓ_d is that computed in accordance with the provisions of 12.8 without regard to the 6 in. minimum. Splice requirements are illustrated in Fig. R12.19.

Notes

PART 5 — STRUCTURAL SYSTEMS OR ELEMENTS

CHAPTER 13 — TWO-WAY SLAB SYSTEMS

CODE

13.0 — Notation

b_1 = width of the critical section defined in 11.12.1.2 measured in the direction of the span for which moments are determined, in.

b_2 = width of the critical section defined in 11.12.1.2 measured in the direction perpendicular to b_1, in.

c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in.

c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction of the span for which moments are being determined, in.

C = cross-sectional constant to define torsional properties

$$= \sum\left(1-0.63\frac{x}{y}\right)\frac{x^3y}{3}$$

The constant C for T- or L-sections shall be permitted to be evaluated by dividing the section into separate rectangular parts and summing the values of C for each part

E_{cb} = modulus of elasticity of beam concrete

E_{cs} = modulus of elasticity of slab concrete

h = overall thickness of member, in.

I_b = moment of inertia about centroidal axis of gross section of beam as defined in 13.2.4

I_s = moment of inertia about centroidal axis of gross section of slab

= $h^3/12$ times width of slab defined in notations α and β_t

K_t = torsional stiffness of torsional member; moment per unit rotation. See R13.7.5.

ℓ_n = length of clear span in direction that moments are being determined, measured face-to-face of supports

ℓ_1 = length of span in direction that moments are being determined, measured center-to-center of supports

ℓ_2 = length of span transverse to ℓ_1, measured center-to-center of supports. See also 13.6.2.3 and 13.6.2.4

M_o = total factored static moment

M_u = factored moment at section

V_c = nominal shear strength provided by concrete. See 11.12.2.1

COMMENTARY

The design methods given in Chapter 13 are based on analysis of the results of an extensive series of tests[13.1-13.7] and the well established performance record of various slab systems. Much of Chapter 13 is concerned with the selection and distribution of flexural reinforcement. It is, therefore, advisable before discussing the various rules for design, to caution the designer that the problem related to safety of a slab system is the transmission of load from the slab to the columns by flexure, torsion, and shear. Design criteria for shear and torsion in slabs are given in Chapter 11.

In the 1995 code, some sections have been renumbered (13.3, 13.4, and 13.5 were 13.4, 13.5, and 13.3, respectively, in the 1989 code) for ease of use in the design process, and parts of the commentary (especially in R13.6 and R13.7) have been removed.

Design aids for use in the engineering analysis and design of two-way slab systems are given in ACI 340.4R [SP-17(S)].[13.8] Design aids are provided to simplify application of the Direct Design and Equivalent Frame Methods of Chapter 13.

V_u = factored shear force at section
w_d = factored dead load per unit area
w_ℓ = factored live load per unit area
w_u = factored load per unit area
x = shorter overall dimension of rectangular part of cross section
y = longer overall dimension of rectangular part of cross section
α = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of the beam
= $\dfrac{E_{cb} I_b}{E_{cs} I_s}$
α_1 = α in direction of ℓ_1
α_2 = α in direction of ℓ_2
β_t = ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports
= $\dfrac{E_{cb} C}{2 E_{cs} I_s}$
γ_f = fraction of unbalanced moment transferred by flexure at slab-column connections. See 13.5.3.2
γ_v = fraction of unbalanced moment transferred by eccentricity of shear at slab-column connections
= $1 - \gamma_f$
ρ = ratio of nonprestressed tension reinforcement
ρ_b = reinforcement ratio producing balanced strain conditions
ϕ = strength reduction factor

13.1 — Scope

13.1.1—Provisions of Chapter 13 shall apply for design of slab systems reinforced for flexure in more than one direction, with or without beams between supports.

13.1.2 — For a slab system supported by columns or walls, the dimensions c_1 and c_2 and the clear span ℓ_n shall be based on an effective support area defined by the intersection of the bottom surface of the slab, or of the drop panel if there is one, with the largest right circular cone, right pyramid, or tapered wedge whose surfaces are located within the column and capital or bracket and are oriented no greater than 45 deg to the axis of the column.

13.1.3 — Solid slabs and slabs with recesses or pockets made by permanent or removable fillers between ribs or joists in two directions are included within the scope of Chapter 13.

R13.1 — Scope

The fundamental design principles contained in Chapter 13 are applicable to all planar structural systems subjected to transverse loads. However, some of the specific design rules, as well as historical precedents, limit the types of structures to which Chapter 13 is applicable. General characteristics of slab systems which may be designed according to Chapter 13 are described in this section. These systems include "flat slabs," "flat plates," "two-way slabs," and "waffle slabs." Slabs with paneled ceilings are two-way wide-band beam systems.

True "one-way slabs," slabs reinforced to resist flexural stresses in only one direction, are excluded. Also excluded are soil supported slabs, such as "slabs on grade," which do not transmit vertical loads from other parts of the structure to the soil.

For slabs with beams, the explicit design procedures of Chapter 13 apply only when the beams are located at the

CODE

13.1.4 — Minimum thickness of slabs designed in accordance with Chapter 13 shall be as required by 9.5.3.

13.2 — Definitions

13.2.1 — Column strip is a design strip with a width on each side of a column centerline equal to 0.25 ℓ_2 or 0.25ℓ_1, whichever is less. Column strip includes beams, if any.

13.2.2 — Middle strip is a design strip bounded by two column strips.

13.2.3 — A panel is bounded by column, beam, or wall centerlines on all sides.

13.2.4 — For monolithic or fully composite construction, a beam includes that portion of slab on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.

13.3 — Slab reinforcement

13.3.1 — Area of reinforcement in each direction for two-way slab systems shall be determined from moments at critical sections, but shall not be less than required by 7.12.

13.3.2 — Spacing of reinforcement at critical sections shall not exceed two times the slab thickness, except for portions of slab area of cellular or ribbed construc-

COMMENTARY

edges of the panel and when the beams are supported by columns or other essentially nondeflecting supports at the corners of the panel. Two-way slabs with beams in one direction with both slab and beams supported by girders in the other direction may be designed under the general requirements of Chapter 13. Such designs must be based upon analysis compatible with the deflected position of the supporting beams and girders.

For slabs supported on walls, the explicit design procedures in this chapter envision the wall as a beam of infinite stiffness; therefore, each wall should support the entire length of an edge of the panel. (See 13.2.3). Wall-like columns less than a full panel length can be treated as columns.

R13.2 — Definitions

R13.2.3 — A panel, by definition, includes all flexural elements between column centerlines. Thus, the column strip includes the beam, if any.

R13.2.4 — For monolithic or fully composite construction, the beams include portions of the slab as flanges. Two examples of the rule in this section are provided in Fig. R13.2.4.

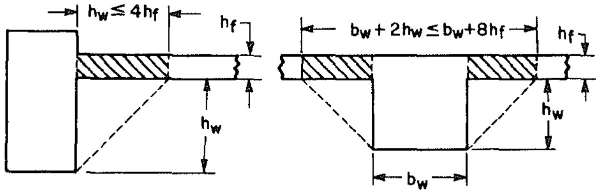


Fig. R13.2.4—Examples of the portion of slab to be included with the beam under 13.2.4

R13.3 — Slab reinforcement

R13.3.2 — The requirement that the center-to-center spacing of the reinforcement be not more than two times the slab thickness applies only to the reinforcement in solid slabs,

CODE

tion. In the slab over cellular spaces, reinforcement shall be provided as required by 7.12.

13.3.3 — Positive moment reinforcement perpendicular to a discontinuous edge shall extend to the edge of slab and have embedment, straight or hooked, at least 6 in. in spandrel beams, columns, or walls.

13.3.4 — Negative moment reinforcement perpendicular to a discontinuous edge shall be bent, hooked, or otherwise anchored, in spandrel beams, columns, or walls, to be developed at face of support according to provisions of Chapter 12.

13.3.5 — Where a slab is not supported by a spandrel beam or wall at a discontinuous edge, or where a slab cantilevers beyond the support, anchorage of reinforcement shall be permitted within the slab.

13.3.6 — In slabs with beams between supports with a value of α greater than 1.0, special top and bottom slab reinforcement shall be provided at exterior corners in accordance with the following:

13.3.6.1 — The special reinforcement in both top and bottom of slab shall be sufficient to resist a moment equal to the maximum positive moment (per foot of width) in the slab.

13.3.6.2 — The moment shall be assumed to be about an axis perpendicular to the diagonal from the corner in the top of the slab and about an axis parallel to the diagonal from the corner in the bottom of the slab.

13.3.6.3 — The special reinforcement shall be provided for a distance in each direction from the corner equal to one-fifth the longer span.

13.3.6.4 — The special reinforcement shall be placed in a band parallel to the diagonal in the top of the slab and a band perpendicular to the diagonal in the bottom of the slab. Alternatively, the special reinforcement shall be placed in two layers parallel to the sides of the slab in either the top or bottom of the slab.

13.3.7 — Where a drop panel is used to reduce amount of negative moment reinforcement over the column of a flat slab, size of drop panel shall be in accordance with the following:

13.3.7.1 — Drop panel shall extend in each direction from centerline of support a distance not less than one-sixth the span length measured from center-to-center of supports in that direction.

COMMENTARY

and not to that in joists or waffle slabs. This limitation is intended to ensure slab action and reduce cracking and to provide for the possibility of loads concentrated on small areas of the slab. See also R10.6.

R13.3.3-R13.3.5 — Bending moments in slabs at spandrel beams can be subject to great variation. If spandrel beams are built solidly into walls, the slab approaches complete fixity. Without an integral wall, the slab could be largely simply supported, depending on the torsional rigidity of the spandrel beam or slab edge. These requirements provide for unknown conditions that might normally occur in a structure.

CODE

13.3.7.2 — Projection of drop panel below the slab shall be at least one-quarter the slab thickness beyond the drop.

13.3.7.3 — In computing required slab reinforcement, thickness of drop panel below the slab shall not be assumed greater than one-quarter the distance from edge of drop panel to edge of column or column capital.

13.3.8 — Details of reinforcement in slabs without beams

13.3.8.1 — In addition to the other requirements of 13.3, reinforcement in slabs without beams shall have minimum extensions as prescribed in Fig. 13.3.8.

13.3.8.2 — Where adjacent spans are unequal, extensions of negative moment reinforcement beyond the face of support as prescribed in Fig. 13.3.8 shall be based on requirements of the longer span.

13.3.8.3 — Bent bars shall be permitted only when depth-span ratio permits use of bends 45 deg or less.

13.3.8.4 — For slabs in frames not braced against sidesway, lengths of reinforcement shall be determined by analysis but shall not be less than those prescribed in Fig. 13.3.8.

13.3.8.5 — All bottom bars or wires within the column strip, in each direction, shall be continuous or spliced with Class A splices located as shown in Fig. 13.3.8. At least two of the column strip bottom bars or wires in each direction shall pass within the column core and shall be anchored at exterior supports.

13.3.8.6 — In slabs with shearheads and in lift-slab construction, at least two bonded bottom bars or wires in each direction shall pass through the shearhead or lifting collar as close to the column as practicable and be continuous or spliced with a Class A splice. At exterior columns, the reinforcement shall be anchored at the shearhead or lifting collar.

COMMENTARY

R13.3.8 — Details of reinforcement in slabs without beams

In 1989, bent bars were removed from Fig. 13.3.8 of this code. This was done because bent bars are seldom used and are difficult to place properly. Bent bars placed in accordance with Fig. 13.4.8 of the 1983 ACI Code are permitted.

R13.3.8.4 — Where two-way slabs act as primary members of a laterally unbraced frame resisting lateral loads, the resulting moments due to the combined lateral and gravity loadings preclude use of the arbitrary lengths and minimum extensions of bars in Fig. 13.3.8.

R13.3.8.5 — The continuous column strip bottom reinforcement provides the slab some residual ability to span to the adjacent supports should a single support be damaged. The two continuous column strip bottom bars or wires through the column may be termed "integrity steel," and are provided to give the slab some residual capacity following a single punching shear failure.[13.9]

R13.3.8.6 — In 1992, this provision was added to require the same "integrity" steel as for other two-way slabs without beams in case of a single punching shear failure at a support. (See Fig. R13.3.8.6 for location of integrity steel.)

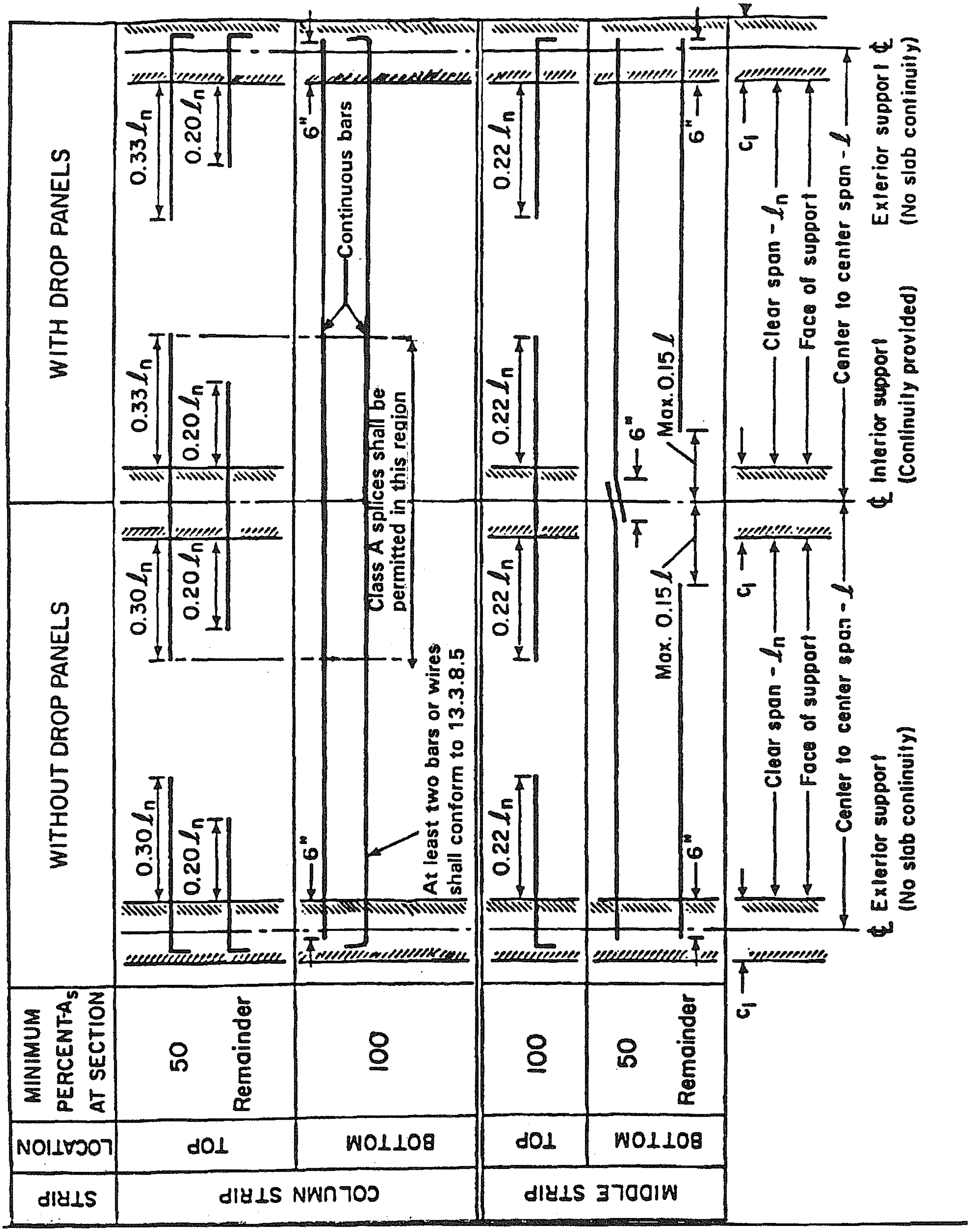

STRIP
LOCATION
MINIMUM PERCENT-A_s AT SECTION
WITHOUT DROP PANELS
WITH DROP PANELS
COLUMN STRIP
TOP
50
Remainder
BOTTOM
100
MIDDLE STRIP
TOP
100
BOTTOM
50
Remainder
$0.30\ell_n$
$0.20\ell_n$
$0.33\ell_n$
$0.22\ell_n$
6"
Continuous bars
Class A splices shall be permitted in this region
At least two bars or wires shall conform to 13.3.8.5
Max. 0.15ℓ
c_1
Clear span - ℓ_n
Face of support
Center to center span - ℓ
℄ Exterior support (No slab continuity)
℄ Interior support (Continuity provided)
Exterior support ℄ (No slab continuity)

Fig. 13.3.8—Minimum extensions for reinforcement in slabs without beams. (See 12.11.1 for reinforcement extension into supports)

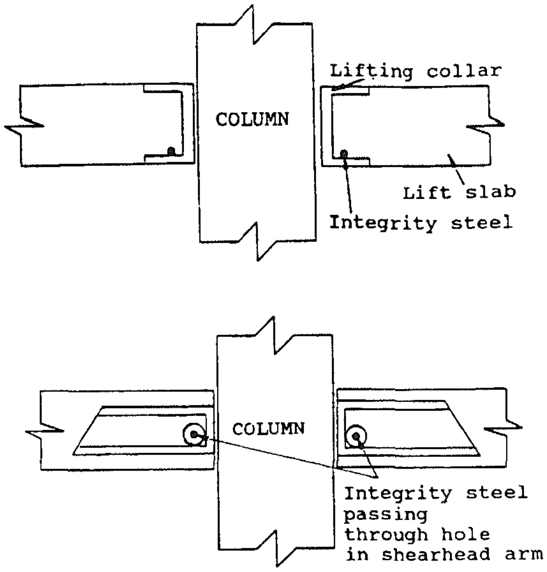


Fig. R13.3.8.6—Location of integrity steel

13.4 — Openings in slab systems

13.4.1 — Openings of any size shall be permitted in slab systems if shown by analysis that the design strength is at least equal to the required strength considering 9.2 and 9.3, and that all serviceability conditions, including the specified limits on deflections, are met.

13.4.2 — As an alternate to special analysis as required by 13.4.1, openings shall be permitted in slab systems without beams only in accordance with the following:

13.4.2.1 — Openings of any size shall be permitted in the area common to intersecting middle strips, provided total amount of reinforcement required for the panel without the opening is maintained.

13.4.2.2 — In the area common to intersecting column strips, not more than one-eighth the width of column strip in either span shall be interrupted by openings. An amount of reinforcement equivalent to that interrupted by an opening shall be added on the sides of the opening.

13.4.2.3 — In the area common to one column strip and one middle strip, not more than one-quarter of the reinforcement in either strip shall be interrupted by openings. An amount of reinforcement equivalent to that interrupted by an opening shall be added on the sides of the opening.

13.4.2.4 — Shear requirements of 11.12.5 shall be satisfied.

R13.4 — Openings in slab systems

See R11.12.5.

CODE

13.5 — Design procedures

13.5.1 — A slab system shall be designed by any procedure satisfying conditions of equilibrium and geometric compatibility, if shown that the design strength at every section is at least equal to the required strength considering 9.2 and 9.3, and that all serviceability conditions, including specified limits on deflections, are met.

13.5.1.1 — Design of a slab system for gravity loads, including the slab and beams (if any) between supports and supporting columns or walls forming orthogonal frames, by either the Direct Design Method of 13.6 or the Equivalent Frame Method of 13.7 shall be permitted.

13.5.1.2 — For lateral loads, analysis of unbraced frames shall take into account effects of cracking and reinforcement on stiffness of frame members.

COMMENTARY

R13.5 — Design procedures

R13.5.1 — This section permits a designer to base a design directly on fundamental principles of structural mechanics, provided it can be demonstrated explicitly that all safety and serviceability criteria are satisfied. The design of the slab may be achieved through the combined use of classic solutions based on a linearly elastic continuum, numerical solutions based on discrete elements, or yield-line analyses, including, in all cases, evaluation of the stress conditions around the supports in relation to shear and torsion as well as flexure. The designer must consider that the design of a slab system involves more than its analysis, and justify any deviations in physical dimensions of the slab from common practice on the basis of knowledge of the expected loads and the reliability of the calculated stresses and deformations of the structure.

R13.5.1.1 — For gravity load analysis of two-way slab systems, two analysis methods are specified in 13.6 and 13.7. The specific provisions of both design methods are limited in application to orthogonal frames subject to gravity loads only. Both methods apply to two-way slabs with beams as well as to flat slabs and flat plates. In both methods, the distribution of moments to the critical sections of the slab reflects the effects of reduced stiffness of elements due to cracking and support geometry.

R13.5.1.2 — For lateral load analysis, moment magnification is proportional to actual lateral displacement (drift). During the life of a structure, construction loads, ordinary occupancy loads, anticipated overloads, and volume changes will cause cracking of slabs. Cracking reduces stiffness of the slab members, especially slabs of unbraced frames. When lateral loads are considered to act on a structure, slab cracking increases drift (lateral deflection). To assure that lateral drift caused by wind or small earthquakes is not underestimated, cracking of slabs must be considered in stiffness assumptions.

If stiffness values are not obtained by a comprehensive analysis taking into account effects of cracking and reinforcement on stiffness, an effective moment of inertia of slab members may be computed by use of a fully cracked section. Alternatively, test results for full-size specimens indicated that stiffness values based on Eq. (9-7) are reasonable.[13.10,13.11]

The designer may select any approach that is shown to satisfy equilibrium and geometrical compatibility and to be in reasonable agreement with available test data. Some of the available design procedures are summarized in Reference 13.12. For slabs of unbraced frames, an "equivalent width" in the range of 25 to 50 percent of the full panel width has been used to reflect reduced stiffness due to cracking of slab members. The stiffness of slab members depends also on other parameters such as ℓ_2/ℓ_1, c_1/ℓ_1, c_2/c_1, and on concentration of reinforcement in the slab width defined in 13.5.3.2

CODE

13.5.1.3 — Combining the results of the gravity load analysis with the results of the lateral load analysis shall be permitted.

13.5.2 — The slab and beams (if any) between supports shall be proportioned for factored moments prevailing at every section.

13.5.3 — When gravity load, wind, earthquake, or other lateral forces cause transfer of moment between slab and column, a fraction of the unbalanced moment shall be transferred by flexure in accordance with 13.5.3.2 and 13.5.3.3.

13.5.3.1 — Fraction of unbalanced moment not transferred by flexure shall be transferred by eccentricity of shear in accordance with 11.12.6.

13.5.3.2 — A fraction of the unbalanced moment given by $\gamma_f \boldsymbol{M_u}$ shall be considered to be transferred by flexure within an effective slab width between lines that are one and one-half slab or drop panel thicknesses **(1.5*h*)** outside opposite faces of the column or capital, where $\boldsymbol{M_u}$ is the moment to be transferred and

$$\gamma_f = \frac{1}{1 + (2/3)\sqrt{b_1/b_2}} \qquad (13\text{-}1)$$

13.5.3.3 — For unbalanced moments about an axis parallel to the edge at exterior supports, the value of γ_f by Eq. (13-1) shall be permitted to be increased up to 1.0 provided that $\boldsymbol{V_u}$ at an edge support does not exceed $\mathbf{0.75}\phi \boldsymbol{V_c}$ or at a corner support does not exceed $\mathbf{0.5}\phi \boldsymbol{V_c}$. For unbalanced moments at interior supports, and for unbalanced moments about an axis transverse to the edge at exterior supports, the value of γ_f in Eq. (13-1) shall be permitted to be increased by up to 25 percent provided that $\boldsymbol{V_u}$ at the support does not exceed $\mathbf{0.4}\phi \boldsymbol{V_c}$. The reinforcement ratio ρ, within the effective slab width defined in 13.5.3.2, shall not exceed $\mathbf{0.375}\rho_b$. No adjustments to γ_f shall be permitted for prestressed slab systems.

COMMENTARY

for unbalanced moment transfer by flexure. This added concentration of reinforcement will increase stiffness[13.12] by preventing premature yielding and softening in the slab and column region.

In unbraced frames a "lower bound" stiffnesses assumption for the slab-beams should be used to assure that a reasonable estimate of drift is obtained. In framing systems with frame-shear wall interaction, a more rigorous calculation of stiffnesses is desirable so that the frame members are properly proportioned to resist the lateral forces and moments their relative stiffnesses will attract.

R13.5.3 — This section is concerned primarily with slab systems without beams. Tests and experience have shown that, unless special measures are taken to resist the torsional and shear stresses, all reinforcement resisting that part of the moment to be transferred to the column by flexure should be placed between lines that are one and one-half the slab or drop panel thickness, **1.5*h***, on each side of the column. The calculated shear stresses in the slab around the column must conform to the requirements of 11.12.2. See R11.12.2.1 and R11.12.2 for more details on application of this section.

R13.5.3.3 — The 1989 code procedures remain unchanged, except that under certain conditions the designer is permitted to adjust the level of moment transferred by shear without revising member sizes. Recent evaluation of available tests indicate that some flexibility in distribution of unbalanced moments transferred by shear and flexure at both exterior and interior supports is possible. Changes in the 1995 Code were made to recognize, to some extent, design practices prior to the 1971 code.[13.13]

At exterior supports, for unbalanced moments about an axis parallel to the edge, the portion of moment transferred by eccentricity of shear $\gamma_v M_u$ may be reduced provided that the factored shear at the support (excluding the shear produced by moment transfer) does not exceed 75 percent of the shear capacity ϕV_c as defined in 11.12.2.1 for edge columns or 50 percent for corner columns. Tests indicate that

CODE

13.5.3.4 — Concentration of reinforcement over the column by closer spacing or additional reinforcement shall be used to resist moment on the effective slab width defined in 13.5.3.2.

13.5.4 — Design for transfer of load from slabs to supporting columns or walls through shear and torsion shall be in accordance with Chapter 11.

13.6 — Direct design method

COMMENTARY

there is no significant interaction between shear and unbalanced moment at the exterior support in such cases.[13.14,13.15] Note that as $\gamma_v M_u$ is decreased, $\gamma_f M_u$ is increased.

Evaluation of tests of interior supports indicates that some flexibility in distributing unbalanced moments by shear and flexure is also possible, but with more severe limitations than for exterior supports. For interior supports, the unbalanced moment transferred by flexure is permitted to be increased up to 25 percent provided that the factored shear (excluding the shear caused by the moment transfer) at the interior supports does not exceed 40 percent of the shear capacity ϕV_c as defined in 11.12.2.1.

Tests of slab-column connections indicate that a large degree of ductility is required, because the interaction between shear and unbalanced moment is critical. When the factored shear is large, the column-slab joint cannot always develop all of the reinforcement provided in the effective width. The modifications for edge, corner, or interior slab-column connections specified in 13.5.3.3 are permitted only when the reinforcement ratio (within the effective width) required to develop the unbalanced moment $\gamma_f M_u$ does not exceed $0.375\rho_b$. The use of Eq. (13-1), without the modification permitted in 13.5.3.3, will generally indicate overstress conditions on the joint. The provisions of 13.5.3.3 are intended to improve ductile behavior of the column-slab joint. When a reversal of moments occurs at opposite faces of an interior support, both top and bottom reinforcement should be concentrated within the effective width. A ratio of top to bottom reinforcement of about 2 has been observed to be appropriate.

R13.6 — Direct design method

The Direct Design Method consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously. Three fundamental steps are involved as follows:

(1) Determination of the total factored static moment (see 13.6.2).

(2) Distribution of the total factored static moment to negative and positive sections (see 13.6.3).

(3) Distribution of the negative and positive factored moments to the column and middle strips and to the beams, if any (see 13.6.4 through 13.6.6). The distribu-

CODE

13.6.1 — Limitations

Design of slab systems within the following limitations by the Direct Design Method shall be permitted.

13.6.1.1 — There shall be a minimum of three continuous spans in each direction.

13.6.1.2 — Panels shall be rectangular, with a ratio of longer to shorter span center-to-center of supports within a panel not greater than 2.

13.6.1.3 — Successive span lengths center-to-center of supports in each direction shall not differ by more than one-third the longer span.

13.6.1.4 — Offset of columns by a maximum of 10 percent of the span (in direction of offset) from either axis between centerlines of successive columns shall be permitted.

13.6.1.5 — All loads shall be due to gravity only and uniformly distributed over an entire panel. Live load shall not exceed two times dead load.

13.6.1.6 — For a panel with beams between supports on all sides, the relative stiffness of beams in two perpendicular directions

$$\frac{\alpha_1 \ell_2^2}{\alpha_2 \ell_1^2} \qquad (13\text{-}2)$$

shall not be less than 0.2 nor greater than 5.0.

COMMENTARY

tion of moments to column and middle strips is also used in the Equivalent Frame Method (see 13.7).

R13.6.1 — Limitations

The direct design method was developed from considerations of theoretical procedures for the determination of moments in slabs with and without beams, requirements for simple design and construction procedures, and precedents supplied by performance of slab systems. Consequently, the slab systems to be designed using the direct design method must conform to the limitations in this section.

R13.6.1.1 — The primary reason for the limitation in this section is the magnitude of the negative moments at the interior support in a structure with only two continuous spans. The rules given for the direct design method assume tacitly that the slab system at the first interior negative moment section is neither fixed against rotation nor discontinuous.

R13.6.1.2 — If the ratio of the two spans (long span/short span) of a panel exceeds two, the slab resists the moment in the shorter span essentially as a one-way slab.

R13.6.1.3 — The limitation in this section is related to the possibility of developing negative moments beyond the point where negative moment reinforcement is terminated, as prescribed in Fig. 13.3.8 of the code.

R13.6.1.4 — Columns can be offset within specified limits from a regular rectangular array. A cumulative total offset of 20 percent of the span is established as the upper limit.

R13.6.1.5 — The direct design method is based on tests for uniform gravity loads and resulting column reactions determined by statics.[13.16] Lateral loads such as wind or seismic require a frame analysis. Inverted foundation mats designed as two-way slabs (see 15.10) involve application of known column loads. Therefore, even where the soil reaction is assumed to be uniform, a frame analysis is required.

In the 1995 code, the limit of applicability of the direct design method for ratios of live load to dead load has been reduced from 3 to 2. In most slab systems, the live to dead load ratio will be less than 2 and it is not necessary to check the effects of pattern loading. Section 13.6.10 in the 1989 code has been eliminated because it is no longer needed.

R13.6.1.6 — The elastic distribution of moments will deviate significantly from those assumed in the direct design method unless the given requirements for stiffness are satisfied.

CODE

13.6.1.7 — Moment redistribution as permitted by 8.4 shall not be applied for slab systems designed by the Direct Design Method. See 13.6.7.

13.6.1.8 — Variations from the limitations of 13.6.1 shall be permitted if demonstrated by analysis that requirements of 13.5.1 are satisfied.

13.6.2 — Total factored static moment for a span

13.6.2.1 — Total factored static moment for a span shall be determined in a strip bounded laterally by centerline of panel on each side of centerline of supports.

13.6.2.2 — Absolute sum of positive and average negative factored moments in each direction shall not be less than

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} \qquad (13\text{-}3)$$

13.6.2.3 — Where the transverse span of panels on either side of the centerline of supports varies, ℓ_2 in Eq. (13-3) shall be taken as the average of adjacent transverse spans.

13.6.2.4 — When the span adjacent and parallel to an edge is being considered, the distance from edge to panel centerline shall be substituted for ℓ_2 in Eq. (13-3).

13.6.2.5 — Clear span ℓ_n shall extend from face to face of columns, capitals, brackets, or walls. Value of ℓ_n used in Eq. (13-3) shall not be less than **0.65** ℓ_1. Circular or regular polygon shaped supports shall be treated as square supports with the same area.

13.6.3 — Negative and positive factored moments

13.6.3.1 — Negative factored moments shall be located at face of rectangular supports. Circular or

COMMENTARY

R13.6.1.7 — Moment redistribution as permitted by 8.4 is not intended where approximate values for bending moments are used. For the direct design method, 10 percent modification is allowed by 13.6.7.

R13.6.1.8 — The designer is permitted to use the direct design method even if the structure does not fit the limitations in this section, provided it can be shown by analysis that the particular limitation does not apply to that structure. For example, in the case of a slab system carrying a nonmovable load (such as a water reservoir in which the load on all panels is expected to be the same), the designer need not satisfy the live load limitation of 13.6.1.5.

R13.6.2 — Total factored static moment for a span

R13.6.2.2 — Eq. (13-3) follows directly from Nichol's derivation[13.17] with the simplifying assumption that the reactions are concentrated along the faces of the support perpendicular to the span considered. In general, the designer will find it expedient to calculate static moments for two adjacent half panels, which include a column strip with a half middle strip along each side.

R13.6.2.5 — If a supporting member does not have a rectangular cross section or if the sides of the rectangle are not parallel to the spans, it is to be treated as a square support having the same area, as illustrated in Fig. R13.6.2.5.

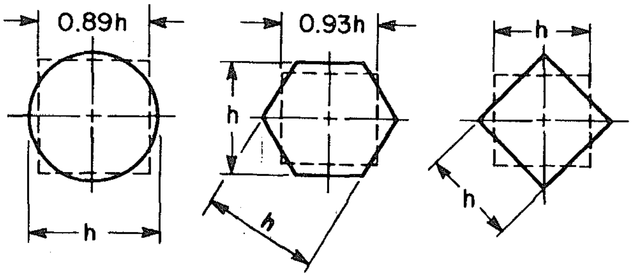


Fig. R13.6.2.5—Examples of equivalent square section for supporting members

R13.6.3 — Negative and positive factored moments

CODE

regular polygon shaped supports shall be treated as square supports with the same area.

13.6.3.2 — In an interior span, total static moment M_o shall be distributed as follows:

Negative factored moment 0.65

Positive factored moment..................................0.35

13.6.3.3 — In an end span, total factored static moment M_o shall be distributed as follows:

	(1)	(2)	(3)	(4)	(5)
	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

13.6.3.4 — Negative moment sections shall be designed to resist the larger of the two interior negative factored moments determined for spans framing into a common support unless an analysis is made to distribute the unbalanced moment in accordance with stiffnesses of adjoining elements.

13.6.3.5 — Edge beams or edges of slab shall be proportioned to resist in torsion their share of exterior negative factored moments.

13.6.3.6 — The gravity load moment to be transferred between slab and edge column in accordance with 13.5.3.1 shall be $0.3M_o$.

COMMENTARY

R13.6.3.3 — The moment coefficients for an end span are based on the equivalent column stiffness expressions from References 13.18, 13.19, and 13.20. The coefficients for an unrestrained edge would be used, for example, if the slab were simply supported on a masonry or concrete wall. Those for a restrained edge would apply if the slab were constructed integrally with a concrete wall having a flexural stiffness so large compared to that of the slab that little rotation occurs at the slab-to-wall connection.

For other than unrestrained or fully restrained edges, coefficients in the table were selected to be near the upper bound of the range for positive moments and interior negative moments. As a result, exterior negative moments were usually closer to a lower bound. The exterior negative moment capacity for most slab systems is governed by minimum reinforcement to control cracking. The final coefficients selected have been adjusted so that the absolute sum of the positive and average moments equal M_o.

For two-way slab systems with beams between supports on all sides (two-way slabs), moment coefficients of Column (2) apply. For slab systems without beams between interior supports (flat plates and flat slabs), the moment coefficients of Column (3) or (4) apply, without or with an edge (spandrel) beam, respectively.

In the 1977 ACI Building Code, distribution factors defined as a function of the stiffness ratio of the equivalent exterior support were used for proportioning the total static moment M_o in an end span. The approach may be used in place of values in 13.6.3.3.

R13.6.3.4 — The differences in slab moment on either side of a column or other type of support must be accounted for in the design of the support. If an analysis is made to distribute unbalanced moments, flexural stiffness may be obtained on the basis of the gross concrete section of the members involved.

R13.6.3.5 — Moments perpendicular to, and at the edge of, the slab structure must be transmitted to the supporting columns or walls. Torsional stresses caused by the moment assigned to the slab must be investigated.

CODE

13.6.4 — Factored moments in column strips

13.6.4.1 — Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

13.6.4.2 — Column strips shall be proportioned to resist the following portions in percent of exterior negative factored moments:

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Linear interpolations shall be made between values shown.

13.6.4.3 — Where supports consist of columns or walls extending for a distance equal to or greater than three-quarters the span length ℓ_2 used to compute M_o, negative moments shall be considered to be uniformly distributed across ℓ_2.

13.6.4.4 — Column strips shall be proportioned to resist the following portions in percent of positive factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

13.6.4.5 — For slabs with beams between supports, the slab portion of column strips shall be proportioned to resist that portion of column strip moments not resisted by beams.

13.6.5 — Factored moments in beams

13.6.5.1 — Beams between supports shall be pro-

COMMENTARY

R13.6.4, R13.6.5, and R13.6.6 — Factored moments in column strips, beams, and middle strips

The rules given for assigning moments to the column strips, beams, and middle strips are based on studies of moments in linearly elastic slabs with different beam stiffness[13.21] tempered by the moment coefficients that have been used successfully in the past.

For the purpose of establishing moments in the half column strip adjacent to an edge supported by a wall, ℓ_n in Eq. (13-3) may be assumed equal to ℓ_n of the parallel adjacent column to column span, and the wall may be considered as a beam having a moment of inertia I_b equal to infinity.

R13.6.4.2 — The effect of the torsional stiffness parameter β_t is to assign all of the exterior negative factored moment to the column strip, and none to the middle strip, unless the beam torsional stiffness is high relative to the flexural stiffness of the supported slab. In the definition of β_t, the shear modulus has been taken as $E_{cb}/2$.

Where walls are used as supports along column lines, they can be regarded as very stiff beams with an $\alpha_1\ell_2/\ell_1$ value greater than one. Where the exterior support consists of a wall perpendicular to the direction in which moments are being determined, β_t may be taken as zero if the wall is of masonry without torsional resistance, and β_t may be taken as 2.5 for a concrete wall with great torsional resistance which is monolithic with the slab.

R13.6.5 — Factored moments in beams

Loads assigned directly to beams are in addition to the uni-

CODE

portioned to resist 85 percent of column strip moments if ($\alpha_1 \ell_2/\ell_1$) is equal to or greater than 1.0.

13.6.5.2 — For values of $\alpha_1 \ell_2/\ell_1$ between 1.0 and zero, proportion of column strip moments resisted by beams shall be obtained by linear interpolation between 85 and zero percent.

13.6.5.3 — In addition to moments calculated for uniform loads according to 13.6.2.2, 13.6.5.1, and 13.6.5.2, beams shall be proportioned to resist all moments caused by concentrated or linear loads applied directly to beams, including weight of projecting beam stem above or below the slab.

13.6.6 — Factored moments in middle strips

13.6.6.1 — That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips.

13.6.6.2 — Each middle strip shall be proportioned to resist the sum of the moments assigned to its two half middle strips.

13.6.6.3 — A middle strip adjacent to and parallel with an edge supported by a wall shall be proportioned to resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports.

13.6.7 — Modification of factored moments

Modification of negative and positive factored moments by 10 percent shall be permitted provided the total static moment for a panel in the direction considered is not less than that required by Eq. (13-3).

13.6.8 — Factored shear in slab systems with beams

13.6.8.1 — Beams with $\alpha_1 \ell_2/\ell_1$ equal to or greater than 1.0 shall be proportioned to resist shear caused by factored loads on tributary areas bounded by 45 deg lines drawn from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides.

13.6.8.2 — In proportioning of beams with $\alpha_1 \ell_2/\ell_1$ less than 1.0 to resist shear, linear interpolation, assuming beams carry no load at $\alpha_1 = 0$, shall be permitted.

13.6.8.3 — In addition to shears calculated according to 13.6.8.1 and 13.6.8.2, beams shall be proportioned to resist shears caused by factored loads applied directly on beams.

13.6.8.4 — Computation of slab shear strength on

COMMENTARY

form dead load of slab, uniform superimposed dead loads such as ceiling, floor finish, or assumed equivalent partition loads, and uniform live loads; all of which are normally included with w_u in Eq. (13-3). Loads applied directly to beams include line loads such as partition walls over (or along) beam centerlines, concentrated loads such as posts above or hangers below the beams, plus additional dead (line) load of the projecting beam stem. For the purpose of assigning directly applied loads, only loads located within the width of beam stem should be considered as directly applied to the beams. (The effective width of a beam as defined in 13.2.4 is solely for strength and relative stiffness calculations.) Line loads and concentrated loads located on the slab away from the beam stem require special consideration to determine their apportionment to slab and beams.

R13.6.8 — Factored shear in slab systems with beams

The tributary area for computing shear on an interior beam is shown shaded in Fig. R13.6.8. If the stiffness for the beam $\alpha_1 \ell_2/\ell_1$ is less than one, the shear on the beam may be

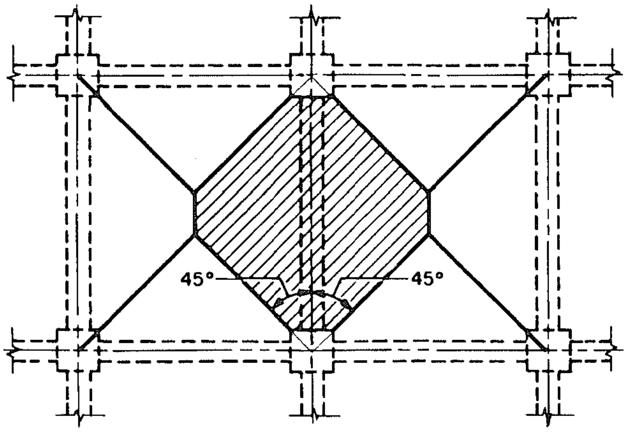


Fig. R13.6.8—Tributary area for shear on an interior beam

the assumption that load is distributed to supporting beams in accordance with 13.6.8.1 or 13.6.8.2 shall be permitted. Resistance to total shear occurring on a panel shall be provided.

13.6.8.5 — Shear strength shall satisfy requirements of Chapter 11.

13.6.9 — Factored moments in columns and walls

13.6.9.1 — Columns and walls built integrally with a slab system shall resist moments caused by factored loads on the slab system.

13.6.9.2 — At an interior support, supporting elements above and below the slab shall resist the moment specified by Eq. (13-4) in direct proportion to their stiffnesses unless a general analysis is made.

$$M = 0.07[(w_d + 0.5w_l)\ell_2 \ell_n^2 - w_d' \ell_2' (\ell_n')^2] \qquad (13\text{-}4)$$

where w_d', ℓ_2', and ℓ_n' refer to shorter span.

13.7 — Equivalent frame method

13.7.1 — Design of slab systems by the Equivalent Frame Method shall be based on assumptions given in 13.7.2 through 13.7.6, and all sections of slabs and supporting members shall be proportioned for moments and shears thus obtained.

13.7.1.1 — Where metal column capitals are used, it shall be permitted to take account of their contributions to stiffness and resistance to moment and to shear.

13.7.1.2 — Neglecting the change in length of columns and slabs due to direct stress, and deflections due to shear, shall be permitted.

13.7.2 — Equivalent frame

13.7.2.1 — The structure shall be considered to be made up of equivalent frames on column lines taken longitudinally and transversely through the building.

13.7.2.2 — Each frame shall consist of a row of columns or supports and slab-beam strips, bounded laterally by the centerline of panel on each side of the centerline of columns or supports.

13.7.2.3 — Columns or supports shall be assumed

obtained by linear interpolation. In such cases, the beams framing into the column will not account for all the shear force applied on the column. The remaining shear force will produce shear stresses in the slab around the column which must be checked in the same manner as for flat slabs, as required by 13.6.8.4. Sections 13.6.8.1 through 13.6.8.3 do not apply to the calculation of torsional moments on the beams. These moments must be based on the calculated flexural moments acting on the sides of the beam.

R13.6.9 — Factored moments in columns and walls

Eq. (13-4) refers to two adjoining spans, with one span longer than the other, with full dead load plus one-half live load applied on the longer span and only dead load applied on the shorter span.

Design and detailing of the reinforcement transferring the moment from the slab to the edge column is critical to both the performance and the safety of flat slabs or flat plates without edge beams or cantilever slabs. It is important that complete design details be shown on design drawings, such as concentration of reinforcement over the column by closer spacing or additional reinforcement.

R13.7 — Equivalent frame method

The Equivalent Frame Method involves the representation of the three-dimensional slab system by a series of two-dimensional frames which are then analyzed for loads acting in the plane of the frames. The negative and positive moments so determined at the critical design sections of the frame are distributed to the slab sections in accordance with 13.6.4 (column strips), 13.6.5 (beams), and 13.6.6 (middle strips). The equivalent frame method is based on studies reported in References 13.18, 13.19, and 13.20. Many of the details of the equivalent frame method given in the Commentary to ACI 318-89 have been removed in ACI 318-95. Computer programs based on the equivalent frame method are available. Most reinforced concrete design textbooks include details of the equivalent frame method.

R13.7.2 — Equivalent frame

Application of the equivalent frame to a regular structure is illustrated in Fig. R13.7.2. The three-dimensional building is divided into a series of two-dimensional frame bents (equivalent frames) centered on column or support centerlines with each frame extending the full height of the building. The width of each equivalent frame is bounded by the centerlines of the adjacent panels. The complete analysis of a slab system for a building consists of analyzing a series of equivalent (interior and exterior) frames spanning longitudinally and transversely through the building.

CODE

to be attached to slab-beam strips by torsional members (see 13.7.5) transverse to the direction of the span for which moments are being determined and extending to bounding lateral panel centerlines on each side of a column.

13.7.2.4 — Frames adjacent and parallel to an edge shall be bounded by that edge and the centerline of adjacent panel.

13.7.2.5 — Analysis of each equivalent frame in its entirety shall be permitted. Alternatively for gravity loading, a separate analysis of each floor or roof with far ends of columns considered fixed shall be permitted.

13.7.2.6 — Where slab-beams are analyzed separately, determination of moment at a given support assuming that the slab-beam is fixed at any support two panels distant therefrom shall be permitted, provided the slab continues beyond that point.

13.7.3 — Slab-beams

13.7.3.1—Determination of the moment of inertia of slab-beams at any cross section outside of joints or column capitals using the gross area of concrete shall be permitted.

13.7.3.2 — Variation in moment of inertia along axis of slab-beams shall be taken into account.

13.7.3.3 — Moment of inertia of slab-beams from center of column to face of column, bracket, or capital shall be assumed equal to the moment of inertia of the slab-beam at face of column, bracket, or capital divided by the quantity $(1 - c_2/\ell_2)^2$ where c_2 and ℓ_2 are measured transverse to the direction of the span for which moments are being determined.

13.7.4 — Columns

13.7.4.1 — Determination of the moment of inertia of columns at any cross section outside of joints or column capitals using the gross area of concrete shall be permitted.

COMMENTARY

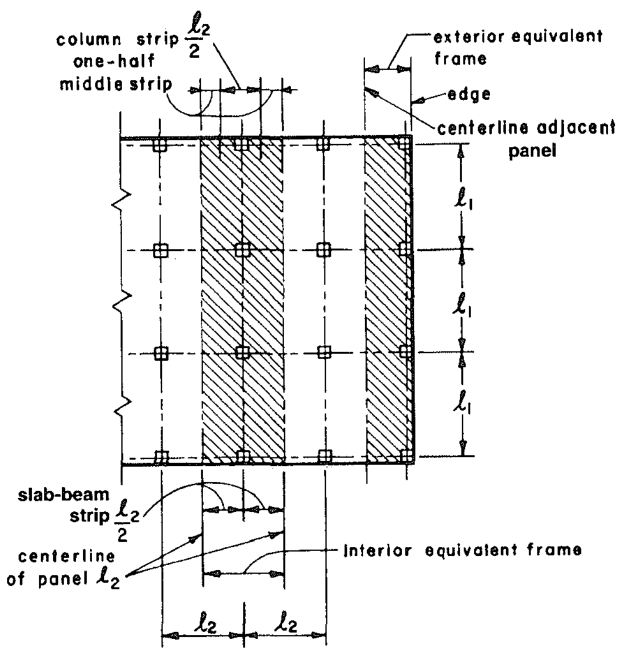

column strip $\frac{\ell_2}{2}$
one-half middle strip
exterior equivalent frame
edge
centerline adjacent panel
ℓ_1
ℓ_1
ℓ_1
slab-beam strip $\frac{\ell_2}{2}$
centerline of panel ℓ_2
interior equivalent frame
ℓ_2
ℓ_2

Fig. R13.7.2—Definitions of equivalent frame

The equivalent frame comprises three parts: (1) the horizontal slab strip, including any beams spanning in the direction of the frame, (2) the columns or other vertical supporting members, extending above and below the slab and, (3) the elements of the structure that provide moment transfer between the horizontal and vertical members.

R13.7.3 — Slab-beams

R13.7.3.3 — A support is defined as a column, capital, bracket, or wall. Note that a beam is not considered a support member for the equivalent frame.

R13.7.4 — Columns

Column stiffness is based on the length of the column from middepth of slab above to middepth of slab below. Column moment of inertia is computed on the basis of its cross section, taking into account the increase in stiffness provided by the capital, if any.

CODE

13.7.4.2 — Variation in moment of inertia along axis of columns shall be taken into account.

13.7.4.3 — Moment of inertia of columns from top to bottom of the slab-beam at a joint shall be assumed infinite.

13.7.5 — Torsional members

13.7.5.1 — Torsional members (see 13.7.2.3) shall be assumed to have a constant cross section throughout their length consisting of the largest of

(a) A portion of slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined, or

(b) For monolithic or fully composite construction, the portion of slab specified in (a) plus that part of the transverse beam above and below the slab, and

(c) The transverse beam as defined in 13.2.4.

13.7.5.2 — Where beams frame into columns in the direction of the span for which moments are being determined, the torsional stiffness shall be multiplied by the ratio of moment of inertia of slab with such beam to moment of inertia of slab without such beam.

COMMENTARY

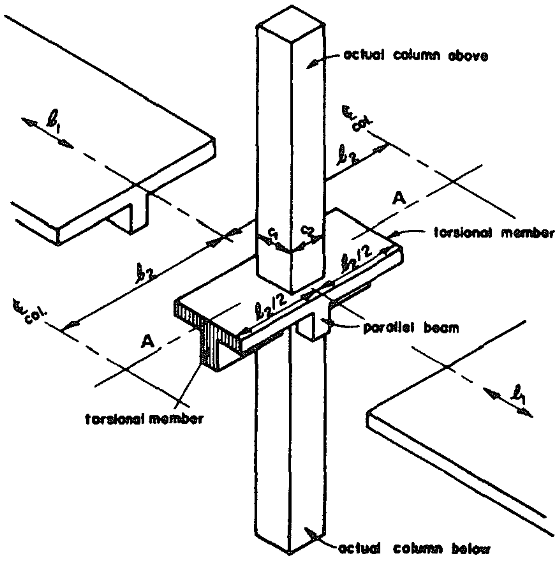


Fig. R13.7.4—Equivalent column (column plus torsional members)

When slab-beams are analyzed separately for gravity loads, the concept of an equivalent column, combining the stiffness of the slab-beam and torsional member into a composite element, is used. The column flexibility is modified to account for the torsional flexibility of the slab-to-column connection which reduces its efficiency for transmission of moments. The equivalent column consists of the actual columns above and below the slab-beam plus "attached" torsional members on each side of the columns extending to the centerline of the adjacent panels as shown in Fig. R13.7.4.

R13.7.5 — Torsional members

Computation of the stiffness of the torsional member requires several simplifying assumptions. If no beam frames into the column, a portion of the slab equal to the width of the column or capital is assumed as the effective beam. If a beam frames into the column, T-beam or L-beam action is assumed, with the flanges extending on each side of the beam a distance equal to the projection of the beam above or below the slab but not greater than four times the thickness of the slab. Furthermore, it is assumed that no torsional rotation occurs in the beam over the width of the support.

The member sections to be used for calculating the torsional stiffness are defined in 13.7.5.1. In the 1989 code, Eq. (13-6) specified the stiffness coefficient K_t of the torsional members. The approximate expression for K_t has been moved to the commentary and the expression for the torsional constant (Eq. 13-7 in the 1989 code) is now defined in 13.0.

Studies of three-dimensional analyses of various slab configurations suggest that a reasonable value of the torsional

13.7.6 — Arrangement of live load

13.7.6.1 — When loading pattern is known, the equivalent frame shall be analyzed for that load.

13.7.6.2 — When live load is variable but does not exceed three-quarters of the dead load, or the nature of live load is such that all panels will be loaded simultaneously, it shall be permitted to assume that maximum factored moments occur at all sections with full factored live load on entire slab system.

13.7.6.3 — For loading conditions other than those defined in 13.7.6.2, it shall be permitted to assume that maximum positive factored moment near midspan of a panel occurs with three-quarters of the full factored live load on the panel and on alternate panels; and it shall be permitted to assume that maximum negative factored moment in the slab at a support occurs with three-quarters of the full live load on adjacent panels only.

13.7.6.4 — Factored moments shall be taken not less than those occurring with full factored live load on all panels.

13.7.7 — Factored moments

13.7.7.1 — At interior supports, critical section for negative factored moment (in both column and middle

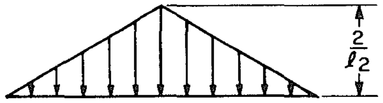


Fig. R13.7.5—Distribution of unit twisting moment along column centerline AA shown in Fig. R13.7.4

stiffness can be obtained by assuming a moment distribution along the torsional member that varies linearly from a maximum at the center of the column to zero at the middle of the panel. The assumed distribution of unit twisting moment along the column centerline is shown in Fig. R13.7.5.

An approximate expression for the stiffness of the torsional member, based on the results of three-dimensional analyses of various slab configurations (References 13.18, 13.19, and 13.20) is given below as

$$K_t = \sum \frac{9E_{cs}C}{\ell_2\left(1 - \frac{c_2}{\ell_2}\right)^3}$$

where an expression for C is given in 13.0.

R13.7.6 — Arrangement of live load

The use of only three-quarters of the full factored live load for maximum moment loading patterns is based on the fact that maximum negative and maximum positive live load moments cannot occur simultaneously and that redistribution of maximum moments is thus possible before failure occurs. This procedure, in effect, permits some local overstress under the full factored live load if it is distributed in the prescribed manner, but still ensures that the ultimate capacity of the slab system after redistribution of moment is not less than that required to carry the full factored dead and live loads on all panels.

R13.7.7 — Factored moments

R13.7.7.1-R13.7.7.3 — These sections correct the negative factored moments to the face of the supports. The cor-

CODE

strips) shall be taken at face of rectilinear supports, but not greater than $0.175\ell_1$ from center of a column.

13.7.7.2 — At exterior supports provided with brackets or capitals, critical section for negative factored moment in the span perpendicular to an edge shall be taken at a distance from face of supporting element not greater than one-half the projection of bracket or capital beyond face of supporting element.

13.7.7.3 — Circular or regular polygon shaped supports shall be treated as square supports with the same area for location of critical section for negative design moment.

13.7.7.4 — When slab systems within limitations of 13.6.1 are analyzed by the Equivalent Frame Method, it shall be permitted to reduce the resulting computed moments in such proportion that the absolute sum of the positive and average negative moments used in design need not exceed the value obtained from Eq. (13-3).

13.7.7.5 — Distribution of moments at critical sections across the slab-beam strip of each frame to column strips, beams, and middle strips as provided in 13.6.4, 13.6.5, and 13.6.6 shall be permitted if the requirement of 13.6.1.6 is satisfied.

COMMENTARY

rection is modified at an exterior support in order not to result in undue reductions in the exterior negative moment. Fig. R13.6.2.5 illustrates several equivalent rectangular supports for use in establishing faces of supports for design with nonrectangular supports.

R13.7.7.4 — This section is a holdover from many previous codes and is based on the principle that if two different methods are prescribed to obtain a particular answer, the code should not require a value greater than the least acceptable value. Due to the long satisfactory experience with designs having total factored static moments not exceeding those given by Eq. (13-3) it is considered that these values are satisfactory for design when applicable limitations are met.

CHAPTER 14 — WALLS

CODE

14.0 — Notation

A_g = gross area of section, in.^{2}
f_c' = specified compressive strength of concrete, psi
h = overall thickness of member, in.
k = effective length factor
ℓ_c = vertical distance between supports, in.
P_{nw} = nominal axial load strength of wall designed by 14.4
ϕ = strength reduction factor. See 9.3

14.1 — Scope

14.1.1 — Provisions of Chapter 14 shall apply for design of walls subjected to axial load, with or without flexure.

14.1.2 — Cantilever retaining walls are designed according to flexural design provisions of Chapter 10 with minimum horizontal reinforcement according to 14.3.3.

14.2 — General

14.2.1 — Walls shall be designed for eccentric loads and any lateral or other loads to which they are subjected.

14.2.2 — Walls subject to axial loads shall be designed in accordance with 14.2, 14.3, and either 14.4 or 14.5.

14.2.3 — Design for shear shall be in accordance with 11.10.

14.2.4 — Unless demonstrated by a detailed analysis, horizontal length of wall to be considered as effective for each concentrated load shall not exceed center-to-center distance between loads, nor width of bearing plus four times the wall thickness.

14.2.5 — Compression members built integrally with walls shall conform to 10.8.2.

14.2.6 — Walls shall be anchored to intersecting elements such as floors and roofs, or to columns, pilasters, buttresses, and intersecting walls, and to footings.

COMMENTARY

R14.1 — Scope

Chapter 14 applies generally to walls as vertical load carrying members. Cantilever retaining walls are designed according to the flexural design provisions of Chapter 10. Walls designed to resist shear forces, such as shearwalls, shall be designed in accordance with Chapter 14 and 11.10 as applicable.

In the 1977 code, walls could be designed according to Chapter 14 or 10.15. In the 1983 code these two were combined in Chapter 14.

R14.2 — General

Walls must be designed to resist all loads to which they are subjected, including eccentric axial loads and lateral forces. Design must be carried out in accordance with 14.4 unless the wall meets the requirements of 14.5.1.

In either case, walls may be designed using either the strength design method of the code or the alternate design method of Appendix A in accordance with A.6.3.

CODE

14.2.7 — Quantity of reinforcement and limits of thickness required by 14.3 and 14.5 shall be permitted to be waived where structural analysis shows adequate strength and stability.

14.2.8 — Transfer of force to footing at base of wall shall be in accordance with 15.8.

14.3 — Minimum reinforcement

14.3.1 — Minimum vertical and horizontal reinforcement shall be in accordance with 14.3.2 and 14.3.3 unless a greater amount is required for shear by 11.10.8 and 11.10.9.

14.3.2 — Minimum ratio of vertical reinforcement area to gross concrete area shall be:

(a) 0.0012 for deformed bars not larger than No. 5 with a specified yield strength not less than 60,000 psi, or

(b) 0.0015 for other deformed bars, or

(c) 0.0012 for welded wire fabric (plain or deformed) not larger than W31 or D31.

14.3.3 — Minimum ratio of horizontal reinforcement area to gross concrete area shall be:

(a) 0.0020 for deformed bars not larger than No. 5 with a specified yield strength not less than 60,000 psi, or

(b) 0.0025 for other deformed bars, or

(c) 0.0020 for welded wire fabric (plain or deformed) not larger than W31 or D31.

14.3.4 — Walls more than 10 in. thick, except basement walls, shall have reinforcement for each direction placed in two layers parallel with faces of wall in accordance with the following:

(a) One layer consisting of not less than one-half and not more than two-thirds of total reinforcement required for each direction shall be placed not less than 2 in. nor more than one-third the thickness of wall from exterior surface.

(b) The other layer, consisting of the balance of required reinforcement in that direction, shall be placed not less than $^3/_4$ in. nor more than one-third the thickness of wall from interior surface.

COMMENTARY

R14.3 — Minimum reinforcement

The requirements of 14.3 are similar to those in previous ACI Building Codes. These apply to walls designed according to 14.4 or 14.5. For walls resisting horizontal shear forces in the plane of the wall, reinforcement designed according to 11.10.9.2 and 11.10.9.4 may exceed the minimum reinforcement specified in 14.3.

CODE

14.3.5 — Vertical and horizontal reinforcement shall not be spaced farther apart than three times the wall thickness, nor 18 in.

14.3.6 — Vertical reinforcement need not be enclosed by lateral ties if vertical reinforcement area is not greater than 0.01 times gross concrete area, or where vertical reinforcement is not required as compression reinforcement.

14.3.7 — In addition to the minimum reinforcement required by 14.3.1, not less than two No. 5 bars shall be provided around all window and door openings. Such bars shall be extended to develop the bar beyond the corners of the openings but not less than 24 in.

14.4 — Walls designed as compression members

Except as provided in 14.5, walls subject to axial load or combined flexure and axial load shall be designed as compression members in accordance with provisions of 10.2, 10.3, 10.10, 10.11, 10.12, 10.13, 10.14, 10.17, 14.2, and 14.3.

14.5 — Empirical design method

14.5.1 — Walls of solid rectangular cross section shall be permitted to be designed by the empirical provisions of 14.5 if resultant of all factored loads is located within the middle third of the overall thickness of wall and all limits of 14.2, 14.3, and 14.5 are satisfied.

14.5.2 — Design axial load strength ϕP_{nw} of a wall satisfying limitations of 14.5.1 shall be computed by Eq. (14-1) unless designed in accordance with 14.4.

$$\phi P_{nw} = 0.55\phi f_c' A_g \left[1 - \left(\frac{k\ell_c}{32h}\right)^2\right] \tag{14-1}$$

where ϕ = **0.70** and effective length factor k shall be:

For walls braced top and bottom against lateral translation and

(a) restrained against rotation at one or both ends (top and/or bottom) 0.8

(b) unrestrained against rotation at both ends..... 1.0

For walls not braced against lateral translation....... 2.0

COMMENTARY

R14.5 — Empirical design method

The empirical design method applies only to solid rectangular cross sections. All other shapes must be designed according to 14.4.

Eccentric loads and lateral forces are used to determine the total eccentricity of the factored axial load P_u. When the resultant load for all applicable load combinations falls within the middle third of the wall thickness (eccentricity not greater than $h/6$) at all sections along the length of the undeformed wall, the empirical design method may be used. The design is then carried out considering P_u as the concentric load. The factored axial load P_u must be less than or equal to the design axial load strength ϕP_{nw} computed by Eq. (14-1), $P_u \leq \phi P_{nw}$.

With the 1980 supplement, (Eq. 14-1) was revised to reflect the general range of end conditions encountered in wall designs. The wall strength equation in the 1977 code edition was based on the assumption of a wall with top and bottom fixed against lateral movement, and with moment restraint at one end corresponding to an effective length factor between 0.8 and 0.9. Axial load strength values determined from the original equation were unconservative when compared to test results[14.1] for walls with pinned conditions at both ends as can occur in some precast and tilt-up applications, or when the top of the wall is not effectively braced against translation, as occurs with many free-standing walls

CODE

14.5.3 — Minimum thickness of walls designed by empirical design method

14.5.3.1 — Thickness of bearing walls shall not be less than $^1/_{25}$ the supported height or length, whichever is shorter, nor less than 4 in.

14.5.3.2 — Thickness of exterior basement walls and foundation walls shall not be less than $7^1/_2$ in.

14.6 — Nonbearing walls

14.6.1 — Thickness of nonbearing walls shall not be less than 4 in., nor less than $^1/_{30}$ the least distance between members that provide lateral support.

14.7 — Walls as grade beams

14.7.1 — Walls designed as grade beams shall have top and bottom reinforcement as required for moment in accordance with provisions of 10.2 through 10.7. Design for shear shall be in accordance with provisions of Chapter 11.

14.7.2 — Portions of grade beam walls exposed above grade shall also meet requirements of 14.3.

COMMENTARY

or in large structures where significant roof diaphragm deflections occur due to wind and seismic loads. Eq. (14-1) gives the same results as the 1977 code for walls braced against translation and with reasonable base restraint against rotation.[14.2] Values of effective vertical length factors ***k*** are given for commonly occurring wall end conditions. The end condition "restrained against rotation" required for a ***k***-factor of 0.8 implies attachment to a member having flexural stiffness EI/ℓ at least as large as that of the wall.

The slenderness portion of Eq. (14-1) results in relatively comparable strengths by either 14.3 or 14.4 for members loaded at the middle third of the thickness with different braced and restrained end conditions. See Fig. R14.5.

R14.5.3 — Minimum thickness of walls designed by empirical design method

The minimum thickness requirements need not be applied to walls designed according to 14.4.

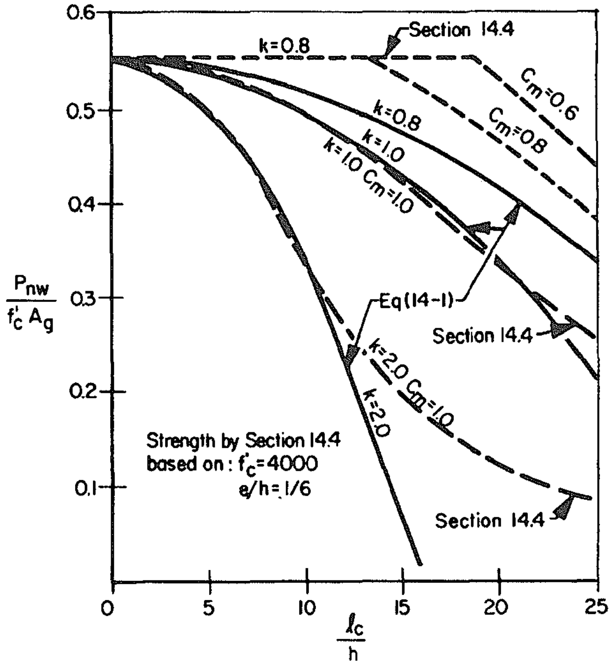


Fig. R14.5—Empirical design of walls, Eq. (14-1), versus 14.4

CHAPTER 15 — FOOTINGS

CODE

15.0 — Notation

A_g = gross area of section, in.^{2}
d_p = diameter of pile at footing base
β = ratio of long side to short side of footing

15.1 — Scope

15.1.1 — Provisions of Chapter 15 shall apply for design of isolated footings and, where applicable, to combined footings and mats.

15.1.2 — Additional requirements for design of combined footings and mats are given in 15.10.

15.2 — Loads and reactions

15.2.1 — Footings shall be proportioned to resist the factored loads and induced reactions, in accordance with the appropriate design requirements of this code and as provided in Chapter 15.

15.2.2 — Base area of footing or number and arrangement of piles shall be determined from unfactored forces and moments transmitted by footing to soil or piles and permissible soil pressure or permissible pile capacity selected through principles of soil mechanics.

15.2.3 — For footings on piles, computations for moments and shears may be based on the assumption that the reaction from any pile is concentrated at pile center.

COMMENTARY

R15.1 — Scope

While the provisions of Chapter 15 apply to isolated footings supporting a single column or wall, most of the provisions are generally applicable to combined footings and mats supporting several columns or walls or a combination thereof.[15.1,15.2]

R15.2 — Loads and reactions

Footings are required to be proportioned to sustain the applied factored loads and induced reactions which include axial loads, moments, and shears that have to be resisted at the base of the footing or pile cap.

After the permissible soil pressure or the permissible pile capacity has been determined by principles of soil mechanics and in accord with the general building code, the size of the base area of a footing on soil or the number and arrangement of the piles must be established on the basis of unfactored (service) loads (***D, L, W, E***, etc.) in whatever combination that will govern the design.

Only the computed end moments that exist at the base of a column (or pedestal) need be transferred to the footing; the minimum moment requirement for slenderness considerations given in 10.12.3.2 need not be considered for transfer of forces and moments to footings.

In cases in which eccentric loads or moments must be considered, the extreme soil pressure or pile reaction obtained from this loading must be within the permissible values. Similarly, the resultant reactions due to service loads combined with moments and/or shears caused by wind or earthquake loads must not exceed the increased values that may be permitted by the general building code.

To proportion a footing or pile cap for strength, the contact soil pressure or pile reaction due to the applied factored loading (see 8.1.1) must be determined. For a single concentrically loaded spread footing, the soil reaction q_s due to the factored loading is $q_s = U/A_f$ where U is the factored con-

CODE

15.3 — Footings supporting circular or regular polygon shaped columns or pedestals

For location of critical sections for moment, shear, and development of reinforcement in footings, it shall be permitted to treat circular or regular polygon shaped concrete columns or pedestals as square members with the same area.

15.4 — Moment in footings

15.4.1 — External moment on any section of a footing shall be determined by passing a vertical plane through the footing, and computing the moment of the forces acting over entire area of footing on one side of that vertical plane.

15.4.2 — Maximum factored moment for an isolated footing shall be computed as prescribed in 15.4.1 at critical sections located as follows:

(a) At face of column, pedestal, or wall, for footings supporting a concrete column, pedestal, or wall.

(b) Halfway between middle and edge of wall, for footings supporting a masonry wall.

COMMENTARY

centric load to be resisted by the footing, and A_f is the base area of the footing as determined by the principles stated previously using the unfactored loads and the permissible soil pressure.

It is important to note that q_s is only a calculated reaction to the factored loading, used to produce in the footing or pile cap the same required strength conditions regarding flexure, shear, and development of reinforcement as in any other member.

In the case of eccentric loading, load factors may cause eccentricities and reactions that are different from those obtained by unfactored loads.

When the alternate design method of Appendix A is used for design of footings, the soil bearing pressures or pile reactions are those caused by the service loads (without load factors). The permissible soil pressures or permissible pile reactions are equated directly with the applied service load pressures or reactions to determine base area of footing or number and arrangement of piles. When lateral loads due to wind or earthquake are included in the governing load combination for footings, advantage may be taken of the 25 percent reduction in required strength in accordance with Section A.2.2.

R15.4 — Moment in footings

CODE

(c) Halfway between face of column and edge of steel base plate, for footings supporting a column with steel base plate.

15.4.3 — In one-way footings, and two-way square footings, reinforcement shall be distributed uniformly across entire width of footing.

15.4.4 — In two-way rectangular footings, reinforcement shall be distributed as follows:

15.4.4.1 — Reinforcement in long direction shall be distributed uniformly across entire width of footing.

15.4.4.2 — For reinforcement in short direction, a portion of the total reinforcement given by Eq. (15-1) shall be distributed uniformly over a band width (centered on centerline of column or pedestal) equal to the length of short side of footing. Remainder of reinforcement required in short direction shall be distributed uniformly outside center band width of footing.

$$\frac{\text{Reinforcement in band width}}{\text{Total reinforcement in short direction}} = \frac{2}{(\beta + 1)} \qquad (15\text{-}1)$$

15.5 — Shear in footings

15.5.1 — Shear strength of footings shall be in accordance with 11.12.

15.5.2 — Location of critical section for shear in accordance with Chapter 11 shall be measured from face of column, pedestal, or wall, for footings supporting a column, pedestal, or wall. For footings supporting a column or pedestal with steel base plates, the critical section shall be measured from location defined in 15.4.2(c).

COMMENTARY

R15.4.4 — As in previous ACI Building Codes, the reinforcement in the short direction of rectangular footings must be distributed so that an area of steel given by Eq. (15-1) is provided in a band width equal to the length of the short side of the footing. The band width is centered about the column centerline.

The remaining reinforcement required in the short direction is to be distributed equally over the two segments outside the band width, one-half to each segment.

R15.5 — Shear in footings

R15.5.1 and R15.5.2 — The shear strength of footings must be determined for the more severe condition of 11.12.1.1 or 11.12.1.2. The critical section for shear is "measured" from the face of supported member (column, pedestal, or wall), except for supported members on steel base plates.

Computation of shear requires that the soil bearing pressure q_s be obtained from the factored loads and the design be in accordance with the appropriate equations of Chapter 11.

Where necessary, shear around individual piles may be investigated in accordance with 11.12.1.2. If shear perime-

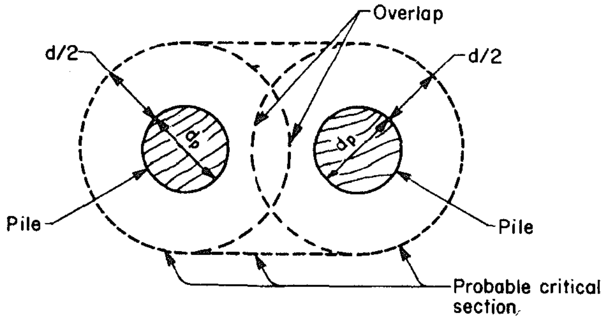


Fig. R15.5—Modified critical section for shear with overlapping critical perimeters

CODE

15.5.3 — Computation of shear on any section through a footing supported on piles shall be in accordance with the following:

15.5.3.1 — Entire reaction from any pile whose center is located $d_p/2$ or more outside the section shall be considered as producing shear on that section.

15.5.3.2 — Reaction from any pile whose center is located $d_p/2$ or more inside the section shall be considered as producing no shear on that section.

15.5.3.3 — For intermediate positions of pile center, the portion of the pile reaction to be considered as producing shear on the section shall be based on straight-line interpolation between full value at $d_p/2$ outside the section and zero value at $d_p/2$ inside the section.

15.6 — Development of reinforcement in footings

15.6.1 — Development of reinforcement in footings shall be in accordance with Chapter 12.

15.6.2 — Calculated tension or compression in reinforcement at each section shall be developed on each side of that section by embedment length, hook (tension only) or mechanical device, or a combination thereof.

15.6.3 — Critical sections for development of reinforcement shall be assumed at the same locations as defined in 15.4.2 for maximum factored moment, and at all other vertical planes where changes of section or reinforcement occur. See also 12.10.6.

15.7 — Minimum footing depth

Depth of footing above bottom reinforcement shall not be less than 6 in. for footings on soil, nor less than 12 in. for footings on piles.

15.8 — Transfer of force at base of column, wall, or reinforced pedestal

15.8.1 — Forces and moments at base of column, wall, or pedestal shall be transferred to supporting pedestal or footing by bearing on concrete and by reinforcement, dowels, and mechanical connectors.

COMMENTARY

ters overlap, the critical perimeter b_o should be taken as that portion of the smallest envelope of individual shear perimeter which will actually resist the critical shear for the group under consideration. One such situation is illustrated in Fig. R15.5.

R15.5.3 — When piles are located inside the critical sections d or $d/2$ from face of column, for one-way or two-way shear, respectively, an upper limit on the shear strength at a section adjacent to the face of the column should be considered. The CRSI Handbook[15.3] offers guidance for this situation.

R15.8 — Transfer of force at base of column, wall, or reinforced pedestal

Section 15.8 provides the specific requirements for force transfer from a column, wall, or pedestal (supported member) to a pedestal or footing (supporting member). Force transfer must be by bearing on concrete (compressive force

CODE

15.8.1.1 — Bearing on concrete at contact surface between supported and supporting member shall not exceed concrete bearing strength for either surface as given by 10.17.

15.8.1.2 — Reinforcement, dowels, or mechanical connectors between supported and supporting members shall be adequate to transfer:

(a) all compressive force that exceeds concrete bearing strength of either member,

(b) any computed tensile force across interface.

In addition, reinforcement, dowels or mechanical connectors shall satisfy 15.8.2 or 15.8.3.

15.8.1.3 — If calculated moments are transferred to supporting pedestal or footing, reinforcement, dowels or mechanical connectors shall be adequate to satisfy 12.17.

COMMENTARY

only) and by reinforcement (tensile or compressive force). Reinforcement may consist of extended longitudinal bars, dowels, anchor bolts, or suitable mechanical connectors.

The requirements of 15.8.1 apply to both cast-in-place construction and precast construction. Additional requirements for cast-in-place construction are given in 15.8.2. Section 15.8.3 gives additional requirements for precast construction.

R15.8.1.1 — Compressive force may be transmitted to a supporting pedestal or footing by bearing on concrete. For strength design, unit bearing strength on the actual loaded area will be equal to $0.85\phi f_c'$ (where $\phi = 0.7$), if the loaded area is equal to the area on which it is supported.

In the common case of a column bearing on a footing larger than the column, bearing strength must be checked at the base of the column and the top of the footing. Strength in the lower part of the column must be checked since the column reinforcement cannot be considered effective near the column base because the force in the reinforcement is not developed for some distance above the base, unless dowels are provided, or the column reinforcement is extended into the footing. The unit bearing stress on the column will normally be $0.85\ \phi f_c'$ (with $\phi = 0.7$, this becomes $0.6 f_c'$). The permissible bearing strength on the footing may be increased in accordance with 10.17 and will usually be two times $0.85\phi f_c'$. The compressive force which exceeds that developed by the permissible bearing strength at the base of the column or at the top of the footing must be carried by dowels or extended longitudinal bars.

For the alternate design method of Appendix A, permissible bearing stresses are limited to 50 percent of the values in 10.17.

R15.8.1.2 — All tensile forces, whether created by uplift, moment, or other means, must be transferred to supporting pedestal or footing entirely by reinforcement or suitable mechanical connectors. Generally, mechanical connections would be used only in precast construction.

R15.8.1.3 — If computed moments are transferred from the column to the footing, the concrete in the compression zone of the column will generally be stressed to $0.85 f_c'$ under factored load conditions and, as a result, all the reinforcement will generally have to be doweled into the footing.

CODE

15.8.1.4 — Lateral forces shall be transferred to supporting pedestal or footing in accordance with shear-friction provisions of 11.7, or by other appropriate means.

15.8.2 — In cast-in-place construction, reinforcement required to satisfy 15.8.1 shall be provided either by extending longitudinal bars into supporting pedestal or footing, or by dowels.

15.8.2.1 — For cast-in-place columns and pedestals, area of reinforcement across interface shall be not less than 0.005 times gross area of supported member.

15.8.2.2 — For cast-in-place walls, area of reinforcement across interface shall be not less than minimum vertical reinforcement given in 14.3.2.

15.8.2.3 — At footings, No. 14 and No. 18 longitudinal bars, in compression only, may be lap spliced with dowels to provide reinforcement required to satisfy 15.8.1. Dowels shall not be larger than No. 11 bar and shall extend into supported member a distance not less than the development length of No. 14 or No. 18 bars or the splice length of the dowels, whichever is greater, and into the footing a distance not less than the development length of the dowels.

15.8.2.4 — If a pinned or rocker connection is provided in cast-in-place construction, connection shall conform to 15.8.1 and 15.8.3.

15.8.3 — In precast construction, anchor bolts or suitable mechanical connectors shall be permitted for satisfying 15.8.1.

15.8.3.1 — Connection between precast columns or pedestals and supporting members shall meet the requirements of 16.5.1.3(a).

COMMENTARY

R15.8.1.4 — The shear-friction method given in 11.7 may be used to check for transfer of lateral forces to supporting pedestal or footing. Shear keys may be used, provided that the reinforcement crossing the joint satisfies 15.8.2.1, 15.8.3.1, and the shear-friction requirements of 11.7. In precast construction, resistance to lateral forces may be provided by shear-friction, shear keys, or mechanical devices.

R15.8.2.1 and R15.8.2.2 — A minimum amount of reinforcement is required between all supported and supporting members to ensure ductile behavior. The code does not require that all bars in a column be extended through and be anchored into a footing. However, reinforcement with an area of 0.005 times the column area or an equal area of properly spliced dowels must extend into the footing with proper anchorage. This reinforcement is required to provide a degree of structural integrity during the construction stage and during the life of the structure.

R15.8.2.3 — Lap splices of No. 14 and No. 18 longitudinal bars in compression only to dowels from a footing are specifically permitted in 15.8.2.3. The dowel bars must be No. 11 or smaller in size. The dowel lap splice length must meet the larger of the two criteria: (a) be able to transfer the stress in the No. 14 and No. 18 bars, and (b) fully develop the stress in the dowels as a splice.

This provision is an exception to 12.14.2.1, which prohibits lap splicing of No. 14 and No. 18 bars. This exception results from many years of successful experience with the lap splicing of these large column bars with footing dowels of the smaller size. The reason for the restriction on dowel bar size is recognition of the anchorage length problem of the large bars, and to encourage use of the smaller size dowels with probable resulting economies in footing depths. A similar exception is allowed for compression splices between different size bars in 12.16.2.

R15.8.3.1 and R15.8.3.2 — For cast-in-place columns, 15.8.2.1 requires a minimum area of reinforcement equal to $\mathbf{0.005}A_g$ across the column-footing interface to provide

CODE

15.8.3.2 — Connection between precast walls and supporting members shall meet the requirements of 16.5.1.3(b) and (c).

15.8.3.3 — Anchor bolts and mechanical connectors shall be designed to reach their design strength prior to anchorage failure or failure of surrounding concrete.

15.9 — Sloped or stepped footings

15.9.1 — In sloped or stepped footings, angle of slope or depth and location of steps shall be such that design requirements are satisfied at every section. (See also 12.10.6.)

15.9.2 — Sloped or stepped footings designed as a unit shall be constructed to assure action as a unit.

15.10 — Combined footings and mats

15.10.1 — Footings supporting more than one column, pedestal, or wall (combined footings or mats) shall be proportioned to resist the factored loads and induced reactions, in accordance with appropriate design requirements of this code.

15.10.2 — The Direct Design Method of Chapter 13 shall not be used for design of combined footings and mats.

15.10.3 — Distribution of soil pressure under combined footings and mats shall be consistent with properties of the soil and the structure and with established principles of soil mechanics.

COMMENTARY

some degree of structural integrity. For precast columns this requirement is expressed in terms of an equivalent tensile force which must be transferred. Thus, across the joint, $A_s f_y$ = $200A_g$ [see 16.5.1.3(a)]. The minimum tensile strength required for precast wall-to-footing connection [see 16.5.1.3(b)] is somewhat less than that required for columns, since an overload would be distributed laterally and a sudden failure would be less likely. Since the tensile strength values of 16.5.1.3 have been arbitrarily chosen, it is not necessary to include a strength reduction factor ϕ for these calculations.

R15.8.3.3 — Anchor bolts and mechanical connectors must be designed to reach their design strength before the bolt or connector yields, slips, or pulls out of the concrete.

R15.10 — Combined footings and mats

R15.10.1 — Any reasonable assumption with respect to the distribution of soil pressure or pile reactions can be used as long as it is consistent with the type of structure and the properties of the soil, and conforms with established principles of soil mechanics (see 15.1). Similarly, as prescribed in 15.2.2 for isolated footings, the base area or pile arrangement of combined footings and mats should be determined using the unfactored forces and/or moments transmitted by the footing to the soil, considering permissible soil pressures and pile reactions.

Design methods using factored loads and strength reduction factors ϕ can be applied to combined footings or mats, regardless of the soil pressure distribution.

Detailed recommendations for design of combined footings and mats are given in **"Suggested Design Procedures for Combined Footings and Mats"** reported by ACI Committee 336.[15.1] See also Reference 15.2.

Notes

CHAPTER 16 — PRECAST CONCRETE

CODE

16.0 — Notation

A_g = gross area of column, in.^{2}
ℓ = clear span, in.

16.1 — Scope

16.1.1 — All provisions of this code, not specifically excluded and not in conflict with the provisions of Chapter 16, shall apply to structures incorporating precast concrete structural members.

16.2 — General

16.2.1 — Design of precast members and connections shall include loading and restraint conditions from initial fabrication to end use in the structure, including form removal, storage, transportation, and erection.

16.2.2 — When precast members are incorporated into a structural system, the forces and deformations occurring in and adjacent to connections shall be included in the design.

16.2.3 — Tolerances for both precast members and interfacing members shall be specified. Design of precast members and connections shall include the effects of these tolerances.

COMMENTARY

R16.1 — Scope

R16.1.1 — See 2.1 for definition of precast concrete.

Design and construction requirements for precast concrete structural members differ in some respects from those for cast-in-place concrete structural members and these differences are addressed in this chapter. Where provisions for cast-in-place concrete apply equally to precast concrete, they have not been repeated. Similarly, items related to composite concrete in Chapter 17 and to prestressed concrete in Chapter 18 that apply to precast concrete are not restated.

More detailed recommendations concerning precast concrete are given in References 16.1 through 16.7. Tilt-up concrete construction is a form of precast concrete. It is recommended that Reference 16.8 be reviewed for tilt-up structures.

R16.2 — General

R16.2.1 — Stresses developed in precast members during the period from casting to final connection may be greater than the service load stresses. Handling procedures may cause undesirable deformations. Hence, care must be given to the methods of storing, transporting, and erecting precast members so that performance at service loads and strength under factored loads meet code requirements.

R16.2.2 — The structural behavior of precast members may differ substantially from that of similar members that are cast-in-place. Design of connections to minimize or transmit forces due to shrinkage, creep, temperature change, elastic deformation, differential settlement, wind, and earthquake require special consideration in precast construction.

R16.2.3 — Design of precast members and connections is particularly sensitive to tolerances on the dimensions of individual members and on their position in the structure. To prevent misunderstanding, the tolerances used in design should be specified in the contract documents. The designer may specify the tolerance standard assumed in design. It is especially important to specify any deviations from accepted standards.

CODE

16.2.4 — In addition to the requirements for drawings and specifications in 1.2, the following shall be included in either the contract documents or shop drawings:

(a) Details of reinforcement, inserts and lifting devices required to resist temporary loads from handling, storage, transportation, and erection.

(b) Required concrete strength at stated ages or stages of construction.

16.3 — Distribution of forces among members

16.3.1 — Distribution of forces that are perpendicular to the plane of members shall be established by analysis or by test.

16.3.2 — Where the system behavior requires in-plane forces to be transferred between the members of a precast floor or wall system, the following shall apply:

16.3.2.1 — In-plane force paths shall be continuous through both connections and members.

16.3.2.2 — Where tension forces occur, a continuous path of steel or steel reinforcement shall be provided.

COMMENTARY

The tolerances required by 7.5 are considered to be a minimum acceptable standard for reinforcement in precast concrete. The designer should refer to publications of the Precast/Prestressed Concrete Institute (References 16.9, 16.10, 16.11) for guidance on industry established standard product and erection tolerances. Added guidance is given in Reference 16.12.

R16.2.4 — The additional requirements may be included in either contract documents or shop drawings depending on the assignment of responsibility for design.

R16.3 — Distribution of forces among members

R16.3.1 — Concentrated point and line loads can be distributed among members provided they have sufficient torsional stiffness and that shear can be transferred across joints. Torsionally stiff members such as hollow-core or solid slabs have more favorable load distribution properties than do torsionally flexible members such as double tees with thin flanges. The actual distribution of the load depends on many factors discussed in detail in References 16.13 through 16.19. Large openings can cause significant changes in distribution of forces.

R16.3.2 — In-plane forces result primarily from diaphragm action in floors and roofs, causing tension or compression in the chords and shear in the body of the diaphragm. A continuous path of steel and/or steel reinforcement, using lap splices or positive mechanical connections, must be provided to carry the tension, whereas the shear and compression may be carried by the net concrete section. A continuous path of steel through a connection may include bolts, weld plates, headed studs or other steel devices. Tension forces in the connections are to be transferred to the primary reinforcement in the members.

In-plane forces in precast wall systems result primarily from diaphragm reactions and external lateral loads.

Connection details should provide for the forces and deformations due to shrinkage, creep, and thermal effects. Connection details may be selected to accommodate volume changes and rotations caused by temperature gradients and long-term deflections. When these effects are restrained, connections and members should be designed to provide adequate strength and ductility.

CODE

16.4 — Member design

16.4.1 — In one-way precast floor and roof slabs and in one-way precast, prestressed wall panels, all not wider than 12 ft, and where members are not mechanically connected to cause restraint in the transverse direction, the shrinkage and temperature reinforcement requirements of 7.12 in the direction normal to the flexural reinforcement shall be permitted to be waived. This waiver shall not apply to members which require reinforcement to resist transverse flexural stresses.

16.4.2 — For precast, nonprestressed walls the reinforcement shall be designed in accordance with the provisions of Chapters 10 or 14 except that the area of horizontal and vertical reinforcement shall each be not less than 0.001 times the gross cross-sectional area of the wall panel. Spacing of reinforcement shall not exceed 5 times the wall thickness or 30 in. for interior walls or 18 in. for exterior walls.

16.5 — Structural integrity

16.5.1 — Except where the provisions of 16.5.2 govern, the following minimum provisions for structural integrity shall apply to all precast concrete structures:

16.5.1.1 — Longitudinal and transverse ties required by 7.13.3 shall connect members to a lateral load resisting system.

COMMENTARY

R16.4 — Member design

R16.4.1 — For prestressed concrete members, not wider than 12 ft, such as hollow-core slabs, solid slabs, or slabs with closely spaced ribs, there is usually no need to provide transverse reinforcement to withstand shrinkage and temperature stresses in the short direction. This is generally true also for nonprestressed floor and roof slabs. The 12 ft width is less than that in which shrinkage and temperature stresses can build up to a magnitude requiring transverse reinforcement. In addition, much of the shrinkage occurs before the members are tied into the structure. Once in the final structure, the members are usually not as rigidly connected transversely as monolithic concrete, thus, the transverse restraint stresses due to both shrinkage and temperature change are significantly reduced.

The waiver does not apply, for example, to members such as single and double tees with thin, wide flanges.

R16.4.2 — This minimum area of wall reinforcement, in lieu of the minimum values in 14.3, has generally been used for many years with no problems, and is recommended by the Precast/Prestressed Concrete Institute [16.4] and the Canadian building code.[16.20] The provisions for reduced minimum reinforcement and greater spacing recognize that precast wall panels have very little restraint at their edges during early stages of curing and, therefore, develop less shrinkage stress than comparable cast-in-place walls.

R16.5 — Structural integrity

R16.5.1 — The general provisions of 7.13.3 apply to all precast concrete structures. Sections 16.5.1 and 16.5.2 give minimum requirements to satisfy 7.13.3. It is not intended that these minimum requirements override other applicable provisions of the code for design of precast concrete structures.

The overall integrity of a structure can be substantially enhanced by minor changes in the amount, location, and detailing of member reinforcement and in the detailing of connection hardware.

R16.5.1.1 — Individual members may be connected into this lateral load resisting system by alternative methods. For example, a load-bearing spandrel could be connected to a diaphragm (part of the lateral load resisting system). Structural integrity could be achieved by connecting the spandrel into all or a portion of the deck members forming the diaphragm. Alternatively, the spandrel could be connected only to its supporting columns, which in turn must be connected to the diaphragm.

CODE

16.5.1.2 — Where precast elements form floor or roof diaphragms, the connections between diaphragm and those members being laterally supported shall have a nominal tensile strength capable of resisting not less than 300 lb per lin ft.

16.5.1.3 — Vertical tension tie requirements of 7.13.3 shall apply to all vertical structural members, except cladding, and shall be achieved by providing connections at horizontal joints in accordance with the following:

(a) Precast columns shall have a nominal strength in tension not less than **200** A_g in pounds. For columns with a larger cross section than required by consideration of loading, a reduced effective area A_g, based on cross section required but not less than one-half the total area, shall be permitted.

(b) Precast wall panels shall have a minimum of two ties per panel, with a nominal tensile strength not less than 10,000 lb per tie.

(c) When design forces result in no tension at the base, the ties required by 16.5.1.3(b) shall be permitted to be anchored into an appropriately reinforced concrete floor slab on grade.

16.5.1.4 — Connection details that rely solely on friction caused by gravity loads shall not be used.

16.5.2 — For precast concrete bearing wall structures three or more stories in height, the following minimum provisions shall apply:

COMMENTARY

R16.5.1.2 — Diaphragms are typically provided as part of the lateral load resisting system. The ties prescribed in 16.5.1.2 are the minimum required to attach members to the floor or roof diaphragms. The tie force is equivalent to the service load value of 200 lb/ft given in the Uniform Building Code.

R16.5.1.3 — Base connections and connections at horizontal joints in precast columns and wall panels, including shear walls, must be designed to transfer all design forces and moments. The minimum tie requirements of 16.5.1.3 are not additive to these design requirements. Industry practice is to place the wall ties symmetrically about the vertical centerline of the wall panel and within the outer quarters of the panel width, wherever possible.

R16.5.1.4 — In the event of damage to a beam, it is important that displacement of its supporting members be minimized, so that other members will not lose their load-carrying capacity. This is a situation that shows why connection details which rely solely on friction caused by gravity loads are not to be used. An exception could be heavy modular unit structures (one or more cells in cell-type structures) where resistance to overturning or sliding in any direction has a large factor of safety. Acceptance of such systems should be based on the provisions of 1.4.

R16.5.2 — The structural integrity minimum tie provisions for bearing wall structures, often called large panel structures, are intended to provide catenary hanger supports in case of loss of a bearing wall support, as shown by test.[16.21] Forces induced by loading, temperature change, creep, and wind or seismic action may require a larger amount of tie force. It is intended that the general precast concrete provisions of 16.5.1 apply to bearing wall structures less than three stories in height.

Minimum ties in structures three or more stories in height, in accordance with 16.5.2.1, 16.5.2.2, 16.5.2.3, 16.5.2.4, and 16.5.2.5, are required for structural integrity (Fig. R16.5.2). These provisions are based on the Precast/Prestressed Concrete Institute's recommendations for design of precast concrete bearing wall buildings.[16.22] Tie capacity is based on yield strength.

CODE

16.5.2.1 — Longitudinal and transverse ties shall be provided in floor and roof systems to provide a nominal strength of 1500 lb per foot of width or length. Ties shall be provided over interior wall supports and between members and exterior walls. Ties shall be positioned in or within 2 ft of the plane of the floor or roof system.

16.5.2.2 — Longitudinal ties parallel to floor or roof slab spans shall be spaced not more than 10 ft on centers. Provisions shall be made to transfer forces around openings.

16.5.2.3 — Transverse ties perpendicular to floor or roof slab spans shall be spaced not greater than the bearing wall spacing.

16.5.2.4 — Ties around the perimeter of each floor and roof, within 4 ft of the edge, shall provide a nominal strength in tension not less than 16,000 lb.

16.5.2.5 — Vertical tension ties shall be provided in all walls and shall be continuous over the height of the building. They shall provide a nominal tensile strength not less than 3000 lb per horizontal foot of wall. Not less than two ties shall be provided for each precast panel.

16.6 — Connection and bearing design

16.6.1 — Forces shall be permitted to be transferred between members by grouted joints, shear keys, mechanical connectors, reinforcing steel connections, reinforced topping, or a combination of these means.

COMMENTARY

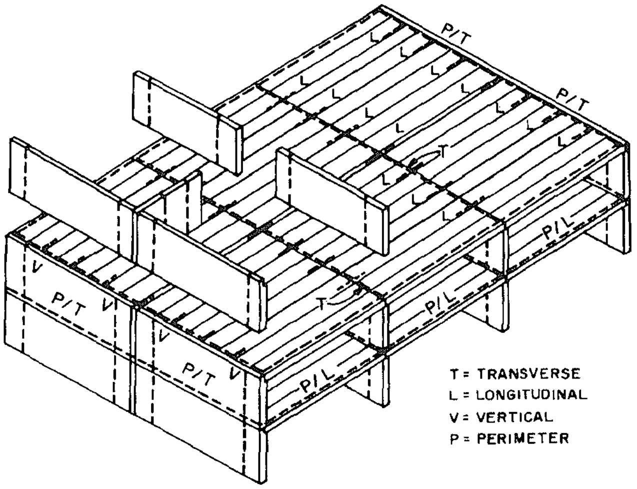


Fig. R16.5.2—Typical arrangement of tensile ties in large panel structures

R16.5.2.1 — Longitudinal ties may project from slabs and be lap spliced, welded, or mechanically connected, or they may be embedded in grout joints, with sufficient length and cover to develop the required force. Bond length for unstressed prestressing steel should be sufficient to develop the yield strength.[16.23] It is not uncommon to have ties positioned in the walls reasonably close to the plane of the floor or roof system.

R16.5.2.3 — Transverse ties may be uniformly spaced either encased in the panels or in a topping, or they may be concentrated at the transverse bearing walls.

R16.5.2.4 —The perimeter tie requirements need not be additive with the longitudinal and transverse tie requirements.

R16.6 — Connection and bearing design

R16.6.1 — The code permits a variety of methods for connecting members. These are intended for transfer of forces both in-plane and perpendicular to the plane of the members.

CODE

16.6.1.1 — The adequacy of connections to transfer forces between members shall be determined by analysis or by test. Where shear is the primary imposed loading, it shall be permitted to use the provisions of 11.7 as applicable.

16.6.1.2 — When designing a connection using materials with different structural properties, their relative stiffnesses, strengths, and ductilities shall be considered.

16.6.2 — Bearing for precast floor and roof members on simple supports shall satisfy the following:

16.6.2.1—The allowable bearing stress at the contact surface between supported and supporting members and between any intermediate bearing elements shall not exceed the bearing strength for either surface and the bearing element. Concrete bearing strength shall be as given in 10.17.

16.6.2.2 — Unless shown by test or analysis that performance will not be impaired, the following minimum requirements shall be met:

(a) Each member and its supporting system shall have design dimensions selected so that, after consideration of tolerances, the distance from the edge of the support to the end of the precast member in the direction of the span is at least $^1/_{180}$ of the clear span ℓ, but not less than:

For solid or hollow-core slabs .... 2 in.
For beams or stemmed members...................... 3 in.

(b) Bearing pads at unarmored edges shall be set back a minimum of $^1/_2$ in. from the face of the support, or at least the chamfer dimension at chamfered edges.

16.6.2.3 — The requirements of 12.11.1 shall not apply to the positive bending moment reinforcement for statically determinate precast members, but at least one-third of such reinforcement shall extend to the center of the bearing length.

COMMENTARY

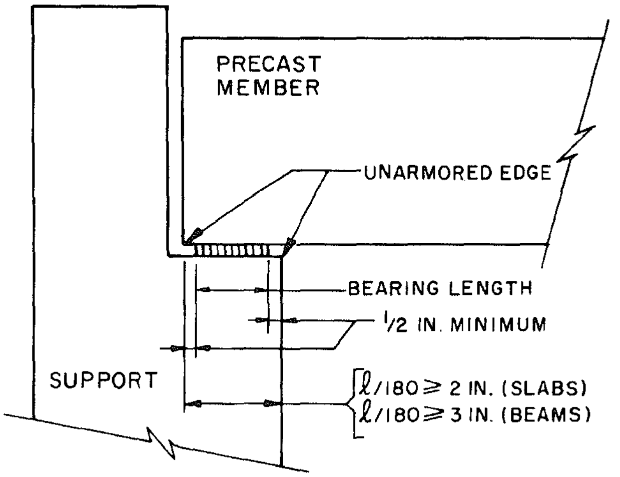


Fig. R16.6.2—Bearing length versus length of member on support

R16.6.1.2 — The designer should be aware that the various components in a connection (e.g. bolts, welds, plates, inserts, etc.) have different properties that can affect the overall behavior of the connection.

R16.6.2.1 — When tensile forces occur in the plane of the bearing, it may be desirable to reduce the allowable bearing stress and/or provide confinement reinforcement. Guidelines are provided in Reference 16.4.

R16.6.2.2 — This section differentiates between bearing length and length of the end of a precast member over the support (Fig. R16.6.2).

Bearing pads distribute concentrated loads and reactions over the bearing area, and allow limited horizontal and rotational movements for stress relief. To prevent spalling under heavily loaded bearing areas, bearing pads should not extend to the edge of the support unless the edge is armored. Edges can be armored with anchored steel plates or angles. Section 11.9.7 gives requirements for bearing on brackets or corbels.

R16.6.2.3 — It is unnecessary to develop positive bending moment reinforcement beyond the ends of the precast element if the system is statically determinate.

CODE

16.7 — Items embedded after concrete placement

16.7.1 — When approved by the engineer, embedded items (such as dowels or inserts) that either protrude from the concrete or remain exposed for inspection shall be permitted to be embedded while the concrete is in a plastic state provided that:

16.7.1.1 — Embedded items are not required to be hooked or tied to reinforcement within the concrete.

16.7.1.2 — Embedded items are maintained in the correct position while the concrete remains plastic.

16.7.1.3 — The concrete is properly consolidated around the embedded item.

16.8 — Marking and identification

16.8.1 — Each precast member shall be marked to indicate its location and orientation in the structure and date of manufacture.

16.8.2 — Identification marks shall correspond to placing drawings.

16.9 — Handling

16.9.1 — Member design shall consider forces and distortions during curing, stripping, storage, transportation, and erection so that precast members are not overstressed or otherwise damaged.

16.9.2 — Precast members and structures shall be adequately supported and braced during erection to ensure proper alignment and structural integrity until permanent connections are completed.

16.10 — Strength evaluation of precast construction

16.10.1 — A precast element to be made composite with cast-in-place concrete shall be permitted to be tested in flexure as a precast element alone in accordance with the following:

COMMENTARY

R16.7 — Items embedded after concrete placement

R16.7.1 — Section 16.7.1 is an exception to the provisions of 7.5.1. Many precast products are manufactured in such a way that it is difficult, if not impossible, to position reinforcement which protrudes from the concrete before the concrete is placed. Experience has shown that such items as ties for horizontal shear and inserts can be placed while the concrete is plastic, if proper precautions are taken. This exception is not applicable to reinforcement which is completely embedded, or to embedded items which must be hooked or tied to embedded reinforcement.

R16.9 — Handling

R16.9.1 — The code requires acceptable performance at service loads and adequate strength under factored loads. However, handling loads should not produce permanent stresses, strains, cracking, or deflections inconsistent with the provisions of the code. A precast member should not be rejected for minor cracking or spalling where strength and durability are not affected. Guidance on assessing cracks in precast members is given in two Precast/Prestressed Concrete Institute reports on fabrication and shipment cracks.[16.24, 16.25]

R16.9.2 — It is important that all temporary erection connections, bracing, and shoring be shown on contract or erection drawings, as well as the sequencing of removal of these items.

R16.10 — Strength evaluation of precast construction

When the strength of a precast member in a structure is in doubt, the strength evaluation procedures of Chapter 20 are applicable. This section amplifies Chapter 20 to include the testing and evaluation of individual precast members before they are integrated into the structure.

CODE

16.10.1.1 — Test loads shall be applied only when calculations indicate the isolated precast element will not be critical in compression or buckling.

16.10.1.2 — The test load shall be that load which, when applied to the precast member alone, induces the same total force in the tension reinforcement as would be induced by loading the composite member with the test load required by 20.3.2.

16.10.2 — The provisions of 20.5 shall be the basis for acceptance or rejection of the precast element.

COMMENTARY

CHAPTER 17 — COMPOSITE CONCRETE FLEXURAL MEMBERS

CODE

17.0 — Notation

A_c = area of contact surface being investigated for horizontal shear, in.^{2}
A_v = area of ties within a distance s, in.^{2}
b_v = width of cross section at contact surface being investigated for horizontal shear
d = distance from extreme compression fiber to centroid of tension reinforcement for entire composite section, in.
h = overall thickness of composite member, in.
s = spacing of ties measured along the longitudinal axis of the member, in.
V_{nh} = nominal horizontal shear strength
V_u = factored shear force at section
λ = correction factor related to unit weight of concrete
ρ_v = ratio of tie reinforcement area to area of contact surface
= $A_v/b_v s$
ϕ = strength reduction factor. See 9.3

17.1 — Scope

17.1.1 — Provisions of Chapter 17 shall apply for design of composite concrete flexural members defined as precast and/or cast-in-place concrete elements constructed in separate placements but so interconnected that all elements respond to loads as a unit.

17.1.2 — All provisions of this code shall apply to composite concrete flexural members, except as specifically modified in Chapter 17.

17.2 — General

17.2.1 — The use of an entire composite member or portions thereof for resisting shear and moment shall be permitted.

17.2.2 — Individual elements shall be investigated for all critical stages of loading.

17.2.3 — If the specified strength, unit weight, or other properties of the various elements are different, properties of the individual elements or the most critical values, shall be used in design.

COMMENTARY

R17.1 — Scope

R17.1.1 — The scope of Chapter 17 is intended to include all types of composite concrete flexural members. In some cases with fully cast-in-place concrete, it may be necessary to design the interface of consecutive placements of concrete as required for composite members. Composite structural steel-concrete members are not covered in this chapter, because design provisions for such composite members are covered in Reference 17.1.

R17.2 — General

CODE

17.2.4 — In strength computations of composite members, no distinction shall be made between shored and unshored members.

17.2.5 — All elements shall be designed to support all loads introduced prior to full development of design strength of composite members.

17.2.6 — Reinforcement shall be provided as required to control cracking and to prevent separation of individual elements of composite members.

17.2.7 — Composite members shall meet requirements for control of deflections in accordance with 9.5.5.

17.3 — Shoring

When used, shoring shall not be removed until supported elements have developed design properties required to support all loads and limit deflections and cracking at time of shoring removal.

17.4 — Vertical shear strength

17.4.1 — When an entire composite member is assumed to resist vertical shear, design shall be in accordance with requirements of Chapter 11 as for a monolithically cast member of the same cross-sectional shape.

17.4.2 — Shear reinforcement shall be fully anchored into interconnected elements in accordance with 12.13.

17.4.3 — Extended and anchored shear reinforcement shall be permitted to be included as ties for horizontal shear.

17.5 — Horizontal shear strength

17.5.1 — In a composite member, full transfer of horizontal shear forces shall be assured at contact surfaces of interconnected elements.

17.5.2 — Unless calculated in accordance with 17.5.3, design of cross sections subject to horizontal shear shall be based on

COMMENTARY

R17.2.4 — Tests have indicated that the strength of a composite member is the same whether or not the first element cast is shored during casting and curing of the second element.

R17.2.6 — The extent of cracking permitted is dependent on such factors as environment, aesthetics, and occupancy. In addition, composite action should not be impaired.

R17.2.7 — The premature loading of precast elements can cause excessive deflections as the result of creep and shrinkage. This is especially so at early ages when the moisture content is high and the strength low.

The transfer of shear by direct bond is important if excessive deflection from slippage is to be prevented. A shear key is an added mechanical factor of safety but it cannot operate until slippage occurs.

R17.3 — Shoring

The provisions of 9.5.5 cover the requirements pertaining to deflections of shored and unshored members.

R17.5 — Horizontal shear strength

R17.5.1 — Full transfer of horizontal shear between segments of composite members should be assured by horizontal shear strength at contact surfaces or properly anchored ties, or both.

R17.5.2 — The nominal horizontal shear strengths V_{nh} apply when the design is based on the load factors and ϕ-factors of Chapter 9.

CODE

$$V_u \leq \phi V_{nh} \qquad (17\text{-}1)$$

where V_u is factored shear force at section considered and V_{nh} is nominal horizontal shear strength in accordance with the following.

17.5.2.1 — When contact surfaces are clean, free of laitance, and intentionally roughened, shear strength V_{nh} shall not be taken greater than $80b_vd$ in pounds.

17.5.2.2 — When minimum ties are provided in accordance with 17.6, and contact surfaces are clean and free of laitance, but not intentionally roughened, shear strength V_{nh} shall not be taken greater than $80\, b_vd$ in pounds.

17.5.2.3 — When ties are provided in accordance with 17.6, and contact surfaces are clean, free of laitance, and intentionally roughened to a full amplitude of approximately $^1/_4$ in., shear strength V_{nh} shall be taken equal to $(260 + 0.6\rho_v f_y)\lambda b_v d$ in pounds, but not greater than $500b_vd$ in pounds. Values for λ in 11.7.4.3 shall apply.

17.5.2.4 — When factored shear force V_u at section considered exceeds $\phi\ (500b_vd)$, design for horizontal shear shall be in accordance with 11.7.4.

17.5.2.5 — When determining nominal horizontal shear strength over prestressed concrete elements, d shall be as defined or $0.8h$, whichever is greater.

17.5.3 — As an alternative to 17.5.2, horizontal shear shall be determined by computing the actual change in compressive or tensile force in any segment, and provisions shall be made to transfer that force as horizontal shear to the supporting element. The factored horizontal shear force shall not exceed horizontal shear strength ϕV_{nh} as given in 17.5.2.1 through 17.5.2.4, where area of contact surface A_c shall be substituted for b_vd.

17.5.3.1 — When ties provided to resist horizontal shear are designed to satisfy 17.5.3, the tie area to tie spacing ratio along the member shall approximately reflect the distribution of shear forces in the member.

COMMENTARY

When the alternate design method of Appendix A is used for design of composite members, V_u is the shear due to service loads, and 55 percent of the values given in 17.5.2 are applicable. See A.7.3. Also, when gravity loads are combined with lateral loads due to wind or earthquake in the governing load combination for horizontal shear, advantage may be taken of the 25 percent reduction in required strength in accordance with A.2.2.

In reviewing composite concrete flexural members for handling and construction loads, V_u may be replaced by the handling service load shear in Eq. (17-1). The handling load horizontal shear should be compared with a nominal horizontal shear strength value of $0.55V_{nh}$ (as provided in Appendix A for the Alternate Design Method) to ensure that an adequate factor of safety results for handling and construction loads.

Prestressed members used in composite construction may have variations in depth of tension reinforcement along member length due to draped or depressed tendons. Because of this variation, the definition of d used in Chapter 11 for determination of vertical shear strength is also appropriate when determining horizontal shear strength.

R17.5.2.3 — The permitted horizontal shear strengths and the requirement of $^1/_4$ in. amplitude for intentional roughness are based on tests discussed in References 17.2 through 17.4.

R17.5.3.1 — The distribution of horizontal shear stresses along the contact surface in a composite member will reflect the distribution of shear along the member. Horizontal shear failure will initiate where the horizontal shear stress is a

CODE

17.5.4 — When tension exists across any contact surface between interconnected elements, shear transfer by contact shall be permitted only when minimum ties are provided in accordance with 17.6.

17.6 — Ties for horizontal shear

17.6.1 — When ties are provided to transfer horizontal shear, tie area shall not be less than that required by 11.5.5.3, and tie spacing shall not exceed four times the least dimension of supported element, nor 24 in.

17.6.2 — Ties for horizontal shear shall consist of single bars or wire, multiple leg stirrups, or vertical legs of welded wire fabric (plain or deformed).

17.6.3 — All ties shall be fully anchored into interconnected elements in accordance with 12.13.

COMMENTARY

maximum and will spread to regions of lower stress. Because the slip at peak horizontal shear resistance is small for a concrete to concrete contact surface, longitudinal redistribution of horizontal shear resistance is very limited. The spacing of the ties along the contact surface should, therefore, be such as to provide horizontal shear resistance distributed approximately as the shear acting on the member is distributed.

R17.5.4 — Proper anchorage of ties extending across interfaces is required to maintain contact of the interfaces.

R17.6 — Ties for horizontal shear

The minimum areas and maximum spacings are based on test data given in References 17.2 through 17.6.

CHAPTER 18 — PRESTRESSED CONCRETE

CODE

COMMENTARY

18.0 — Notation

A = area of that part of cross section between flexural tension face and center of gravity of gross section, in.^{2}
A_{ps} = area of prestressed reinforcement in tension zone, in.^{2}
A_s = area of nonprestressed tension reinforcement, in.^{2}
A_s' = area of compression reinforcement, in.^{2}
b = width of compression face of member, in.
d = distance from extreme compression fiber to centroid of nonprestressed tension reinforcement, in.
d' = distance from extreme compression fiber to centroid of compression reinforcement, in.
d_p = distance from extreme compression fiber to centroid of prestressed reinforcement
D = dead loads, or related internal moments and forces
e = base of Napierian logarithms
f_c' = specified compressive strength of concrete, psi
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi
f_{ci}' = compressive strength of concrete at time of initial prestress, psi
$\sqrt{f_{ci}'}$ = square root of compressive strength of concrete at time of initial prestress, psi
f_{pc} = average compressive stress in concrete due to effective prestress force only (after allowance for all prestress losses), psi
f_{ps} = stress in prestressed reinforcement at nominal strength, psi
f_{pu} = specified tensile strength of prestressing tendons, psi
f_{py} = specified yield strength of prestressing tendons, psi
f_r = modulus of rupture of concrete, psi
f_{se} = effective stress in prestressed reinforcement (after allowance for all prestress losses), psi
f_y = specified yield strength of nonprestressed reinforcement, psi
h = overall thickness of member, in.
K = wobble friction coefficient per foot of prestressing tendon
ℓ = length of span of two-way flat plates in direction parallel to that of the reinforcement being determined, in. See Eq. (18-8)
ℓ_x = length of prestressing tendon element from jacking end to any point x, ft. See Eq. (18-1) and (18-2)

CODE

L = live loads, or related internal moments and forces
N_c = tensile force in concrete due to unfactored dead load plus live load ($D + L$)
P_s = prestressing tendon force at jacking end
P_x = prestressing tendon force at any point x
α = total angular change of prestressing tendon profile in radians from tendon jacking end to any point x
β_1 = factor defined in 10.2.7.3
γ_p = factor for type of prestressing tendon
= 0.55 for f_{py}/f_{pu} not less than 0.80
= 0.40 for f_{py}/f_{pu} not less than 0.85
= 0.28 for f_{py}/f_{pu} not less than 0.90
μ = curvature friction coefficient
ρ = ratio of nonprestressed tension reinforcement
= A_s/bd
ρ' = ratio of compression reinforcement
= A_s'/bd
ρ_p = ratio of prestressed reinforcement
= A_{ps}/bd_p
ϕ = strength reduction factor. See 9.3
ω = $\rho f_y/f_c'$
ω' = $\rho' f_y/f_c'$
ω_p = $\rho_p f_{ps}/f_c'$
$\omega_w, \omega_{pw}, \omega_w'$ = reinforcement indices for flanged sections computed as for ω, ω_p, and ω' except that b shall be the web width, and reinforcement area shall be that required to develop compressive strength of web only

18.1 — Scope

18.1.1 — Provisions of Chapter 18 shall apply to members prestressed with wire, strands, or bars conforming to provisions for prestressing tendons in 3.5.5.

18.1.2 — All provisions of this code not specifically excluded, and not in conflict with provisions of Chapter 18, shall apply to prestressed concrete.

18.1.3 — The following provisions of this code shall not apply to prestressed concrete, except as specifically noted: Sections 7.6.5, 8.4, 8.10.2, 8.10.3, 8.10.4, 8.11, 10.3.2, 10.3.3, 10.5, 10.6, 10.9.1, and 10.9.2; Chapter 13; and Sections 14.3, 14.5, and 14.6.

COMMENTARY

R18.1 — Scope

R18.1.1 — The provisions of Chapter 18 were developed primarily for structural members such as slabs, beams, and columns which are commonly used in buildings. However, many of the provisions may be applied to other types of construction such as pressure vessels, pavements, pipes, and crossties. Application of the provisions is left to the judgment of the engineer in cases not specifically cited in the code.

R18.1.3 — Some sections of the code are excluded from use in the design of prestressed concrete for specific reasons. The following discussion provides explanation for such exclusions:

Section 7.6.5 — Section 7.6.5 of the code is excluded from application to prestressed concrete since the requirements for bonded reinforcement and unbonded tendons for cast-in-place members are provided in 18.9 and 18.12, respectively.

CODE

COMMENTARY

Section 8.4 — Section 8.4 of the code is excluded since moment redistribution for prestressed concrete is provided in 18.10.4.

Sections 8.10.2, 8.10.3, and 8.10.4 — The empirical provisions of 8.10.2, 8.10.3, and 8.10.4 for T-beams were developed for conventionally reinforced concrete and if applied to prestressed concrete would exclude many standard prestressed products in satisfactory use today. Hence, proof by experience permits variations.

By excluding 8.10.2, 8.10.3, and 8.10.4, no special requirements for prestressed concrete T-beams appear in the code. Instead, the determination of an effective width of flange is left to the experience and judgment of the engineer. Where possible, the flange widths in 8.10.2, 8.10.3, and 8.10.4 should be used unless experience has proven that variations are safe and satisfactory. It is not necessarily conservative in elastic analysis and design considerations to use the maximum flange width as permitted in 8.10.2.

Sections 8.10.1 and 8.10.5 provide general requirements for T-beams that are also applicable to prestressed concrete units. The spacing limitations for slab reinforcement are based on flange thickness, which for tapered flanges can be taken as the average thickness.

Section 8.11 — The empirical limits established for conventionally reinforced concrete joist floors are based on successful past performance of joist construction using "standard" joist forming systems. See R8.11. For prestressed joist construction, experience and judgment should be used. The provisions of 8.11 may be used as a guide.

Sections 10.3.2, 10.3.3, 10.5, 10.9.1, and 10.9.2 — For prestressed concrete, the limitations on reinforcement given in 10.3.2, 10.3.3, 10.5, 10.9.1, and 10.9.2 are replaced by those in 18.8, 18.9, and 18.11.2.

Section 10.6 — When originally prepared, the provisions of 10.6 for distribution of flexural reinforcement were not intended for prestressed concrete members. The behavior of a prestressed member is considerably different from that of a nonprestressed member. Experience and judgment must be used for proper distribution of reinforcement in a prestressed member.

Chapter 13 — The design of prestressed concrete slabs requires recognition of secondary moments induced by the undulating profile of the prestressing tendons. Also volume changes due to the prestressing force can create additional loads on the structure that are not adequately covered in Chapter 13. Because of these unique properties associated with prestressing, many of the design procedures of Chapter 13 are not appropriate for prestressed concrete structures and are replaced by the provisions of 18.12.

Sections 14.5 and 14.6 — The requirements for wall design in 14.5 and 14.6 are largely empirical, utilizing considerations not intended to apply to prestressed concrete.

CODE

18.2 — General

18.2.1 — Prestressed members shall meet the strength requirements specified in this code.

18.2.2 — Design of prestressed members shall be based on strength and on behavior at service conditions at all load stages that will be critical during the life of the structure from the time prestress is first applied.

18.2.3 — Stress concentrations due to prestressing shall be considered in design.

18.2.4 — Provisions shall be made for effects on adjoining construction of elastic and plastic deformations, deflections, changes in length, and rotations due to prestressing. Effects of temperature and shrinkage shall also be included.

18.2.5 — Possibility of buckling in a member between points where concrete and prestressing tendons are in contact and of buckling in thin webs and flanges shall be considered.

18.2.6 — In computing section properties prior to bonding of prestressing tendons, effect of loss of area due to open ducts shall be considered.

COMMENTARY

R18.2 — General

R18.2.1 and R18.2.2 — As has been past practice in the design of prestressed concrete, the design investigation should include all load stages that may be significant. The three major stages are: (1) jacking stage, or prestress transfer stage—when the tensile force in the prestressing tendons is transferred to the concrete and stress levels may be high relative to concrete strength, (2) service load stage—after long-term volume changes have occurred, and (3) the factored load stage—when the strength of the member is checked. There may be other load stages that require investigation. For example, if the cracking load is significant, this load stage may require study, or the handling and transporting stage may be critical.

From the standpoint of satisfactory behavior, the two stages of most importance are those for service load and factored load.

Service load stage refers to the loads defined in the general building code (without load factors), such as live load and dead load, while the factored load stage refers to loads multiplied by the appropriate load factors.

Section 18.3.2 provides assumptions that may be used for investigation at service loads and after transfer of the prestressing force.

R18.2.5 — This refers to the type of post-tensioning where the tendon makes contact with the prestressed concrete member intermittently. Precautions should be taken to prevent buckling of such members.

If the tendon is in complete contact with the member being prestressed, or is an unbonded tendon in a duct not excessively larger than the tendon, it is not possible to buckle the member under the prestressing force being introduced.

R18.2.6 — In considering the area of the open ducts, the critical sections should include those which have coupler sheaths which may be of a larger size than the duct containing the tendon. Also, in some instances, the trumpet or transition piece from the conduit to the anchorage may be of such a size as to create a critical section. If the effect of the open duct area on design is deemed negligible, section properties may be based on total area.

In pretensioned members and in post-tensioned members after grouting, section properties may be based on gross sections, net sections, or effective sections using transformed areas of bonded tendons and nonprestressed reinforcement.

CODE

18.3 — Design assumptions

18.3.1 — Strength design of prestressed members for flexure and axial loads shall be based on assumptions given in 10.2, except 10.2.4 shall apply only to reinforcement conforming to 3.5.3.

18.3.2 — For investigation of stresses at transfer of prestress, at service loads, and at cracking loads, straight-line theory shall be used with the following assumptions.

18.3.2.1 — Strains vary linearly with depth through entire load range.

18.3.2.2 — At cracked sections, concrete resists no tension.

18.4 — Permissible stresses in concrete — Flexural members

18.4.1 — Stresses in concrete immediately after prestress transfer (before time-dependent prestress losses) shall not exceed the following:

(a) Extreme fiber stress in compression......... $0.60f_{ci}'$

(b) Extreme fiber stress in tension except as permitted in (c) .... $3\sqrt{f_{ci}'}$

(c) Extreme fiber stress in tension at ends of simply supported members .... $6\sqrt{f_{ci}'}$

Where computed tensile stresses exceed these values, bonded auxiliary reinforcement (nonprestressed or prestressed) shall be provided in the tensile zone to resist the total tensile force in concrete computed with the assumption of an uncracked section.

18.4.2 — Stresses in concrete at service loads (after allowance for all prestress losses) shall not exceed the following:

(a) Extreme fiber stress in compression due to prestress plus sustained loads .... $0.45f_c'$

(b) Extreme fiber stress in compression due to prestress plus total load..................................... $0.60f_c'$

COMMENTARY

R18.4 — Permissible stresses in concrete — Flexural members

Permissible stresses in concrete are given to control serviceability. They do not ensure adequate structural strength, which must be checked in conformance with other code requirements.

R18.4.1 — The concrete stresses at this stage are caused by the force in the prestressing tendons at transfer reduced by the losses due to elastic shortening of the concrete, relaxation of the tendon, anchorage seating, and the stresses due to the weight of the member. Generally, shrinkage is not included at this stage. These stresses apply to both pretensioned and post-tensioned concrete with proper modifications of the losses at transfer.

R18.4.1(b) and (c) — The tension stress limits of $3\sqrt{f_{ci}'}$ and $6\sqrt{f_{ci}'}$ refer to tensile stress at locations other than the precompressed tensile zone. Where the tensile stresses exceed the permissible values, the total force in the tensile stress zone may be calculated and reinforcement proportioned on the basis of this force at a stress of $0.6f_y$, but not more than 30,000 psi. The effects of creep and shrinkage begin to reduce the tensile stress almost immediately; however, some tension remains in these areas after allowance is made for all prestress losses.

R18.4.2(a) and (b) — The compression stress limit of $0.45f_c'$ was conservatively established to decrease the probability of failure of prestressed concrete members due to repeated loads. In addition, the early code writers felt that this limit was reasonable to preclude excessive creep deformation. At higher values of stress, creep strains tend to increase more rapidly as applied stress increases. This is not consistent with the design assumption that creep strain is proportional to stress in calculating time-dependent camber and deflection and prestress losses.

CODE

(c) Extreme fiber stress in tension in precompressed tensile zone .... $6\sqrt{f_c'}$

(d) Extreme fiber stress in tension in precompressed tensile zone of members (except two-way slab systems), where analysis based on transformed cracked sections and on bilinear moment-deflection relationships shows that immediate and long-term deflections comply with requirements of 9.5.4, and where cover requirements comply with 7.7.3.2.......... $12\sqrt{f_c'}$

COMMENTARY

The change in allowable stress in the 1995 code recognizes that fatigue tests of prestressed concrete have shown that concrete failures are not the controlling criterion, and that designs with large transient live loads compared to sustained dead and live loads have been penalized by the previous single compression stress limit. Therefore, the new stress limit of $0.60f_c'$ permits a one-third increase in allowable compression stress for members subject to transient loads.

Sustained live load is any portion of the service live load which will be sustained for a sufficient period to cause significant time-dependent deflections. Thus, when sustained dead load and live loads are a large percentage of total service load, the $0.45f_c'$ limit of 18.4.2(a) may control. On the other hand, when a large portion of the total service load consists of a transient or temporary service live load, the increased stress limit of 18.4.2(b) may control.

The compression stress limit of $0.45f_c'$ for prestress plus sustained loads will continue to control the long-term behavior of prestressed members.

R18.4.2(c) — The precompressed tensile zone is that portion of the member cross section in which flexural tension occurs under dead and live loads. Prestressed concrete is usually designed so that the prestress force introduces compression into this zone, thus effectively reducing the magnitude of the tensile stress.

The permissible tensile stress of $6\sqrt{f_c'}$ is compatible with the concrete covers required by 7.7.3.1. For conditions of corrosive environments, defined as an environment in which chemical attack such as seawater, corrosive industrial atmosphere, sewer gas, or other highly corrosive environments are encountered, greater cover than that required by 7.7.3.1 should be used, in accordance with 7.7.3.2, and tension stresses reduced to eliminate possible cracking at service loads. The engineer must use judgment to determine the amount of increased cover and whether reduced tension stresses are required.

R18.4.2(c) and (d) — The permissible concrete tensile stress depends on whether or not enough bonded reinforcement is provided to control cracking. Such bonded reinforcement may consist of prestressed or nonprestressed tendons or of reinforcing bars. It should be noted that the control of cracking depends not only on the amount of reinforcement provided but also on its distribution over the tensile zone.

Because of the bonded reinforcement requirements of 18.9, it is considered that the behavior of segmental members generally will be comparable to that of similarly constructed monolithic concrete members. Therefore, the permissible tensile stress limits of 18.4.2(c) and 18.4.2(d) apply to both segmental and monolithic members. If deflections are important, the built-in cracks of segmental members should be considered in the computations.

CODE

18.4.3 — Permissible stresses in concrete of 18.4.1 and 18.4.2 shall be permitted to be exceeded if shown by test or analysis that performance will not be impaired.

18.5 — Permissible stresses in prestressing tendons

18.5.1 — Tensile stress in prestressing tendons shall not exceed the following:

(a) Due to tendon jacking force .... $0.94f_{py}$

but not greater than the lesser of $\mathbf{0.80}f_{pu}$ and the maximum value recommended by the manufacturer of prestressing tendons or anchorages.

(b) Immediately after prestress transfer .... $0.82f_{py}$

but not greater than $\mathbf{0.74}f_{pu}$.

COMMENTARY

R18.4.2(d) — The permissible tensile stress of $12\sqrt{f_c'}$ provides improved service load performance, especially when live loads are of a transient nature. To take advantage of the increased permissible stress, the engineer is required to increase the concrete protection on the reinforcement, as stipulated in 7.7.3.2, and to investigate the deflection characteristics of the member, particularly at the load where the member changes from uncracked behavior to cracked behavior.

The exclusion of two-way slab systems is based on Reference 18.1, which recommends that the permissible tension stress be not greater than $6\sqrt{f_c'}$ for design of prestressed concrete flat plates analyzed by the equivalent frame method or other approximate methods. For flat plate designs based on more exact analyses, or for other two-way slab systems rigorously analyzed and designed for strength and serviceability, the limiting stress may be exceeded in accordance with 18.4.3.

Reference 18.2 provides information on the use of bilinear moment-deflection relationships.

R18.4.3 — This section provides a mechanism whereby development of new products, materials, and techniques in prestressed concrete construction need not be inhibited by limits on stress which represented the most advanced requirements at the time the code provisions were adopted. Approvals for the design should be in accordance with 1.4 of the code.

R18.5 — Permissible stresses in prestressing tendons

The code does not distinguish between temporary and effective prestress tendon stresses. Only one limit on prestress tendon stress is provided because the initial tendon stress (immediately after transfer) can prevail for a considerable time, even after the structure has been put into service. This stress, therefore, must have an adequate safety factor under service conditions and cannot be considered as a temporary stress. Any subsequent decrease in tendon stress due to losses can only improve conditions and, hence, no limit on such stress decrease is provided in the code.

R18.5.1 — With the 1983 code edition, permissible stresses in tendons are revised to recognize the higher yield strength of low-relaxation wire and strand meeting the requirements of ASTM A 421 and A 416 of 3.5.5. For such tendons, it is more appropriate to specify permissible stresses in terms of specified minimum ASTM yield strength rather than specified minimum ASTM tensile strength. For the low-relaxation wire and strands, with f_{py} equal to $\mathbf{0.90}f_{pu}$, the $\mathbf{0.94}f_{py}$ and $\mathbf{0.82}f_{py}$ limits are equivalent to $\mathbf{0.85}f_{pu}$ and $\mathbf{0.74}f_{pu}$, respectively. In the 1986 revision and in the 1989 code, the maximum jacking stress for low-relaxation tendons was reduced to $\mathbf{0.80}f_{pu}$ to ensure closer compatibility with the maximum tendon stress value of $\mathbf{0.74}f_{pu}$ immediately after

CODE

(c) Post-tensioning tendons, at anchorages and couplers, immediately after tendon anchorage.... **$0.70f_{pu}$**

18.6 — Loss of prestress

18.6.1 — To determine effective prestress f_{se}, allowance for the following sources of loss of prestress shall be considered:

(a) Anchorage seating loss

(b) Elastic shortening of concrete

(c) Creep of concrete

(d) Shrinkage of concrete

(e) Relaxation of tendon stress

(f) Friction loss due to intended or unintended curvature in post-tensioning tendons.

18.6.2 — Friction loss in post-tensioning tendons

18.6.2.1 — Effect of friction loss in post-tensioning tendons shall be computed by

$$P_s = P_x e^{(K\ell_x + \mu\alpha)} \qquad (18\text{-}1)$$

When $(K\ell_x + \mu\alpha)$ is not greater than 0.3, effect of friction loss shall be permitted to be computed by

$$P_s = P_x(1 + K\ell_x + \mu\alpha) \qquad (18\text{-}2)$$

18.6.2.2—Friction loss shall be based on experimentally determined wobble K and curvature μ friction coefficients, and shall be verified during tendon stressing operations.

COMMENTARY

prestress transfer. The higher yield strength of the low-relaxation tendons does not change the effectiveness of tendon anchorages; thus, the permissible stress at post-tensioning anchorages (and couplers) is not increased above the previously permitted value of $0.70f_{pu}$. For ordinary tendons (wire, strands, and bars) with f_{py} equal to $0.85f_{pu}$, the $0.94f_{py}$ and $0.82f_{py}$ limits are equivalent to $0.80f_{pu}$ and $0.70f_{pu}$, respectively, the same as permitted in the 1977 code. For bar tendons with f_{py} equal to $0.80f_{pu}$, the same limits are equivalent to $0.75f_{pu}$ and $0.66f_{pu}$, respectively.

Because of the higher allowable initial stresses permitted since the 1983 code, final stresses can be greater. Designers should be concerned with setting a limit on final stress when the structure is subject to corrosive conditions or repeated loadings.

R18.6 — Loss of prestress

R18.6.1 — For an explanation of how to compute prestress losses, see References 18.3 through 18.6. Lump sum values of prestress losses for both pretensioned and post-tensioned members which were indicated in pre-1983 editions of the commentary are considered obsolete. Reasonably accurate estimates of prestress losses can be easily calculated in accordance with the recommendations in Reference 18.6 which include consideration of initial stress level ($0.7f_{pu}$ or higher), type of steel (stress-relieved or low-relaxation; wire, strand, or bar), exposure conditions, and type of construction (pretensioned, bonded post-tensioned, or unbonded post-tensioned).

Actual losses, greater or smaller than the computed values, have little effect on the design strength of the member, but affect service load behavior (deflections, camber, cracking load) and connections. At service loads, overestimation of prestress losses can be almost as detrimental as underestimation, since the former can result in excessive camber and horizontal movement.

R18.6.2 — Friction loss in post-tensioning tendons

The coefficients tabulated in Table R18.6.2 give a range that generally can be expected. Due to the many types of ducts, tendons, and wrapping materials available, these values can only serve as a guide. Where rigid conduit is used, the wobble coefficient K can be considered as zero. For large tendons in semirigid type conduit, the wobble factor can also be considered zero. Values of the coefficients to be used for the particular types of tendons and particular types of ducts should be obtained from the manufacturers of the tendons. An unrealistically low evaluation of the friction loss can lead to improper camber of the member and inadequate prestress. Overestimation of the friction may result in extra prestressing force if the estimated friction values are not attained in the field. This could lead to excessive camber and excessive shortening of a member. If the friction factors

CODE

18.6.2.3 — Values of wobble and curvature friction coefficients used in design shall be shown on design drawings.

18.6.3—Where loss of prestress in a member occurs due to connection of member to adjoining construction, such loss of prestress shall be allowed for in design.

18.7 — Flexural strength

18.7.1 — Design moment strength of flexural members shall be computed by the strength design methods of this code. For prestressing tendons, f_{ps} shall be substituted for f_y in strength computations.

18.7.2 — As an alternative to a more accurate determination of f_{ps} based on strain compatibility, the following approximate values of f_{ps} shall be used if f_{se} is not less than $0.5f_{pu}$.

COMMENTARY

TABLE R18.6.2—FRICTION COEFFICIENTS FOR POST-TENSIONED TENDONS FOR USE IN EQ. (18-1) OR (18-2)

			Wobble coefficient, K	Curvature coefficient, μ
		Wire tendons	0.0010-0.0015	0.15-0.25
		High-strength bars	0.0001-0.0006	0.08-0.30
		7-wire strand	0.0005-0.0020	0.15-0.25
Unbonded tendons	Mastic coated	Wire tendons	0.0010-0.0020	0.05-0.15
		7-wire strand	0.0010-0.0020	0.05-0.15
	Pre-greased	Wire tendons	0.0003-0.0020	0.05-0.15
		7-wire strand	0.0003-0.0020	0.05-0.15

are determined to be less than those assumed in the design, the tendon stressing should be adjusted to give only that prestressing force in the critical portions of the structure required by the design.

R18.6.2.3 — When the safety or serviceability of the structure may be involved, the acceptable range of tendon jacking forces or other limiting requirements should either be given or approved by the structural engineer in conformance with the permissible stresses of 18.4 and 18.5.

R18.7 — Flexural strength

R18.7.1 — Design moment strength of prestressed flexural members may be computed using strength equations similar to those for conventionally reinforced concrete members. Textbooks and ACI 318R-83[18.7] provide strength equations for rectangular and flanged sections, with tension reinforcement only and with tension and compression reinforcement. When part of the prestressed reinforcement is in the compression zone, a method based on applicable conditions of equilibrium and compatibility of strains at a factored load condition should be used.

For other cross sections, the design moment strength ϕM_n is computed by a general analysis based on stress and strain compatibility, using the stress-strain properties of the prestressing tendons and the assumptions given in 10.2.

R18.7.2 — Eq. (18-3) may underestimate the strength of beams with high percentages of reinforcement and, for more accurate evaluations of their strength, the strain compatibility and equilibrium method should be used. Use of Eq.

CODE

(a) For members with bonded prestressing tendons:

$$f_{ps} = f_{pu}\left\{1 - \frac{\gamma_p}{\beta_1}\left[\rho_p\frac{f_{pu}}{f_c'} + \frac{d}{d_p}(\omega - \omega')\right]\right\} \qquad (18\text{-}3)$$

If any compression reinforcement is taken into account when calculating f_{ps} by Eq. (18-3), the term

$$\left[\rho_p\frac{f_{pu}}{f_c'} + \frac{d}{d_p}(\omega - \omega')\right]$$

shall be taken not less than 0.17 and d' shall be no greater than $0.15d_p$.

(b) For members with unbonded prestressing tendons and with a span-to-depth ratio of 35 or less:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{100\rho_p} \qquad (18\text{-}4)$$

but f_{ps} in Eq. (18-4) shall not be taken greater than f_{py}, nor $(f_{se} + 60{,}000)$.

(c) For members with unbonded prestressing tendons and with a span-to-depth ratio greater than 35:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{300\rho_p} \qquad (18\text{-}5)$$

but f_{ps} in Eq. (18-5) shall not be taken greater than f_{py}, nor $(f_{se} + 30{,}000)$.

18.7.3 — Nonprestressed reinforcement conforming to 3.5.3, if used with prestressing tendons, shall be permitted to be considered to contribute to the tensile force and to be included in moment strength computations at a stress equal to the specified yield strength f_y. Other nonprestressed reinforcement shall be permitted to be included in strength computations only if a strain compatibility analysis is made to determine stresses in such reinforcement.

COMMENTARY

(18-3) is appropriate when all of the prestressed reinforcement is in the tension zone. When part of the prestressed reinforcement is in the compression zone, a strain compatibility and equilibrium method should be used.

By inclusion of the ω' term, Eq. (18-3) reflects the increased value of f_{ps} obtained when compression reinforcement is provided in a beam with a large reinforcement index. When the term $[\rho_p f_{pu}/f_c' + (d/d_p)(\omega - \omega')]$ in Eq. (18-3) is small, the neutral axis depth is small, hence the compressive reinforcement does not develop its yield strength and Eq. (18-3) becomes unconservative. This is the reason why the term $[\rho_p f_{pu}/f_c' + (d/d_p)(\omega - \omega')]$ in Eq. (18-3) may not be taken less than 0.17 if compression reinforcement is taken into account when computing f_{ps}. (Note that if the compression reinforcement is neglected when using Eq. (18-3), i.e.,ω' is taken as zero, then the term $[\rho_p f_{pu}/f_c' + (d/d_p)\omega]$ may be less than 0.17 and hence an increased and correct value of f_{ps} is obtained.)

When d' is large, the strain in compression reinforcement can be considerably less than its yield strain. In such a case, the compression reinforcement does not influence f_{ps} as favorably as implied by Eq. (18-3). It is for this reason that the applicability of Eq. (18-3) is limited to beams in which d' is less than or equal to $0.15d_p$.

The term $[\rho_p f_{pu}/f_c' + (d/d_p)(\omega - \omega')$ in Eq. (18-3) may also be written $[\rho_p f_{pu}/f_c' + A_s f_y/(bd_p f_c') - A_s' f_y/(bd_p f_c')]$. This form may sometimes be more conveniently used, e.g., when there is no unprestressed tension reinforcement.

Eq. (18-5) reflects results of tests on members with unbonded tendons and span-to-depth ratios greater than 35 (one-way slabs, flat plates, and flat slabs).[18.8] These tests also indicate that Eq. (18-4), formerly used for all span-depth ratios, would overestimate the amount of stress increase in such members. Although these same tests indicate that the moment strength of those shallow members designed using Eq. (18-4) meets the factored load strength requirements, this result reflects the code requirements for minimum bonded reinforcement, as well as the limitation on concrete tensile stress which often controls the amount of prestressing force provided.

CODE

18.8 — Limits for reinforcement of flexural members

18.8.1 — Ratio of prestressed and nonprestressed reinforcement used for computation of moment strength of a member, except as provided in 18.8.2, shall be such that ω_p, $[\omega_p + (d/d_p)(\omega - \omega')]$, or $[\omega_{pw} + (d/d_p)(\omega_w - \omega_w')]$ is not greater than $0.36\,\beta_1$.

18.8.2 — When a reinforcement ratio in excess of that specified in 18.8.1 is provided, design moment strength shall not exceed the moment strength based on the compression portion of the moment couple.

18.8.3 — Total amount of prestressed and nonprestressed reinforcement shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture f_r specified in 9.5.2.3, except for flexural members with shear and flexural strength at least twice that required by 9.2.

18.9 — Minimum bonded reinforcement

18.9.1 — A minimum area of bonded reinforcement shall be provided in all flexural members with unbonded prestressing tendons as required by 18.9.2 and 18.9.3.

18.9.2 — Except as provided in 18.9.3, minimum area of bonded reinforcement shall be computed by

$$A_s = 0.004A \qquad (18\text{-}6)$$

18.9.2.1 — Bonded reinforcement required by Eq. (18-6) shall be uniformly distributed over precompressed tensile zone as close as practicable to extreme tension fiber.

COMMENTARY

R18.8 — Limits for reinforcement of flexural members

R18.8.1 — It can be shown that the terms ω_p, $[\omega_p + (d/d_p)(\omega - \omega')]$, and $[\omega_{pw} + (d/d_p)(\omega_w - \omega_w')]$ are each equal to $0.85\,a/d_p$, where a is the depth of the equivalent rectangular stress distribution for the section under consideration, as defined in 10.2.7.1. Use of this relationship can simplify the calculations necessary to check compliance with 18.8.1.

R18.8.2 — Design moment strength of overreinforced members may be computed using strength equations similar to those for conventionally reinforced concrete members. Textbooks and ACI 318R-83[18.7] provide strength equations for rectangular and flanged sections.

R18.8.3 — This provision is a precaution against abrupt flexural failure developing immediately after cracking. A flexural member designed according to code provisions requires considerable additional load beyond cracking to reach its flexural strength. Thus, considerable deflection would warn that the member strength is approaching. If the flexural strength should be reached shortly after cracking, the warning deflection would not occur.

R18.9 — Minimum bonded reinforcement

R18.9.1 — Some bonded reinforcement is required by the code in members prestressed with unbonded tendons to ensure flexural performance at ultimate member strength, rather than behavior as a tied arch, and to control cracking at service load when tensile stresses exceed the modulus of rupture of the concrete. Providing minimum bonded reinforcement, as specified in 18.9, helps to ensure adequate performance.

Research has shown that unbonded post-tensioned members do not inherently provide large capacity for energy dissipation under severe earthquake loadings because the member response is primarily elastic. For this reason, unbonded post-tensioned structural elements reinforced in accordance with the provisions of this section should be assumed to carry only vertical loads and to act as horizontal diaphragms between energy dissipating elements under earthquake loadings of the magnitude defined in 21.2.1.1. The minimum bonded reinforcement areas required by Eq. (18-6) and (18-8) are absolute minimum areas independent of grade of steel or design yield strength.

R18.9.2 — The minimum amount of bonded reinforcement for members other than two-way flat plates is based on research comparing the behavior of bonded and unbonded post-tensioned beams.[18.9] Although research is limited for members other than beams and flat plates, it is advisable to apply the provisions of 18.9.2 to beams and slab systems not specifically reported in Reference 18.9. The need for applying Eq. (18-6) to two-way flat plates has not been sub-

CODE

18.9.2.2 — Bonded reinforcement shall be required regardless of service load stress conditions.

18.9.3 — For two-way flat plates, defined as solid slabs of uniform thickness, minimum area and distribution of bonded reinforcement shall be as follows:

18.9.3.1 — Bonded reinforcement shall not be required in positive moment areas where computed tensile stress in concrete at service load (after allowance for all prestress losses) does not exceed $2\sqrt{f_c'}$.

18.9.3.2 — In positive moment areas where computed tensile stress in concrete at service load exceeds $2\sqrt{f_c'}$, minimum area of bonded reinforcement shall be computed by

$$A_s = \frac{N_c}{0.5 f_y} \qquad (18\text{-}7)$$

where design yield strength f_y shall not exceed 60,000 psi. Bonded reinforcement shall be uniformly distributed over precompressed tensile zone as close as practicable to extreme tension fiber.

18.9.3.3 — In negative moment areas at column supports, minimum area of bonded reinforcement in each direction shall be computed by

$$A_s = 0.00075 h \ell \qquad (18\text{-}8)$$

where ℓ is length of span in direction parallel to that of the reinforcement being determined. Bonded reinforcement required by Eq. (18-8) shall be distributed within a slab width between lines that are **1.5*h*** outside opposite faces of the column support. At least four bars or wires shall be provided in each direction. Spacing of bonded reinforcement shall not exceed 12 in.

18.9.4 — Minimum length of bonded reinforcement required by 18.9.2 and 18.9.3 shall be as follows:

18.9.4.1 — In positive moment areas, minimum length of bonded reinforcement shall be one-third the clear span length and centered in positive moment area.

COMMENTARY

stantiated by test data and, therefore, the requirements originally contained in ACI 318-71 were subsequently modified in the 1977 code to reflect this information.

R18.9.3 — The minimum amount of bonded reinforcement in two-way flat plates is based on reports by ACI-ASCE Committee 423.[18.3,18.10] Limited research available for two-way flat slabs with drop panels[18.11] or waffle slabs[18.12] indicates that behavior of these particular systems is similar to the behavior of flat plates. However, until more complete information is available, 18.9.3 should be applied only to two-way flat plates (solid slabs of uniform thickness) and 18.9.2 should be applied to all other two-way slab systems.

R18.9.3.1 — For usual loads and span lengths, flat plate tests summarized in the Committee 423 report[18.3] and experience since the 1963 ACI Building Code was adopted indicate satisfactory performance without bonded reinforcement.

R18.9.3.2 — In positive moment areas, where tensile stresses are between $2\sqrt{f_c'}$ and $6\sqrt{f_c'}$, a minimum bonded reinforcement area proportioned according to Eq. (18-7) is required.The tensile force N_c is computed at service load on the basis of an uncracked, homogeneous section.

R18.9.3.3 — Research evaluated by ACI-ASCE Committee 423[18.3] shows that bonded reinforcement in negative moment regions of two-way flat plates, proportioned on the basis of 0.15 percent of the cross-sectional area of the column strip, provides adequate crack control and sufficient ductility. Eq. (18-8) is modified to require the larger amount of bonded reinforcement to be placed in the direction of the larger span at supports of rectangular panels. Concentration of this reinforcement in the top of the slab directly over and immediately adjacent to the column is important. Research also shows that where low tensile stresses occur at service load, satisfactory behavior has been achieved at factored load without bonded reinforcement. However, current practice calls for the code specified minimum bonded reinforcement regardless of service load stress levels to help ensure flexural continuity and ductility, and to control cracking due to overload, temperature, or shrinkage.

R18.9.4 — Bonded reinforcement should be adequately anchored to develop factored load forces. The requirements of Chapter 12 will ensure that bonded reinforcement required for flexural strength under factored loads in accordance with 18.7.3, or for tensile stress conditions at service load in accordance with 18.9.3.2, will be adequately anchored to develop tension or compression forces. For

CODE

18.9.4.2 — In negative moment areas, bonded reinforcement shall extend one-sixth the clear span on each side of support.

18.9.4.3 — Where bonded reinforcement is provided for design moment strength in accordance with 18.7.3, or for tensile stress conditions in accordance with 18.9.3.2, minimum length also shall conform to provisions of Chapter 12.

18.10 — Statically indeterminate structures

18.10.1 — Frames and continuous construction of prestressed concrete shall be designed for satisfactory performance at service load conditions and for adequate strength.

18.10.2 — Performance at service load conditions shall be determined by elastic analysis, considering reactions, moments, shears, and axial forces produced by prestressing, creep, shrinkage, temperature change, axial deformation, restraint of attached structural elements, and foundation settlement.

18.10.3 — Moments to be used to compute required strength shall be the sum of the moments due to reactions induced by prestressing (with a load factor of 1.0) and the moments due to factored loads. Adjustment of the sum of these moments shall be permitted as allowed in 18.10.4.

18.10.4 — Redistribution of negative moments in continuous prestressed flexural members

18.10.4.1 — Where bonded reinforcement is provided at supports in accordance with 18.9.2, negative

COMMENTARY

bonded reinforcement required by 18.9.2 or 18.9.3.3, but not required for flexural strength in accordance with 18.7.3, the minimum lengths apply. Research[18.1] on continuous spans shows that these minimum lengths provide adequate behavior under service load and factored load conditions.

R18.10 — Statically indeterminate structures

R18.10.3 — For statically indeterminate structures, the moments due to reactions induced by prestressing forces, generally referred to as secondary moments, are significant in both the elastic and inelastic states. When hinges and full redistribution of moments occur to create a statically determinate structure, secondary moments disappear. However, the elastic deformations caused by a nonconcordant tendon change the amount of inelastic rotation required to obtain a given amount of moment redistribution. Conversely, for a beam with a given inelastic rotational capacity, the amount by which the moment at the support may be varied is changed by an amount equal to the secondary moment at the support due to prestressing. Thus, the code requires that secondary moments be included in determining design moments.

To determine the moments used in design, the order of calculation should be: (a) determine moments due to dead and live load; (b) modify by algebraic addition of secondary moments; (c) redistribute as permitted. A positive secondary moment at the support caused by a tendon transformed downward from a concordant profile will, therefore, reduce the negative moments near the supports and increase the positive moments in the midspan regions. A tendon that is transformed upward will have the reverse effect.

R18.10.4 — Redistribution of negative moments in continuous prestressed flexural members

As member strength is approached, inelastic behavior at some sections can result in a redistribution of moments in

CODE

moments calculated by elastic theory for any assumed loading arrangement shall be permitted to be increased or decreased by not more than

$$20\left[1-\frac{\omega_p+\frac{d}{d_p}(\omega-\omega')}{0.36\beta_1}\right]\text{ percent}$$

18.10.4.2 — The modified negative moments shall be used for calculating moments at sections within spans for the same loading arrangement.

18.10.4.3 — Redistribution of negative moments shall be made only when the section at which moment is reduced is so designed that ω_p, $[\omega_p + (d/d_p)(\omega - \omega')]$, or $[\omega_{pw} + (d/d_p)(\omega_w - \omega'_w)]$, whichever is applicable, is not greater than $0.24\,\beta_1$.

18.11 — Compression members — Combined flexure and axial loads

18.11.1 — Prestressed concrete members subject to combined flexure and axial load, with or without nonprestressed reinforcement, shall be proportioned by the strength design methods of this code for members without prestressing. Effects of prestress, creep, shrinkage, and temperature change shall be included.

18.11.2 — Limits for reinforcement of prestressed compression members

18.11.2.1 — Members with average prestress f_{pc} less than 225 psi shall have minimum reinforcement in accordance with 7.10, 10.9.1 and 10.9.2 for columns, or 14.3 for walls.

18.11.2.2—Except for walls, members with average

COMMENTARY

prestressed concrete beams. Recognition of this behavior can be advantageous in design under certain circumstances. A rigorous design method for moment redistribution is quite complex. However, recognition of moment redistribution can be accomplished with the simple method of permitting a reasonable adjustment of the sum of the elastically calculated factored gravity load moments and the unfactored secondary moments due to prestress. The amount of adjustment must be kept within predetermined safe limits.

The amount of redistribution allowed depends on the ability of the critical sections to deform inelastically by a sufficient amount. Serviceability under service loads is taken care of by the limiting stresses of 18.4. The choice of $0.24\,\beta_1$ as the largest tension reinforcement index, ω_p, $[\omega_p + (d/d_p)(\omega - \omega')]$, or $[\omega_{pw} + (d/d_p)(\omega_w - \omega'_w)]$, for which redistribution of moments is allowed, is in agreement with the requirements for conventionally reinforced concrete of $0.5\rho_b$ stated in 8.4.

It can be shown that the terms ω_p, $[\omega_p + (d/d_p)(\omega - \omega')]$, and $[\omega_{pw} + (d/d_p)(\omega_w - \omega'_w)]$ which appear in 18.10.4.1 and 18.10.4.3 are each equal to $0.85a/d_p$, where a is the depth of the equivalent rectangular stress distribution for the section under consideration, as defined in 10.2.7.1. Use of this relationship can simplify the calculations necessary to determine the amount of moment redistribution permitted by 18.10.4.1 and to check compliance with the limitation on flexural reinforcement contained in 18.10.4.3.

For the moment redistribution principles of 18.10.4 to be applicable to beams with unbonded tendons, it is necessary that such beams contain sufficient bonded reinforcement to ensure they will act as beams after cracking and not as a series of tied arches. The minimum bonded reinforcement requirements of 18.9 will serve this purpose.

R18.11 — Compression members — Combined flexure and axial loads

R18.11.2 — Limits for reinforcement of prestressed compression members

CODE

prestress f_{pc} equal to or greater than 225 psi shall have all prestressing tendons enclosed by spirals or lateral ties in accordance with the following:

(a) Spirals shall conform to 7.10.4.

(b) Lateral ties shall be at least No. 3 in size or welded wire fabric of equivalent area, and spaced vertically not to exceed 48 tie bar or wire diameters, or least dimension of compression member.

(c) Ties shall be located vertically not more than half a tie spacing above top of footing or slab in any story, and shall be spaced as provided herein to not more than half a tie spacing below lowest horizontal reinforcement in members supported above.

(d) Where beams or brackets frame into all sides of a column, it shall be permitted to terminate ties not more than 3 in. below lowest reinforcement in such beams or brackets.

18.11.2.3 — For walls with average prestress f_{pc} equal to or greater than 225 psi, minimum reinforcement required by 14.3 shall not apply where structural analysis shows adequate strength and stability.

18.12 — Slab systems

18.12.1 — Factored moments and shears in prestressed slab systems reinforced for flexure in more than one direction shall be determined in accordance with provisions of 13.7 (excluding 13.7.7.4 and 13.7.7.5), or by more detailed design procedures.

18.12.2 — Moment strength of prestressed slabs at every section shall be at least equal to the required strength considering 9.2, 9.3, 18.10.3, and 18.10.4. Shear strength of prestressed slabs at columns shall be at least equal to the required strength considering 9.2, 9.3, 11.1, 11.12.2, and 11.12.6.2.

COMMENTARY

R18.11.2.3 — The minimum amounts of reinforcement, specified in 14.3 for walls, need not apply to prestressed concrete walls, provided the average prestress is 225 psi or greater and a complete structural analysis is made to show adequate strength and stability with lower amounts of reinforcement.

R18.12 — Slab systems

R18.12.1 — Use of the equivalent frame method of analysis (see 13.7) or more precise design procedures is required for determination of both service and factored moments and shears for prestressed slab systems. The equivalent frame method of analysis has been shown by tests of large structural models to satisfactorily predict factored moments and shears in prestressed slab systems. (See References 18.13, 18.14, 18.15, 18.16, 18.17, and 18.18.) The referenced research also shows that analysis using prismatic sections or other approximations of stiffness may provide erroneous results on the unsafe side. Section 13.7.7.4 is excluded from application to prestressed slab systems because it relates to reinforced slabs designed by the direct design method, and because moment redistribution for prestressed slabs is covered in 18.10.4. Section 13.7.7.5 is excluded from application to prestressed slab systems because the distribution of moments between column strips and middle strips required by 13.7.7.5 is based on tests for reinforced concrete slabs. Simplified methods of analysis using average coefficients do not apply to prestressed concrete slab systems.

R18.12.2 — Tests indicate that the moment and shear strength of prestressed slabs is controlled by total tendon strength and by the amount and location of nonprestressed reinforcement, rather than by tendon distribution. (See References 18.13, 18.14, 18.15, 18.16, 18.17, and 18.18.)

CODE

18.12.3 — At service load conditions, all serviceability limitations, including specified limits on deflections, shall be met, with appropriate consideration of the factors listed in 18.10.2.

18.12.4 — For normal live loads and loads uniformly distributed, spacing of prestressing tendons or groups of tendons in one direction shall not exceed 8 times the slab thickness, nor 5 ft. Spacing of tendons also shall provide a minimum average prestress (after allowance for all prestress losses) of 125 psi on the slab section tributary to the tendon or tendon group. A minimum of two tendons shall be provided in each direction through the critical shear section over columns. Special consideration of tendon spacing shall be provided for slabs with concentrated loads.

18.12.5 — In slabs with unbonded prestressing tendons, bonded reinforcement shall be provided in accordance with 18.9.3 and 18.9.4.

18.12.6 — In lift slabs, bonded bottom reinforcement shall be detailed in accordance with 13.3.8.6.

18.13 — Tendon anchorage zones

18.13.1 — Reinforcement shall be provided where required in tendon anchorage zones to resist bursting, splitting, and spalling forces induced by tendon anchorages. Regions of abrupt change in section shall be adequately reinforced.

18.13.2 — End blocks shall be provided where required for support bearing or for distribution of concentrated prestressing forces.

18.13.3 — Post-tensioning anchorages and supporting concrete shall be designed to resist maximum jacking force for strength of concrete at time of prestressing.

18.13.4 — Post-tensioning anchorage zones shall be designed to develop the guaranteed ultimate tensile strength of prestressing tendons using a strength reduction factor ϕ of 0.90 for concrete.

COMMENTARY

R18.12.3 — For prestressed flat slabs continuous over two or more spans in each direction, the span-thickness ratio generally should not exceed 42 for floors and 48 for roofs; these limits may be increased to 48 and 52, respectively, if calculations verify that both short- and long-term deflection, camber, and vibration frequency and amplitude are not objectionable.

Short- and long-term deflection and camber should be computed and checked against the requirements of serviceability of the particular usage of the structure.

The maximum length of a slab between construction joints is generally limited to 100 to 150 ft to minimize the effects of slab shortening, and to avoid excessive loss of prestress due to friction.

R18.12.4 — This section provides specific guidance concerning tendon distribution that will permit the use of banded tendon distributions in one direction. This method of tendon distribution has been shown to provide satisfactory performance by structural research.

R18.13 — Tendon anchorage zones

Because the actual stresses are quite complicated around post-tensioning anchorages, a refined strength analysis should be used whenever possible, with ϕ being taken as 0.9.

Appropriate formulas from Reference 18.19 and ACI 318R-83[18.7] may be used as a guide to size tendon anchorages when experimental data or more refined analysis are not available. Additional guidance on design and details for post-tensioning anchorage zones is given in Reference 18.20.

CODE

18.14 — Corrosion protection for unbonded prestressing tendons

18.14.1 — Unbonded tendons shall be completely coated with suitable material to ensure corrosion protection.

18.14.2 — Tendon covering shall be continuous over entire length to be unbonded, and shall prevent intrusion of cement paste or loss of coating materials during concrete placement.

18.14.3 — Unbonded single strand tendons shall be protected against corrosion in accordance with "Specification for Unbonded Single Strand Tendons," revised July 1993, published by the Post-Tensioning Institute.

18.15 — Post-tensioning ducts

18.15.1 — Ducts for grouted or unbonded tendons shall be mortar-tight and nonreactive with concrete, tendons, or filler material.

18.15.2 — Ducts for grouted single wire, strand, or bar tendons shall have an inside diameter at least $^1/_4$ in. larger than tendon diameter.

18.15.3 — Ducts for grouted multiple wire, strand, or bar tendons shall have an inside cross-sectional area at least two times area of tendons.

18.15.4 — Ducts shall be maintained free of water if members to be grouted are exposed to temperatures below freezing prior to grouting.

18.16 — Grout for bonded prestressing tendons

18.16.1 — Grout shall consist of portland cement and water; or portland cement, sand, and water.

COMMENTARY

R18.14 — Corrosion protection for unbonded prestressing tendons

R18.14.1 — Suitable material for corrosion protection of unbonded tendons should have the properties identified in Section 5.1 of Reference 18.21.

R18.14.3 — Corrosion protection requirements for unbonded single strand tendons in accordance with the Post-Tensioning Institute's "Specification for Unbonded Single Strand Tendons" were added in ACI 318-89 to the general provisions that appeared in previous editions. That specification included additional corrosion protective measures for single strand tendons used in corrosive environments. A revised and updated report,[18.21] published by the Post-Tensioning Institute in July 1993, is to be used as the guide for corrosion protection of unbonded single strand tendons.

R18.16 — Grout for bonded prestressing tendons

Grout is the means by which bond is provided between the post-tensioning tendons and the concrete and by which corrosion protection of the tendons is assured. Proper grout and grouting procedures, therefore, play an important part in post-tensioned construction.[18.22,18.23]

Past success with grout for bonded prestressing tendons has been with portland cement as the cementing material. A blanket endorsement of all cementitious materials (defined in 2.1) for use with this grout is deemed inappropriate because of a lack of experience or tests with cementitious materials other than portland cement and a concern that some cementitious materials might introduce chemicals

CODE

18.16.2 — Materials for grout shall conform to the following:

18.16.2.1 — Portland cement shall conform to 3.2.

18.16.2.2 — Water shall conform to 3.4.

18.16.2.3 — Sand, if used, shall conform to "Standard Specification for Aggregate for Masonry Mortar" (ASTM C 144) except that gradation shall be permitted to be modified as necessary to obtain satisfactory workability.

18.16.2.4 — Admixtures conforming to 3.6 and known to have no injurious effects on grout, steel, or concrete shall be permitted. Calcium chloride shall not be used.

18.16.3 — Selection of grout proportions

18.16.3.1 — Proportions of materials for grout shall be based on either of the following:

(a) Results of tests on fresh and hardened grout prior to beginning grouting operations, or

(b) Prior documented experience with similar materials and equipment and under comparable field conditions.

18.16.3.2 — Cement used in the work shall correspond to that on which selection of grout proportions was based.

18.16.3.3 — Water content shall be minimum necessary for proper pumping of grout; however, water-cement ratio shall not exceed 0.45 by weight.

18.16.3.4 — Water shall not be added to increase grout flowability that has been decreased by delayed use of grout.

18.16.4 — Mixing and pumping grout

18.16.4.1 — Grout shall be mixed in equipment capable of continuous mechanical mixing and agitation that will produce uniform distribution of materials, passed through screens, and pumped in a manner that will completely fill tendon ducts.

18.16.4.2 — Temperature of members at time of grouting shall be above 35 F and shall be maintained above 35 F until field-cured 2-in. cubes of grout reach a minimum compressive strength of 800 psi.

COMMENTARY

listed as harmful to tendons in R18.16.2. Thus, "portland cement" in 18.16.1 and "water-cement ratio" in 18.16.3.3 are retained in this edition of the code.

R18.16.2 — The limitations on admixtures in 3.6 apply to grout. Substances known to be harmful to prestressing tendons, grout, or concrete are chlorides, fluorides, sulfites, and nitrates. Aluminum powder or other expansive admixtures, when approved, should produce an unconfined expansion of 5 to 10 percent. Neat cement grout is used in almost all building construction. Only with large ducts having large void areas should the advantages of using finely graded sand in the grout be considered.

R18.16.3 — Selection of grout proportions

Grout proportioned in accordance with these provisions will generally lead to 7-day compressive strength on standard 2-in. cubes in excess of 2500 psi and 28-day strengths of about 4000 psi. The handling and placing properties of grout are usually given more consideration than strength when designing grout mixtures.

R18.16.4 — Mixing and pumping grout

In an ambient temperature of 35 F, grout with an initial minimum temperature of 60 F may require as much as 5 days to reach 800 psi. A minimum grout temperature of 60 F is suggested because it is consistent with the recommended minimum temperature for concrete placed at an ambient temperature of 35 F. Quickset grouts, when approved, may require shorter periods of protection and the recommendations of the suppliers should be followed. Test cubes should be cured under temperature and moisture conditions as close as possible to those of the grout in the member. Grout temperatures in excess of 90 F will lead to difficulties in pumping.

CODE

18.16.4.3 — Grout temperatures shall not be above 90 F during mixing and pumping.

18.17 — Protection for prestressing tendons

Burning or welding operations in vicinity of prestressing tendons shall be carefully performed, so that tendons are not subject to excessive temperatures, welding sparks, or ground currents.

18.18 — Application and measurement of prestressing force

18.18.1 — Prestressing force shall be determined by both of the following methods:

(a) Measurement of tendon elongation. Required elongation shall be determined from average load-elongation curves for the prestressing tendons used.

(b) Observation of jacking force on a calibrated gage or load cell or by use of a calibrated dynamometer.

Cause of any difference in force determination between (a) and (b) that exceeds 5 percent for pretensioned elements or 7 percent for post-tensioned construction shall be ascertained and corrected.

18.18.2 — Where transfer of force from bulkheads of pretensioning bed to concrete is accomplished by flame cutting prestressing tendons, cutting points and cutting sequence shall be predetermined to avoid undesired temporary stresses.

18.18.3 — Long lengths of exposed pretensioned strand shall be cut near the member to minimize shock to concrete.

18.18.4 — Total loss of prestress due to unreplaced broken tendons shall not exceed 2 percent of total prestress.

COMMENTARY

R18.18 — Application and measurement of prestressing force

R18.18.1 — Elongation measurements for prestressed elements should be in accordance with the procedures outlined in the "Manual for Quality Control for Plants and Production of Precast and Prestressed Concrete Products," published by the Precast/Prestressed Concrete Institute.[18.24]

ACI 318-89, 18.18.1, was revised to permit 7 percent tolerance in tendon force determined by gage pressure and elongation measurements for post-tensioned construction. Elongation measurements for post-tensioned construction are affected by several factors that are less significant, or that do not exist, for pretensioned elements. The friction along post-tensioning tendons may be affected to varying degrees by placing tolerances and small irregularities in profile due to concrete placement. The friction coefficients between the tendons and the duct are also subject to variation. The 5 percent tolerance that has appeared in the code since ACI 318-63 was proposed by ACI-ASCE Committee 423 in 1958,[18.3] and primarily reflected experience with production of pretensioned concrete elements. Since the tendons for pretensioned elements are usually stressed in air with minimal friction effects, the 5 percent tolerance for such elements was retained.

R18.18.4 — This provision applies to all prestressed concrete members. For cast-in-place post-tensioned slab systems, a "member" should be that portion considered as an element in the design, such as the joist and effective slab width in one-way joist systems, or the column strip or middle strip in two-way flat plate systems.

CODE

18.19 — Post-tensioning anchorages and couplers

18.19.1 — Anchorages and couplers for bonded and unbonded prestressing tendons shall develop at least 95 percent of the specified breaking strength of the tendons, when tested in an unbonded condition, without exceeding anticipated set. For bonded tendons, anchorages and couplers shall be located so that 100 percent of the specified breaking strength of the tendons shall be developed at critical sections after tendons are bonded in the member.

18.19.2 — Couplers shall be placed in areas approved by the engineer and enclosed in housing long enough to permit necessary movements.

18.19.3 — In unbonded construction subject to repetitive loads, special attention shall be given to the possibility of fatigue in anchorages and couplers.

18.19.4 — Anchorages, couplers, and end fittings shall be permanently protected against corrosion.

COMMENTARY

R18.19 — Post-tensioning anchorages and couplers

R18.19.1 — In the 1986 interim code provisions, the separate provisions for strength of unbonded and bonded tendon anchorages and couplers presented in 18.19.1 and 18.19.2 of ACI 318-83 were combined into a single revised 18.19.1 covering anchorages and couplers for both unbonded and bonded tendons. Since the 1989 revision, the required strength of the tendon-anchorage or tendon-coupler assemblies for both unbonded and bonded tendons, when tested in an unbonded state, is based on 95 percent of the specified breaking strength of the tendon material in the test. The tendon material must comply with the minimum provisions of the applicable ASTM specifications as outlined in 3.5.5. The specified strength of anchorages and couplers exceeds the maximum design strength of the tendons by a substantial margin, and, at the same time, recognizes the stress-riser effects associated with most available post-tensioning anchorages and couplers. Anchorage and coupler strength must be attained with a minimum amount of permanent deformation and successive set, recognizing that some deformation and set will occur in testing to failure. Tendon assemblies should conform to the 2 percent elongation requirements in ACI 301[18.25] and industry recommendations.[18.19] Anchorages and couplers for bonded tendons that develop less than 100 percent of the specified breaking strength of the tendon should be used only where the bond transfer length between the anchorage or coupler and critical sections equals or exceeds that required to develop the tendon strength. This bond length may be calculated by the results of tests of bond characteristics of untensioned prestressing strand,[18.26] or by bond tests on other tendon materials, as appropriate.

R18.19.3 — For a more complete discussion on fatigue loading see Reference 18.27.

For detailed recommendations on tests for static and cyclic loading conditions for tendons and anchorage fittings of unbonded tendons, see Section 4.1.3 of Reference 18.10, and Section 15.2.2 of Reference 18.25.

R18.19.4 — For recommendations regarding protection see Sections 4.2 and 4.3 of Reference 18.10, and Sections 3.4, 3.6, 5, 6, and 8.3 of Reference 18.21.

CHAPTER 19 — SHELLS AND FOLDED PLATE MEMBERS

CODE

COMMENTARY

19.0 — Notation

E_c = modulus of elasticity of concrete, psi. See 8.5.1
f_c' = specified compressive strength of concrete, psi
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi
f_y = specified yield strength of nonprestressed reinforcement, psi
h = thickness of shell or folded plate, in.
ℓ_d = development length, in.
ϕ = strength reduction factor. See 9.3

19.1 — Scope and definitions

19.1.1 — Provisions of Chapter 19 shall apply to thin shell and folded plate concrete structures, including ribs and edge members.

19.1.2 — All provisions of this code not specifically excluded, and not in conflict with provisions of Chapter 19 shall apply to thin-shell structures.

19.1.3 — ***Thin shells*** — three-dimensional spatial structures made up of one or more curved slabs or

19.1 — Scope and definitions

This code and commentary provides building code information on the design, analysis, and construction of concrete thin shells and folded plates. The process began in 1964 with the publication of a practice and commentary by ACI Committee 334,[19.1] and continued with the inclusion of Chapter 19 in ACI Building Code ACI 318-71 and in later editions. The revision of ACI 334R.1 in 1982 reflected additional experience in design, analysis, and construction gained since the earlier publications, and was influenced by the publication of the **"Recommendations for Reinforced Concrete Shells and Folded Plates"** of the International Association for Shell and Spatial Structures (IASS) in 1979.[19.2]

Since Chapter 19 applies to concrete thin shells and folded plates of all shapes, extensive discussion of their design, analysis, and construction in the commentary is not possible. Additional information can be obtained from the references listed for this chapter, which are provided for the assistance of the designer. They are not an official part of the code. The designer is responsible for their interpretation and use. Performance of shells and folded plates requires special attention to detail.[19.3]

R19.1.1 — Discussion of the application of thin shells in special structures such as cooling towers and circular prestressed concrete tanks may be found in the reports of ACI-ASCE Committee 334[19.4] and ACI Committee 344.[19.5]

R19.1.3 — Common types of thin shells are domes (surfaces of revolution),[19.6,19.7] cylindrical shells,[19.7] barrel

CODE

folded plates whose thicknesses are small compared to their other dimensions. Thin shells are characterized by their three-dimensional load-carrying behavior which is determined by the geometry of their forms, by the manner in which they are supported, and by the nature of the applied load.

19.1.4 — ***Folded Plates*** — a special class of shell structures formed by joining flat, thin slabs along their edges to create a three-dimensional spatial structure.

19.1.5 — ***Ribbed shells*** — spatial structures with material placed primarily along certain preferred rib lines, with the area between the ribs filled with thin slabs or left open.

19.1.6 — ***Auxiliary members*** — ribs or edge beams which serve to strengthen, stiffen, and/or support the shell; usually, auxiliary members act jointly with the shell.

19.1.7 — ***Elastic analysis*** — an analysis of deformations and internal forces based on equilibrium, compatibility of strains, and assumed elastic behavior, and representing to a suitable approximation the three-dimensional action of the shell together with its auxiliary members.

19.1.8 — ***Inelastic analysis*** — an analysis of deformations and internal forces based on equilibrium, nonlinear stress-strain relations for concrete and

COMMENTARY

vaults,[19.8] conoids,[19.8] elliptical paraboloids,[19.8] hyperbolic paraboloids,[19.9] and groined vaults.[19.9] Considerable information on the experience gained in the design, analysis, and construction of these shells may be found in the cited references. Less experience is available regarding other shell types or shapes, including free-form shells.

R19.1.4 — Folded plates may be prismatic,[19.7,19.10] nonprismatic,[19.10] or faceted. The first two types consist generally of planar thin slabs joined along their longitudinal edges to form a beam-like structure spanning between supports. Faceted folded plates are made up of triangular and/or polygonal planar thin slabs joined along their edges to form three-dimensional spatial structures.

R19.1.5 — Ribbed shells[19.6,19.11] generally have been used for larger spans where the increased thickness of the curved slab alone becomes excessive or uneconomical. Ribbed shells also have been used because of the construction techniques employed and to enhance the aesthetic impact of the completed structure.

R19.1.6 — Most thin shell structures require ribs or edge beams at their boundaries to carry the shell boundary forces, to assist in transmitting them to the supporting structure, and to accommodate the increased amount of reinforcement in these areas.

R19.1.7 — Elastic analysis of thin shells and folded plates means any method of structural analysis which is based on assumptions which provide suitable approximations to the three-dimensional behavior of the structure. The method must provide the internal forces and displacements needed in the design of the shell proper, the rib or edge members, and the supporting structure. Equilibrium of internal forces and external loads and compatibility of deformations must be satisfied.

Methods of elastic analysis based on classical shell theory, simplified mathematical or analytical models, or numerical solutions using finite element,[19.9] finite differences,[19.6] or numerical integration techniques,[19.6] are described in the cited references.

The choice of the method of analysis and the degree of accuracy required depends on certain critical factors. These include: the size of the structure, the geometry of the thin shell or folded plate, the manner in which the structure is supported, the nature of the applied load, and, finally, the extent of personal or documented experience regarding the reliability of the given method of analysis in predicting the behavior of the specific type of shell[19.6] or folded plate.[19.10]

R19.1.8—Inelastic analysis of thin shells and folded plates means a refined method of analysis based on the specific nonlinear material properties, nonlinear behavior due to the cracking of concrete, and time dependent effects such as

CODE

reinforcement, consideration of cracking and time dependent effects, and compatibility of strains. The analysis shall represent to a suitable approximation three-dimensional action of the shell together with its auxiliary members.

19.1.9 — ***Experimental analysis*** — an analysis procedure based on the measurement of deformations and/or strains of the structure or its model; experimental analysis is based on either elastic or inelastic behavior.

19.2 — Analysis and design

19.2.1 — Elastic behavior shall be an accepted basis for determining internal forces and displacements of thin shells. This behavior shall be permitted to be established by computations based on an analysis of the uncracked concrete structure in which the material is assumed linearly elastic, homogeneous, and isotropic. Poisson's ratio of concrete shall be permitted to be taken equal to zero.

19.2.2 — Inelastic analyses shall be permitted to be used where it can be shown that such methods provide a safe basis for design.

19.2.3 — Equilibrium checks of internal resistances and external loads shall be made to ensure consistency of results.

19.2.4 — Experimental or numerical analysis procedures shall be permitted where it can be shown that such procedures provide a safe basis for design.

19.2.5 — Approximate methods of analysis shall be permitted where it can be shown that such methods provide a safe basis for design.

COMMENTARY

creep, shrinkage, temperature, and load history. These effects are incorporated in order to trace the response and crack propagation of a reinforced concrete shell through the elastic, inelastic and ultimate ranges. Such analyses usually require incremental loading and iterative procedures to converge on solutions which satisfy both equilibrium and strain compatibility. Analyses of this type generally require extensive computer time.[19.12,19.13]

R19.2 — Analysis and design

R19.2.1 — For types of shell structures where experience, tests, and analyses have shown that the structure can sustain reasonable overloads without undergoing brittle failure, elastic analysis is a generally acceptable procedure. The designer may assume that reinforced concrete is ideally elastic, homogeneous, and isotropic, having identical properties in all directions. An analysis should be performed for the shell considering service load conditions. The analysis of shells of unusual size, shape, or complexity should consider behavior through the elastic and cracking ranges and into the inelastic range using factored loads.

R19.2.2 — Inelastic analysis procedures will generally require extensive use of computer procedures. Several references[19.12,19.13] indicate possible solution methods.

R19.2.4 — Experimental analysis of elastic models[19.14] has been used as a substitute for an analytical solution of a complex shell structure. Experimental analysis of reinforced micro-concrete models through the elastic, cracking, inelastic, and ultimate ranges should be considered for important shells of unusual size, shape, and/or complexity.

For model analysis, only those portions of the structure which affect significantly the items under study need be simulated. Every attempt should be made to ensure that the experiments reveal the quantitative behavior of the prototype structure.

Wind tunnel tests of a scaled-down model do not necessarily provide usable results and should be conducted by a recognized expert in wind tunnel testing of structural models.

R19.2.5 — In general, solutions that include both membrane and bending effects and satisfy conditions of compatibility and equilibrium are preferred. Approximate solutions

CODE

19.2.6 — In prestressed shells, the analysis shall also consider behavior under loads induced during prestressing, at cracking load, and at factored load. Where prestressing tendons are draped within a shell, design shall take into account force components on the shell resulting from the tendon profile not lying in one plane.

19.2.7 — The thickness of a shell and its reinforcement shall be proportioned for the required strength and serviceability, using either the strength design method of 8.1.1 or the alternate design method of 8.1.2.

19.2.8 — Shell instability shall be investigated and shown by design to be precluded.

COMMENTARY

which satisfy statics but not the compatibility of strains may be used only when extensive experience has proved that safe designs have resulted from their use. Such methods include beam-type analysis for barrel shells and folded plates having large ratios of span to either width or radius of curvature, simple membrane analysis for shells of revolution, and others in which the equations of equilibrium are satisfied, while the strain compatibility equations are not.

R19.2.6 — If the shell is prestressed, the analysis must include its strength at factored loads as well as its adequacy under service loads, under the load which causes cracking, and under loads induced during prestressing. Axial forces due to draped prestressed tendons may not lie in one plane and due consideration must be given to the resulting force components. The effects of post-tensioning of supporting members on the shell must be taken into account.

R19.2.7 — The thin shell's thickness and reinforcement must be proportioned to satisfy the strength provisions of this code, and to resist internal forces obtained from an analysis, an experimental model study, or a combination thereof. Reinforcement sufficient to control and minimize cracking under service load conditions should be provided. The thickness of the shell is often dictated by the required reinforcement and the construction exigencies, by 19.2.8, or by the code minimum thickness requirements.

R19.2.8 — Thin shells, like other structures that experience in-plane membrane compressive forces, are subject to buckling when the applied load reaches a critical value. Because of the surface-like geometry of shells, the problem of calculating buckling load is complex. If one of the principal membrane forces is tensile, the shell is less likely to buckle than if both principal membrane forces are compressive. The kinds of membrane forces that develop in a shell depend on its initial shape and the manner in which the shell is supported and loaded. In some types of shells, post-buckling behavior must be considered in determining safety against instability.[19.2]

Investigation of thin shells for stability shall consider the effect of the following factors: (1) anticipated deviation of the geometry of the shell surface as built from the idealized, perfect geometry, (2) large deflections, (3) creep and shrinkage of concrete, (4) inelastic properties of materials, (5) cracking of concrete, (6) location, amount, and orientation of reinforcement, and (7) possible deformation of supporting elements.

Practical measures to improve resistance to buckling successfully used in the past include the provision of two mats of reinforcement—one near each outer surface of the shell, a local increase of shell curvatures, the use of ribbed shells, and the use of concrete with high tensile strength and low creep.

CODE

COMMENTARY

A practical procedure for determining critical buckling loads of shells is given in the IASS recommendations.[19.2] Some recommendations for buckling design of domes used in industrial applications are given in References 19.5 and 19.15.

19.2.9 — Auxiliary members shall be designed according to the applicable provisions of this code. It shall be permitted to assume that a portion of the shell equal to the flange width, as specified in 8.10, acts with the auxiliary member. In such portions of the shell, the reinforcement perpendicular to the auxiliary member shall be at least equal to that required for the flange of a T-beam by 8.10.5.

R19.2.9 — Strength design can be used for the auxiliary members even though the alternate design method was used for the shell surface as long as serviceability requirements are also met. Portions of the shell may be utilized as flanges for transverse or longitudinal frames or arch-frames and beams.

19.2.10 — Strength design of shell slabs for membrane and bending forces shall be based on the distribution of stresses and strains as determined from either an elastic or an inelastic analysis.

R19.2.10 — The stresses and strains in the shell slab used for design are those determined by analysis (elastic or inelastic) multiplied by appropriate load factors. Because of detrimental effects of membrane cracking, the computed tensile strain in the reinforcement under factored loads should be limited.

19.2.11 — In a region where membrane cracking is predicted, the nominal compressive strength parallel to the cracks shall be taken as $0.4f_c'$.

R19.2.11 — When principal tensile stress produces membrane cracking in the shell, experiments indicate the attainable compressive strength in the direction parallel to the cracks is reduced.[19.16,19.17] For the alternate design method, the compressive strength f_c' parallel to the cracks should be replaced by $0.4f_c'$ in calculations involving A.3.1(a) or A.6.1.

19.3 — Design strength of materials

19.3.1 — Specified compressive strength of concrete f_c' at 28 days shall not be less than 3000 psi.

19.3.2 — Specified yield strength of nonprestressed reinforcement f_y shall not exceed 60,000 psi.

19.4 — Shell reinforcement

R19.4 — Shell reinforcement

19.4.1 — Shell reinforcement shall be provided to resist tensile stresses from internal membrane forces, to resist tension from bending and twisting moments, to control shrinkage and temperature cracking, and as special reinforcement at shell boundaries, load attachments, and shell openings.

R19.4.1 — At any point in a shell, two different kinds of internal forces may occur simultaneously: those associated with membrane action, and those associated with bending of the shell. The membrane forces are assumed to act in the tangential plane midway between the surfaces of the shell, and are the two axial forces and the membrane shears. Flexural effects include bending moments, twisting moments, and the associated transverse shears. Control of membrane cracking due to shrinkage, temperature, and service load conditions is a major design consideration.

19.4.2 — Tensile reinforcement shall be provided in two or more directions and shall be proportioned such

R19.4.2 — The requirement of ensuring strength in every direction is based on safety considerations. Any method

CODE

that its resistance in any direction equals or exceeds the component of internal forces in that direction.

Alternatively, reinforcement for the membrane forces in the slab shall be calculated as the reinforcement required to resist axial tensile forces plus the tensile force due to shear-friction required to transfer shear across any cross section of the membrane. The assumed coefficient of friction shall not exceed 1.0λ where $\lambda = 1.0$ for normal weight concrete, 0.85 for "sand-lightweight" concrete, and 0.75 for "all-lightweight" concrete. Linear interpolation shall be permitted when partial sand replacement is used.

19.4.3 — The area of shell reinforcement at any section as measured in two orthogonal directions shall not be less than the slab shrinkage or temperature reinforcement required by 7.12.

19.4.4 — Reinforcement for shear and bending moments about axes in the plane of the shell slab shall be calculated in accordance with Chapters 10, 11, and 13.

19.4.5 — The area of shell tension reinforcement shall be limited so that the reinforcement will yield before either crushing of concrete in compression or shell buckling can take place.

19.4.6 — In regions of high tension, membrane reinforcement shall, if practical, be placed in the general directions of the principal tensile membrane forces. Where this is not practical, it shall be permitted to

COMMENTARY

which assures sufficient strength consistent with equilibrium is considered acceptable. The direction of the principal membrane tensile force at any point may vary depending on the direction, magnitudes, and combinations of the various applied loads.

The magnitude of the internal membrane forces, acting at any point due to a specific load system, is generally calculated on the basis of an elastic theory in which the shell is assumed as uncracked. The computation of the required amount of reinforcement to resist the internal membrane forces has been traditionally based on the assumption that concrete cannot resist tension. The associated deflections, and the possibility of cracking, should be investigated in the serviceability phase of the design. To achieve the desired results may require a working stress design for steel selection.

Where reinforcement is not placed in the direction of the principal tensile forces and where cracks at the service load level would be objectionable, the computation of reinforcement may have to be based on a more refined approach[19.16,19.18,19.19] which considers the existence of cracks. In the cracked state, the concrete is assumed to be unable to resist either tension or shear. Thus, equilibrium is attained by means of tensile resisting forces in reinforcement and compressive resisting forces in concrete.

The alternative provides a simple way to calculate orthogonal reinforcement using shear-friction. It is based on the assumption that shear integrity of a shell should be maintained at factored loads. It is not necessary to calculate principal stresses if the alternative approach is used.

R19.4.3 — Minimum membrane reinforcement corresponding to slab shrinkage and temperature reinforcement must be provided in at least two approximately orthogonal directions even if the calculated membrane forces are compressive in one or more directions.

R19.4.5 — The requirement that the tensile reinforcement yields before the concrete crushes anywhere is consistent with 10.3.3. Such crushing can also occur in regions near supports and for some shells where the principal membrane forces are approximately equal and opposite in sign.

R19.4.6 — It is generally desirable for all shells, and particularly important in regions of substantial tension, that the orientation of reinforcement approximate the directions of the principal tensile membrane forces. However, in some

CODE

place membrane reinforcement in two or more component directions.

19.4.7 — If the direction of reinforcement varies more than 10 deg from the direction of principal tensile membrane force, the amount of reinforcement shall be reviewed in relation to cracking at service loads.

19.4.8 — Where the magnitude of the principal tensile membrane stress within the shell varies greatly over the area of the shell surface, reinforcement resisting the total tension shall be permitted to be concentrated in the regions of largest tensile stress where it can be shown that this provides a safe basis for design. However, the ratio of shell reinforcement in any portion of the tensile zone shall be not less than 0.0035 based on the overall thickness of the shell.

19.4.9 — Reinforcement required to resist shell bending moments shall be proportioned with due regard to the simultaneous action of membrane axial forces at the same location. Where shell reinforcement is required in only one face to resist bending moments, equal amounts shall be placed near both surfaces of the shell even though a reversal of bending moments is not indicated by the analysis.

19.4.10 — Shell reinforcement in any direction shall not be spaced farther apart than 18 in. nor five times the shell thickness. Where the principal membrane tensile stress on the gross concrete area due to factored loads exceeds $4\phi\sqrt{f_c'}$, reinforcement shall not be spaced farther apart than three times the shell thickness.

19.4.11 — Shell reinforcement at the junction of the shell and supporting members or edge members shall be anchored in or extended through such members in accordance with the requirements of Chapter 12,

COMMENTARY

structures it is not always possible or practical for the reinforcement to follow the stress trajectories. For such cases, orthogonal component reinforcement is allowed.

R19.4.7 — When the directions of reinforcement deviate significantly (more than 10 deg) from the directions of the principal membrane forces, higher strains in the shell must occur to develop the capacity of reinforcement. This might lead to the development of unacceptably wide cracks. The crack width should be estimated and controlled if necessary.

Permissible crack widths for service loads under different environmental conditions are given in the report of ACI Committee 224.[19.20] Crack width can be limited by an increase in the amount of reinforcement used, by reducing the stress at the service load level, by providing reinforcement in three or more directions in the plane of the shell, or by using closer spacing of smaller diameter bars.

R19.4.8 — The practice of concentrating tensile reinforcement in the regions of maximum tensile stress has led to a number of successful and economical designs, primarily for long folded plates, long barrel vault shells, and for domes. The requirement of providing the minimum reinforcement in the remaining tensile zone is intended to control cracking.

R19.4.9 — The design method should assure that the concrete sections, including consideration of the reinforcement, are capable of developing the internal forces required to assure the equations of equilibrium are satisfied.[19.21]The sign of bending moments may change rapidly from point to point of a shell. For this reason, bending reinforcement, where required, is to be placed near both outer surfaces of the shell. In many cases, the thickness required to provide proper cover and spacing for the multiple layers of reinforcement may govern the design of the shell thickness.

R19.4.10 — The value of ϕ to be used is that prescribed in 9.3.2.2(a) for axial tension.

R19.4.11 and R19.4.12 — On curved shell surfaces it is difficult to control the alignment of precut reinforcement. This must be considered to avoid insufficient splice and development lengths. Sections 19.4.11 and 19.4.12 specify

CODE

except that the minimum development length shall be **1.2ℓ_d** but not less than 18 in.

19.4.12 — Splice lengths of shell reinforcement shall be governed by the provisions of Chapter 12, except that the minimum splice length of tension bars shall be 1.2 times the value required by Chapter 12 but not less than 18 in. The number of splices in principal tensile reinforcement shall be kept to a practical minimum. Where splices are necessary they shall be staggered at least ℓ_d with not more than one-third of the reinforcement spliced at any section.

19.5 — Construction

19.5.1 — When removal of formwork is based on a specific modulus of elasticity of concrete because of stability or deflection considerations, the value of the modulus of elasticity E_c shall be determined from flexural tests of field-cured beam specimens. The number of test specimens, the dimensions of test beam specimens, and test procedures shall be specified by the engineer.

19.5.2 — The engineer shall specify the tolerances for the shape of the shell. If construction results in deviations from the shape greater than the specified tolerances, an analysis of the effect of the deviations shall be made and any required remedial actions shall be taken to ensure safe behavior.

COMMENTARY

extra reinforcement length to maintain the minimum lengths on curved surfaces.

R19.5 — Construction

R19.5.1 — When early removal of forms is necessary, the magnitude of the modulus of elasticity at the time of proposed form removal must be investigated in order to ensure safety of the shell with respect to buckling, and to restrict deflections.[19.3,19.22] The value of the modulus of elasticity E_c must be obtained from a flexural test of field-cured specimens. It is not sufficient to determine the modulus from the formula in 8.5.1, even if f_c' is determined for the field-cured specimen.

R19.5.2 — In some types of shells, small local deviations from the theoretical geometry of the shell can cause relatively large changes in local stresses and in overall safety against instability. These changes can result in local cracking and yielding which may make the structure unsafe or can greatly affect the critical load producing instability. The effect of such deviations should be evaluated and any necessary remedial actions should be taken promptly. Special attention is needed when using air supported form systems.[19.23]

PART 6 — SPECIAL CONSIDERATIONS

CHAPTER 20 — STRENGTH EVALUATION OF EXISTING STRUCTURES

CODE

20.0 — Notation

D = dead loads or related internal moments and forces

f_c' = specified compressive strength of concrete, psi

h = overall thickness of member, in.

L = live loads or related internal moments and forces

ℓ_t = span of member under load test, in. (The shorter span for two-way slab systems.) Span is the smaller of (a) distance between centers of supports, and (b) clear distance between supports plus thickness h of member. In Eq. (20-1), span for a cantilever shall be taken as twice the distance from support to cantilever end, in.

Δ_{max} = measured maximum deflection, in. See Eq. (20-1)

Δ_{rmax} = measured residual deflection, in. See Eq. (20-2) and (20-3)

Δ_{fmax} = maximum deflection measured during the second test relative to the position of the structure at the beginning of the second test, in. See Eq. (20-3)

20.1 — Strength evaluation — General

20.1.1 — If there is doubt that a part or all of a structure meets the safety requirements of this code, a strength evaluation shall be carried out as required by the engineer or building official.

COMMENTARY

R20.1 — Strength evaluation — General

Chapter 20 does not cover load testing for the approval of new design or construction methods. (See 16.10 for recommendations on strength evaluation of precast concrete members.) Provisions of Chapter 20 may be used to evaluate whether a structure or a portion of a structure satisfies the safety requirements of this code. A strength evaluation may be required if the materials are considered to be deficient in quality, if there is evidence indicating faulty construction, if a structure has deteriorated, if a building will be used for a new function, or if, for any reason, a structure or a portion of it does not appear to satisfy the requirements of the code. In such cases, Chapter 20 provides guidance for investigating the safety of the structure.

If the safety concerns are related to an assembly of elements or an entire structure, it is not feasible to load test every element and section to the maximum for the applied load intensity. In such cases, it is appropriate that an investigation plan be developed to address the specific safety concerns. If

CODE

COMMENTARY

a load test is described as part of the strength evaluation process, it is desirable for all parties involved to come to an agreement about the region to be loaded, the magnitude of the load, the load test procedure, and acceptance criteria before any load tests are made.

20.1.2 — If the effect of the strength deficiency is well understood and if it is feasible to measure the dimensions and material properties required for analysis, analytical evaluations of strength based on those measurements shall suffice. Required data shall be determined in accordance with 20.2.

R20.1.2 — In the practice of reinforced concrete building design, it is currently assumed that strength considerations related to axial load, flexure, and combined axial load and flexure are well understood. There are reliable theories relating strength and short-term displacement to load in terms of dimensional and material data for the structure.

If it is decided to determine the strength of the structure by analysis, calculations must be based on data gathered on the actual dimensions of the structure, properties of the materials in place, and all pertinent details. Requirements for data collection are in 20.2.

20.1.3 — If the effect of the strength deficiency is not well understood or if it is not feasible to establish the required dimensions and material properties by measurement, a load test shall be required if the structure is to remain in service.

R20.1.3 — If the shear or bond strength of an element is critical in relation to the doubt expressed about safety, a physical test may be the most efficient solution to eliminate or confirm the doubt. A physical test may also be appropriate if it is not possible or feasible to determine the material and dimensional properties required for analysis even if the cause of the concern relates to flexure or axial load.

Wherever possible and appropriate, it is desirable to support the results of the load test by analysis.

20.1.4 — If the doubt about safety of a part or all of a structure involves deterioration, and if the observed response during the load test satisfies the acceptance criteria, the structure or part of the structure shall be permitted to remain in service for a specified time period. If deemed necessary by the engineer, periodic reevaluations shall be conducted.

R20.1.4 — For a deteriorating structure, the acceptance provided by the load test may not be assumed to be without limits in terms of time. In such cases, a periodic inspection program is useful. A program that involves physical tests and periodic inspection can justify a longer period in service. Another option for maintaining the structure in service, while the periodic inspection program continues, is to limit the live load to a level determined to be appropriate.

The length of the specified time period should be based on consideration of (a) the nature of the problem, (b) environmental and load effects, (c) service history of the structure, and (d) scope of the periodic inspection program. At the end of a specified time period, further strength evaluation is required if the structure is to remain in service.

With the agreement of all concerned parties, special procedures may be devised for periodic testing that do not necessarily conform to the loading and acceptance criteria specified in Chapter 20.

20.2 — Determination of required dimensions and material properties

R20.2 — Determination of required dimensions and material properties

This section applies if it is decided to make an analytical evaluation (20.1.2).

CODE

20.2.1 — Dimensions of the structural elements shall be established at critical sections.

20.2.2 — Locations and sizes of the reinforcing bars, welded wire fabric, or tendons shall be determined by measurement. It shall be permitted to base reinforcement locations on available drawings if spot checks are made confirming the information on the drawings.

20.2.3 — If required, concrete strength shall be based on results of cylinder tests or tests of cores removed from the part of the structure where the strength is in doubt. Concrete strengths shall be determined as specified in 5.6.4.

20.2.4 — If required, reinforcement or tendon strength shall be based on tensile tests of representative samples of the material in the structure in question.

20.2.5 — If the required dimensions and material properties are determined through measurements and testing, and if calculations can be made in accordance with 20.1.2, it shall be permitted to increase the strength reduction factor in 9.3, but the strength reduction factor shall not be more than:

Flexure, without axial load	1.0
Axial tension, and axial tension with flexure	1.0
Axial compression and axial compression with flexure:	
Members with spiral reinforcement conforming to 10.9.3	0.9
Other members	0.85
Shear and/or torsion	0.9
Bearing on concrete	0.85

20.3 — Load test procedure

20.3.1 — ***Load arrangement*** — The number and arrangement of spans or panels loaded shall be selected to maximize the deflection and stresses in the critical regions of the structural elements of which strength is in doubt. More than one test load arrangement shall be used if a single arrangement will not simultaneously result in maximum values of the effects (such as deflection, rotation, or stress) necessary to demonstrate the adequacy of the structure.

COMMENTARY

R20.2.1 — Critical sections are those at which each type of stress calculated for the load in question reaches its maximum value.

R20.2.2 — For individual elements, amount, size, arrangement, and location must be determined at the critical sections for reinforcement and/or tendons designed to resist applied load. Nondestructive investigation methods are acceptable. In large structures, determination of these data for approximately five percent of the reinforcement or tendons in critical regions may suffice if these measurements confirm the data provided in the construction drawings.

R20.2.3 — The number of tests may depend on the size of the structure and the sensitivity of structural safety to concrete strength for the problem. In cases where the potential problem involves flexure only, investigation of concrete strength can be minimal for a lightly reinforced section ($\rho f_y / f_c' \leq 0.15$ for rectangular section).

R20.2.4 — The number of tests required depends on the uniformity of the material and is best determined by the engineer for the specific application.

R20.2.5 — Strength reduction factors given in 20.2.5 are larger than those specified in Chapter 9. These increased values are justified by the use of accurate field-obtained material properties, actual in-place dimensions, and well understood methods of analysis.

R20.3 — Load test procedure

R20.3.1 — It is important to apply the load at locations so that its effects on the suspected defect are a maximum and the probability of unloaded members sharing the applied load is a minimum. In cases where it is shown by analysis that adjoining unloaded elements will help carry some of the load, the load must be placed to develop effects consistent with the intent of the load factor.

CODE

20.3.2 — *Load intensity* — The total test load (including dead load already in place) shall not be less than $\mathbf{0.85\ (1.4}\boldsymbol{D}\mathbf{ + 1.7}\boldsymbol{L}\mathbf{)}$. It shall be permitted to reduce $\boldsymbol{L}$ in accordance with the requirements of the applicable general building code.

20.3.3 — A load test shall not be made until that portion of the structure to be subject to load is at least 56 days old. If the owner of the structure, the contractor, and all involved parties agree, it shall be permitted to make the test at an earlier age.

20.4 — Loading criteria

20.4.1 — The initial value for all applicable response measurements (such as deflection, rotation, strain, slip, crack widths) shall be obtained not more than one hour before application of the first load increment. Measurements shall be made at locations where maximum response is expected. Additional measurements shall be made if required.

20.4.2 — Test load shall be applied in not less than four approximately equal increments.

20.4.3 — Uniform test load shall be applied in a manner to ensure uniform distribution of the load transmitted to the structure or portion of the structure being tested. Arching of the applied load shall be avoided.

20.4.4 — A set of response measurements shall be made after each load increment is applied and after the total load has been applied on the structure for at least 24 hr.

20.4.5 — Total test load shall be removed immediately after all response measurements defined in 20.4.4 are made.

20.4.6 — A set of final response measurements shall be made 24 hr after the test load is removed.

20.5 — Acceptance criteria

20.5.1 — The portion of the structure tested shall show no evidence of failure. Spalling and crushing of compressed concrete shall be considered an indication of failure.

COMMENTARY

R20.3.2 — The required load intensity follows previous load test practice. The live load $\boldsymbol{L}$ may be reduced as permitted by the general building code governing safety considerations for the structure. The live load should be increased to compensate for resistance provided by unloaded portions of the structure in questions. The increase in live load is determined from analysis of the loading conditions in relation to the selected pass/fail criterion for the test.

R20.4 — Loading criteria

R20.4.2 — It is advisable to inspect the structure after each load increment.

R20.4.3 — "Arching" refers to the tendency for the load to be transmitted nonuniformly to the flexural element being tested. For example, if a slab is loaded by a uniform arrangement of bricks with the bricks in contact, "arching" would results in reduction of the load on the slab near the midspan of the slab.

R20.5 — Acceptance criteria

R20.5.1 — A general acceptance criterion for the behavior of a structure under the test load is that it shall not show "evidence of failure." Evidence of failure will include cracking, spalling, and/or deflection of such magnitude and extent that the observed result is obviously excessive and incompatible with the safety requirements of the structure. No simple rules can be developed for application to all types of structures and conditions. If sufficient damage has occurred that the structure is considered to have failed that test, retesting is not permitted since it is considered that

CODE

20.5.2 — Measured maximum deflections shall satisfy one of the following conditions:

$$\Delta_{max} \le \frac{\ell_t^2}{20{,}000h} \tag{20-1}$$

$$\Delta_{rmax} \le \frac{\Delta_{max}}{4} \tag{20-2}$$

If the measured maximum and residual deflections do not satisfy Eq. (20-1) or (20-2), it shall be permitted to repeat the load test.

The repeat test shall be conducted not earlier than 72 hr after removal of the first test load. The portion of the structure tested in the repeat test shall be considered acceptable if deflection recovery satisfies the condition:

$$\Delta_{rmax} \le \frac{\Delta_{fmax}}{5} \tag{20-3}$$

where Δ_{fmax} is the maximum deflection measured during the second test relative to the position of the structure at the beginning of the second test.

20.5.3 — Structural members tested shall not have cracks indicating the imminence of shear failure.

COMMENTARY

damaged members should not be put into service even at a lower rating.

Local spalling or flaking of the compressed concrete in flexural elements related to casting imperfections need not indicate overall structural distress. Crack widths are good indicators of the state of the structure and should be observed to help determine whether the structure is satisfactory. However, exact prediction or measurement of crack widths in reinforced concrete elements is not likely to be achieved under field conditions. It is advisable to establish criteria before the test, relative to the types of cracks anticipated, where the cracks will be measured, how they will be measured, and to establish approximate limits or criteria to evaluate new cracks or limits for the changes in crack width.

R20.5.2 — Specified deflection limits and the retest option follow past practice. If the structure shows no evidence of failure, "recovery of deflection" after removal of the test load is used to determine whether the strength of the structure is satisfactory. In the case of a very stiff structure, however, the errors in measurements under field conditions may be of the same order as the actual deflections and recovery. To avoid penalizing a satisfactory structure in such a case, recovery measurements are waived if the maximum deflection is less than $\ell_t^2/(20{,}000h)$. The residual deflection Δ_{rmax} is the difference between the initial and final (after load removal) deflections for the load test or the repeat load test.

R20.5.3 — Forces are transmitted across a shear crack plane by a combination of aggregate interlock at the interface of the crack which is enhanced by clamping action of transverse stirrup reinforcing and by dowel action of stirrups crossing the crack. As crack lengths increase to approach a horizontal projected length equal to the depth of the member and concurrently widen to the extent that aggregate interlock cannot occur, and as transverse stirrups if present begin to yield or display loss of anchorage so as to threaten their integrity, the member is assumed to be approaching imminent shear failure.

CODE

20.5.4 — In regions of structural members without transverse reinforcement, appearance of structural cracks inclined to the longitudinal axis and having a horizontal projection longer than the depth of the member at midpoint of the crack shall be evaluated.

20.5.5 — In regions of anchorage and lap splices, the appearance along the line of reinforcement of a series of short inclined cracks or horizontal cracks shall be evaluated.

20.6 — Provision for lower load rating

If the structure under investigation does not satisfy conditions or criteria of 20.1.2, 20.5.2, or 20.5.3, the structure shall be permitted for use at a lower load rating based on the results of the load test or analysis, if approved by the building official.

20.7 — Safety

20.7.1 — Load tests shall be conducted in such a manner as to provide for safety of life and structure during the test.

20.7.2 — No safety measures shall interfere with load test procedures or affect results.

COMMENTARY

R20.5.4 — The intent of 20.5.4 is to make certain that the professionals in charge of the test will pay attention to the structural implication of observed inclined cracks that may lead to brittle collapse in members without transverse reinforcement.

R20.5.5 — Cracking along the axis of the reinforcement in anchorage zones may be related to high stresses associated with the transfer of forces between the reinforcement and the concrete. These cracks may be indicators of pending brittle failure of the element if they are associated with the main reinforcement. It is important that their causes and consequences be evaluated.

R20.6 — Provision for lower load rating

Except for load tested members that have failed under a test (see 20.5), the building official may permit the use of a structure or member at a lower load rating that is judged to be safe and appropriate on the basis of the test results.

CHAPTER 21 — SPECIAL PROVISIONS FOR SEISMIC DESIGN

CODE

COMMENTARY

21.0 — Notation

A_{ch} = cross-sectional area of a structural member measured out-to-out of transverse reinforcement, in.^{2}
Acp = area of concrete section, resisting shear, of an individual pier or horizontal wall segment, in.^{2}
A_{cv} = net area of concrete section bounded by web thickness and length of section in the direction of shear force considered, in.^{2}
A_g = gross area of section, in.^{2}
A_j = effective cross-sectional area within a joint, see 21.5.3.1, in a plane parallel to plane of reinforcement generating shear in the joint. The joint depth shall be the overall depth of the column. Where a beam frames into a support of larger width, the effective width of the joint shall not exceed the smaller of:
  (a) beam width plus the joint depth
  (b) twice the smaller perpendicular distance from the longitudinal axis of the beam to the column side. See 21.5.3.1
A_{sh} = total cross-sectional area of transverse reinforcement (including crossties) within spacing s and perpendicular to dimension h_c
b = effective compressive flange width of a structural member, in.
b_w = web width, or diameter of circular section, in.
d = effective depth of section
d_b = bar diameter
E = load effects of earthquake, or related internal moments and forces
f_c' = specified compressive strength of concrete, psi
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi
f_y = specified yield strength of reinforcement, psi
f_{yh} = specified yield strength of transverse reinforcement, psi
h_c = cross-sectional dimension of column core measured center-to-center of confining reinforcement
h_w = height of entire wall (diaphragm) or of the segment of wall (diaphragm) considered
ℓ_d = development length for a straight bar
ℓ_{dh} = development length for a bar with a standard hook as defined in Eq. (21-5)
ℓ_o = minimum length, measured from joint face along axis of structural member, over which transverse reinforcement must be provided, in.

CODE

COMMENTARY

ℓ_w = length of entire wall (diaphragm) or of segment of wall (diaphragm) considered in direction of shear force

M_{pr} = probable flexural moment strength of members, with or without axial load, determined using the properties of the member at the joint faces assuming a tensile strength in the longitudinal bars of at least $1.25\ f_y$ and a strength reduction factor ϕ of 1.0

M_s = portion of slab moment balanced by support moment

s = spacing of transverse reinforcement measured along the longitudinal axis of the structural member, in.

s_o = maximum spacing of transverse reinforcement, in.

V_c = nominal shear strength provided by concrete

V_e = design shear force determined from 21.3.4.1 or 21.4.5.1

V_n = nominal shear strength

V_u = factored shear force at section

α_c = coefficient defining the relative contribution of concrete strength to wall strength. See Eq. (21-7)

ρ = ratio of nonprestressed tension reinforcement = A_s/bd

ρ_g = ratio of total reinforcement area to cross-sectional area of column

ρ_n = ratio of distributed shear reinforcement on a plane perpendicular to plane of A_{cv}

ρ_s = ratio of volume of spiral reinforcement to the core volume confined by the spiral reinforcement (measured out-to-out)

ρ_v = A_{sv}/A_{cv}; where A_{sv} is the projection on A_{cv} of area of distributed shear reinforcement crossing the plane of A_{cv}

ϕ = strength reduction factor

21.1 — Definitions

Base of structure — Level at which earthquake motions are assumed to be imparted to a building. This level does not necessarily coincide with the ground level.

Boundary elements — Portions along wall and diaphragm edges strengthened by longitudinal and transverse reinforcement. Boundary elements do not necessarily require an increase in the thickness of the wall or diaphragm. Edges of openings within walls and diaphragms shall be provided with boundary elements if required by 21.6.6 or 21.6.7.

Collector elements — Elements that serve to transmit the inertial forces within the diaphragms to mem-

CODE

bers of the lateral-force resisting systems.

Crosstie — A continuous reinforcing bar having a seismic hook at one end and a hook not less than 90 deg with at least a six-diameter extension at the other end. The hooks shall engage peripheral longitudinal bars. The 90-deg hooks of two successive crossties engaging the same longitudinal bars shall be alternated end for end.

Design load combinations — Combinations of factored loads and forces specified in 9.2.

Development length for a bar with a standard hook — The shortest distance between the critical section (where the strength of the bar is to be developed) and a tangent to the outer edge of the 90-deg hook.

Factored loads and forces — Specified loads and forces modified by the factors in 9.2.

Hoop — A hoop is a closed tie or continuously wound tie. A closed tie can be made up of several reinforcement elements each having seismic hooks at both ends. A continuously wound tie shall have a seismic hook at both ends.

Lateral-force resisting system — That portion of the structure composed of members proportioned to resist forces related to earthquake effects.

Lightweight aggregate concrete — "All-lightweight" or "sand-lightweight" aggregate concrete made with lightweight aggregates conforming to 3.3.

Seismic hook — A hook on a stirrup, hoop, or crosstie having a bend not less than 135 deg with a six-diameter (but not less than 3 in.) extension that engages the longitudinal reinforcement and projects into the interior of the stirrup or hoop.

Shell concrete — Concrete outside the transverse reinforcement confining the concrete.

Specified lateral forces — Lateral forces corresponding to the appropriate distribution of the design base shear force prescribed by the governing code for earthquake-resistant design.

Structural diaphragms — Structural members, such as floor and roof slabs, which transmit inertial forces to lateral-force resisting members.

Structural trusses — Assemblages of reinforced concrete members subjected primarily to axial forces.

Structural walls — Walls proportioned to resist combinations of shears, moments, and axial forces

COMMENTARY

CODE

induced by earthquake motions. A “shearwall” is a “structural wall.”

Strut — An element of a structural diaphragm used to provide continuity around an opening in the diaphragm.

Tie elements — Elements which serve to transmit inertia forces and prevent separation of such building components as footings and walls.

21.2 — General requirements

21.2.1 — Scope

21.2.1.1 — Chapter 21 contains special requirements for design and construction of reinforced concrete members of a structure for which the design forces, related to earthquake motions, have been determined on the basis of energy dissipation in the nonlinear range of response.

21.2.1.2 — The provisions of Chapters 1 through 18 shall apply except as modified by the provisions of this chapter.

21.2.1.3 — In regions of moderate seismic risk, reinforced concrete frames resisting forces induced by earthquake motions shall be proportioned to satisfy only 21.8 of Chapter 21 in addition to the requirements of Chapters 1 through 18.

21.2.1.4 — In regions of high seismic risk, all reinforced concrete structural members shall satisfy 21.2 through 21.7 of Chapter 21 in addition to the requirements of Chapters 1 through 17.

21.2.1.5 — A reinforced concrete structural system not satisfying the requirements of this chapter shall be permitted if it is demonstrated by experimental evidence and analysis that the proposed system will have strength and toughness equal to or exceeding those provided by a comparable monolithic reinforced concrete structure satisfying this chapter.

COMMENTARY

R21.2 — General requirements

R21.2.1 — Scope

Chapter 21 contains specifications which are currently considered to be the minimum requirements for producing a monolithic reinforced concrete structure with adequate proportions and details to enable the structure to sustain a series of oscillations into the inelastic range of response without critical decay in strength. Demand for integrity of the structure in the inelastic range of response is created by the rationalization of forces used for design in documents such as the report of the Seismology Committee of the Structural Engineers Association of California.[21.1] The lateral design forces specified in Reference 21.1 are considered less than those corresponding to linear response of the anticipated earthquake intensity.[21.2-21.5]

As a properly detailed reinforced concrete structure responds to strong ground motion, its effective stiffness decreases and its capability to dissipate energy increases. These developments tend to reduce the response acceleration or lateral inertia forces with respect to those forces calculated for a linearly elastic model of the uncracked and lightly damped structure.[21.6] Thus, the use of design forces representing earthquake effects such as those in Reference 21.1 requires that the building be equipped with a lateral-force resisting system which will retain a substantial portion of its strength as it is subject to displacement reversals into the inelastic range. Toughness of the structure is an essential property for earthquake resistance.

The level of toughness required, and therefore of special detail needs, for a given structure depends on the quantitative relationship between earthquake intensity and structural strength. Recognizing the fact that most of the experience which has led to the development of special details for reinforced concrete construction has been obtained from events with very strong ground motions, it is proper first to consider the question of a trade-off between strength and detail requirements in an environment of high earthquake risk. Given a design earthquake intensity or a design response spectrum indexed by an effective peak acceleration, it appears plausible to soften or relinquish some of the detail requirements if the design strength is increased with respect to the minimum code requirement. However, available knowledge of ground motion and structural response to such motion does not make precise estimates of inelastic displacement possible. Furthermore, it is not currently possible to devise explicit and universally applicable relationships between the required number of inelastic displacements and required reinforcement details. The practical choice is between (a) a system with sufficient strength to respond to the ground motion within the linear or nearly linear range of response, and (b) a system with adequate details to permit nonlinear response without critical loss of strength.

For applications in zones of high earthquake risk, requirements of Chapter 21 in 21.2 through 21.7 have been developed in relation to the second option, on the assumption that the design forces are based on Reference 21.1 or a comparable document[21.2,21.7] with a similar approach to the determination of design forces. The entire building, including the foundation and nonstructural members, must satisfy 21.2 through 21.7 of Chapter 21 (Sections 21.2.1.3 and 21.2.1.4) as outlined in Table R21.2.1.

TABLE R21.2.1—SECTIONS OF CHAPTER 21 TO BE SATISFIED[*]

Earthquake risk level[†]	High	Moderate
Frame members resisting earthquake effects	2,3,4,5	8
Walls, diaphragms, and trusses resisting earthquake effects	2,6	None
Frame members not resisting earthquake effects	7	None

[*] In addition to requirements of Chapters 1-17 in regions of high risk and Chapters 1-18 in regions of moderate risk,

[†] The terms refer to regions with earthquake risk identified in building codes such as American Society of Civil Engineers standard "Minimum Design Loads for Buildings and Other Structures," ASCE 7-88 (formerly ANSI A58.1) and Uniform Building Code.[21.8] Regions of high earthquake risk correspond approximately to Zones 3 and 4, and regions of moderate earthquake risk correspond approximately to Zone 2 in both documents.

Chapter 21 requires a minimum of special details for reinforced concrete buildings in zones of moderate earthquake risk. These requirements, all presented in 21.8, apply only to reinforced concrete frames proportioned to resist forces caused by earthquake motions. There are no special requirements for other structural or nonstructural components of buildings in zones of moderate seismic risk.

Chapter 21 does not apply to construction in regions of low and no seismic risk.

Field and laboratory experience which has led to the special proportioning and detailing requirements in Chapter 21 has been predominantly with monolithic reinforced concrete building structures. Projection of these requirements to other types of reinforced concrete structures, which may differ in concept or fabrication from monolithic construction, must be tempered by relevant physical evidence and analysis. Precast and/or prestressed members may be used for earthquake resistance provided it is demonstrated that the resulting structure will provide the safety and service-

CODE

21.2.2 — Analysis and proportioning of structural members

21.2.2.1 — The interaction of all structural and nonstructural members which materially affect the linear and nonlinear response of the structure to earthquake motions shall be considered in the analysis.

21.2.2.2 — Rigid members assumed not to be a part of the lateral-force resisting system shall be permitted provided their effect on the response of the system is considered and accommodated in the structural design. Consequences of failure of structural and nonstructural members, which are not a part of the lateral-force resisting system, shall also be considered.

21.2.2.3 — Structural members below base of structure required to transmit to the foundation forces resulting from earthquake effects shall also comply with the requirements of Chapter 21.

21.2.2.4 — All structural members assumed not to be part of the lateral-force resisting system shall conform to 21.7.

21.2.3 — Strength reduction factors

Strength reduction factors shall be as given in 9.3.4.

COMMENTARY

ability levels (during and after the earthquake) expected from monolithic construction.

The "toughness" requirements in 21.2.1.5 refer to the concern for the structural integrity of the entire lateral-force resisting structure at lateral displacements anticipated for ground motions corresponding to design intensity. Depending on the energy-dissipation characteristics of the structural system used, such displacements may have to be more than those for a monolithic reinforced concrete structure.

R21.2.2 — Analysis and proportioning of structural members

It is assumed that the distribution of required strength to the various components of a lateral-force resisting system will be guided by the analysis of a linearly elastic model of the system acted on by the factored forces specified by the governing code. If nonlinear response history analyses are to be used, base motions should be selected after a detailed study of the site conditions and local seismic history.

Because the design basis admits nonlinear response, it is necessary to investigate the stability of the lateral-force resisting system as well as its interaction with other structural and nonstructural members at displacements larger than those indicated by linear analysis. To handle this problem without having to resort to nonlinear response analysis, one option is to multiply by a factor of at least two the displacements from linear analysis for the factored lateral forces, unless the governing code specifies the factors to be used as in References 21.2 and 21.8. For lateral displacement calculations, assuming at least all the horizontal structural members to be fully cracked is likely to lead to better estimates of the possible drift than using uncracked stiffness for all members.

The main concern of Chapter 21 is the safety of the structure. The intent of 21.2.2.1 and 21.2.2.2 is to draw attention to the influence of nonstructural members on structural response and to hazards from falling objects.

Section 21.2.2.3 alerts the designer to the fact that the base of the structure as defined in analysis may not necessarily correspond to the foundation or ground level.

In selecting member sizes for earthquake-resistant structures, it is very important to consider problems related to congestion of reinforcement. The designer should assure that all reinforcement can be assembled and placed and that concrete can be cast and consolidated properly. Use of upper limits of reinforcement ratios permitted is likely to lead to insurmountable construction problems especially at frame joints.

CODE

21.2.4 — Concrete in members resisting earthquake-induced forces

21.2.4.1 — Compressive strength f_c' of the concrete shall be not less than 3000 psi.

21.2.4.2 — Compressive strength of lightweight aggregate concrete used in design shall not exceed 4000 psi. Lightweight aggregate concrete with higher design compressive strength shall be permitted if demonstrated by experimental evidence that structural members made with that lightweight aggregate concrete provide strength and toughness equal to or exceeding those of comparable members made with normal weight aggregate concrete of the same strength.

21.2.5 — Reinforcement in members resisting earthquake-induced forces

Reinforcement resisting earthquake-induced flexural and axial forces in frame members and in wall boundary elements shall comply with ASTM A 706. ASTM A 615 Grades 40 and 60 reinforcement shall be permitted in these members if (a) the actual yield strength based on mill tests does not exceed the specified yield strength by more than 18,000 psi (retests shall not exceed this value by more than an additional 3000 psi), and (b) the ratio of the actual ultimate tensile strength to the actual tensile yield strength is not less than 1.25.

21.2.6 — Welded splices and mechanically connected reinforcement

21.2.6.1 — Reinforcement resisting earthquake-induced flexural or axial forces in frame members or in wall boundary elements shall be permitted to be spliced using welded splices or mechanical connectors conforming to 12.14.3.3 or 12.14.3.4 provided not more than alternate bars in each layer of longitudinal reinforcement are spliced at a section and the center-

COMMENTARY

R21.2.4 — Concrete in members resisting earthquake-induced forces

Requirements of this section refer to concrete quality in frames, trusses, or walls proportioned to resist earthquake-induced forces. The maximum design compressive strength of lightweight aggregate concrete to be used in structural design calculations is limited to 4000 psi primarily because of paucity of experimental and field data on the behavior of members made with lightweight aggregate concrete subjected to displacement reversals in the nonlinear range. If convincing evidence is developed for a specific application, the limit on maximum compressive strength of lightweight aggregate concrete may be increased to a level justified by the evidence.

R21.2.5 — Reinforcement in members resisting earthquake-induced forces

Use of longitudinal reinforcement with strength substantially higher than that assumed in design will lead to higher shear and bond stresses at the time of development of yield moments. These conditions may lead to brittle failures in shear or bond and should be avoided even if such failures may occur at higher loads than those anticipated in design. Therefore, a ceiling is placed on the actual yield strength of the steel [see 21.2.5(a)].

The requirement for an ultimate tensile strength larger than the yield strength of the reinforcement [21.2.5(b)] is based on the assumption that the capability of a structural member to develop inelastic rotation capacity is a function of the length of the yield region along the axis of the member. In interpreting experimental results, length of the yield region has been related to the relative magnitudes of ultimate and yield moments.[21.9] According to that interpretation, the larger the ratio of ultimate to yield moment, the longer the yield region. Chapter 21 requires that the ratio of actual tensile strength to actual yield strength is not less than 1.25. Members with reinforcement not satisfying that condition can also develop inelastic rotation, but their behavior is sufficiently different to exclude them from direct consideration on the basis of rules derived from experience with members reinforced with strain-hardening steel.

R21.2.6 — Welded splices and mechanically connected reinforcement

R21.2.6.1 — Welding is permitted on reinforcement resisting earthquake-induced flexural or axial forces when the welding is performed according to a controlled procedure with adequate inspection.

CODE

to-center distance between splices of adjacent bars is 24 in. or more measured along the longitudinal axis of the member.

21.2.6.2 — Welding of stirrups, ties, inserts, or other similar elements to longitudinal reinforcement required by design shall not be permitted.

21.3 — Flexural members of frames

21.3.1 — Scope

Requirements of 21.3 apply to frame members (a) resisting earthquake-induced forces, and (b) proportioned primarily to resist flexure. These frame members shall also satisfy the following conditions:

21.3.1.1 — Factored axial compressive force on the member shall not exceed $(A_g f_c'/10)$.

21.3.1.2 — Clear span for the member shall not be less than four times its effective depth.

21.3.1.3 — The width-to-depth ratio shall not be less than 0.3.

21.3.1.4 — The width shall not be (a) less than 10 in., and (b) more than the width of the supporting member (measured on a plane perpendicular to the longitudinal axis of the flexural member) plus distances on each side of the supporting member not exceeding three-fourths of the depth of the flexural member.

21.3.2 — Longitudinal reinforcement

21.3.2.1 — At any section of a flexural member, except as provided in 10.5.3, for top as well as for bottom reinforcement, the amount of reinforcement shall not be less than that given by Eq. (10-3) but not less than $200 b_w d/f_y$, and the reinforcement ratio ρ shall not exceed 0.025. At least two bars shall be provided continuously both top and bottom.

21.3.2.2 — Positive moment strength at joint face shall be not less than one-half of the negative moment strength provided at that face of the joint. Neither the negative nor the positive moment strength at any section along member length shall be less than one-fourth the maximum moment strength provided at face of either joint.

COMMENTARY

R21.2.6.2 — Welding or tack-welding of crossing reinforcing bars can lead to local embrittlement of the steel. If such welding will facilitate fabrication or field installation, it must be done only on bars added expressly for construction. Provisions for tack-welding of crossing reinforcing bars do not apply to materials that are welded with welding operations under continuous competent control as in the manufacture of welded wire fabric.

R21.3 — Flexural members of frames

R21.3.1 — Scope

This section refers to girders of frames resisting lateral loads induced by earthquake motions. If any frame member is subjected to a factored axial compressive force exceeding $(A_g f_c'/10)$, it is to be proportioned and detailed as described in 21.4.

Experimental evidence[21.10] indicates that, under reversals of displacement into the nonlinear range, behavior of continuous members having length-to-depth ratios of less than four is significantly different from the behavior of relatively slender members. Design rules derived from experience with relatively slender members do not apply directly to members with length-to-depth ratios less than four, especially with respect to shear strength.

Geometric constraints indicated in 21.3.1.3 and 21.3.1.4 were derived from practice with reinforced concrete frames resisting earthquake-induced forces.[21.1]

R21.3.2 — Longitudinal reinforcement

Section 10.3.3 limits the tensile reinforcement ratio in a flexural member to a fraction of the amount that would produce "balanced" conditions. For a section subjected to bending only and loaded monotonically to yielding, this approach is feasible because the likelihood of compressive failure can be estimated reliably with the behavioral model assumed for determining the reinforcement ratio corresponding to "balanced" failure. The same behavioral model (because of incorrect assumptions such as linear strain distribution, well-defined yield point for the steel, limiting compressive strain in the concrete of 0.003, and compressive stresses in the shell concrete) fails to describe the conditions in a flexural member subjected to reversals of displacements well into the inelastic range. Thus, there is little rationale for continuing to refer to "balanced" condi-

CODE

21.3.2.3 — Lap splices of flexural reinforcement shall be permitted only if hoop or spiral reinforcement is provided over the lap length. Maximum spacing of the transverse reinforcement enclosing the lapped bars shall not exceed ***d*/4** or 4 in. Lap splices shall not be used (a) within the joints, (b) within a distance of twice the member depth from the face of the joint, and (c) at locations where analysis indicates flexural yielding caused by inelastic lateral displacements of the frame.

21.3.2.4 — Welded splices and mechanical connections shall conform to 21.2.6.1.

21.3.3 — Transverse reinforcement

21.3.3.1 — Hoops shall be provided in the following regions of frame members:

(1) Over a length equal to twice the member depth measured from the face of the supporting member toward midspan, at both ends of the flexural member.

(2) Over lengths equal to twice the member depth on both sides of a section where flexural yielding is likely to occur in connection with inelastic lateral displacements of the frame.

21.3.3.2 — The first hoop shall be located not more than 2 in. from the face of a supporting member. Maximum spacing of the hoops shall not exceed (a) ***d*/4,** (b) eight times the diameter of the smallest longitudinal bars, (c) 24 times the diameter of the hoop bars, and (d) 12 in.

21.3.3.3 — Where hoops are required, longitudinal bars on the perimeter shall have lateral support conforming to 7.10.5.3.

21.3.3.4 — Where hoops are not required, stirrups with seismic hooks at both ends shall be spaced at a distance not more than ***d*/2** throughout the length of the member.

21.3.3.5 — Stirrups or ties required to resist shear shall be hoops over lengths of members as specified in 21.3.3, 21.4.4, and 21.5.2.

21.3.3.6 — Hoops in flexural members shall be permitted to be made up of two pieces of reinforcement: a stirrup having seismic hooks at both ends and closed by a crosstie. Consecutive crossties engaging the same longitudinal bar shall have their 90-deg hooks at

COMMENTARY

tions in earthquake-resistant design of reinforced concrete structures.

The limiting reinforcement ratio of 0.025 is based primarily on considerations of steel congestion and, indirectly, on limiting shear stresses in girders of typical proportions. The requirement of at least two bars, top and bottom, refers again to construction rather than behavioral requirements.

Lap splices of reinforcement (see 21.3.2.3) are prohibited at regions where flexural yielding is anticipated because such splices are not considered reliable under conditions of cyclic loading into the inelastic range. Transverse reinforcement for lap splices at any location is mandatory because of the likelihood of loss of shell concrete.

R21.3.3 —Transverse reinforcement

This reinforcement is required primarily to confine the concrete and maintain lateral support for the reinforcing bars in regions where yielding is expected. Examples of hoops suitable for flexural members of frames are shown in Fig. R21.3.3.

In case of members with varying strength along the span or members for which the permanent load represents a large proportion of the total design load, concentrations of inelastic rotation may occur within the span. If such a condition is anticipated, transverse reinforcement must be provided also in regions where yielding is expected.

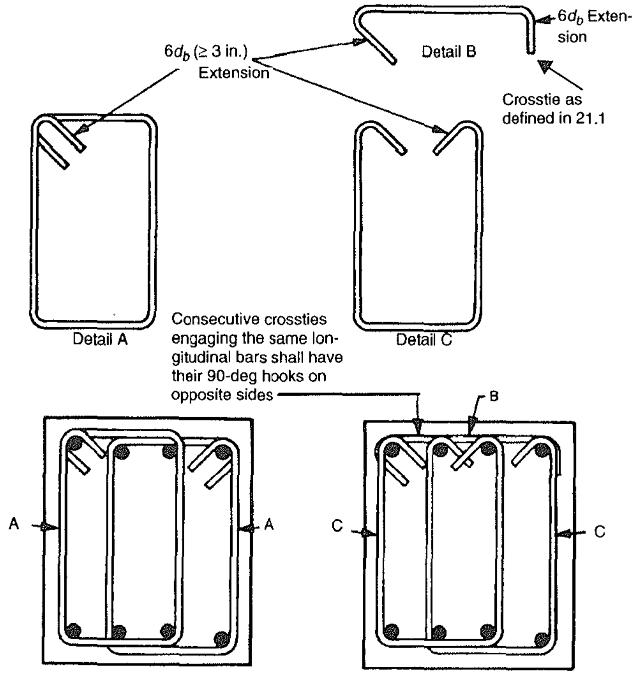

$6d_b$ (≥ 3 in.) Extension
Detail B
$6d_b$ Extension
Crosstie as defined in 21.1
Detail A
Detail C
Consecutive crossties engaging the same longitudinal bars shall have their 90-deg hooks on opposite sides
B
A
A
C
C

Fig. R21.3.3—Examples of overlapping hoops

CODE

opposite sides of the flexural member. If the longitudinal reinforcing bars secured by the crossties are confined by a slab on only one side of the flexural frame member, the 90-deg hooks of the crossties shall be placed on that side.

21.3.4 — Shear strength requirements

21.3.4.1 — Design forces

The design shear force V_e shall be determined from consideration of the statical forces on the portion of the member between faces of the joints. It shall be assumed that moments of opposite sign corresponding to probable strength M_{pr} act at the joint faces and that the member is loaded with the factored tributary gravity load along its span.

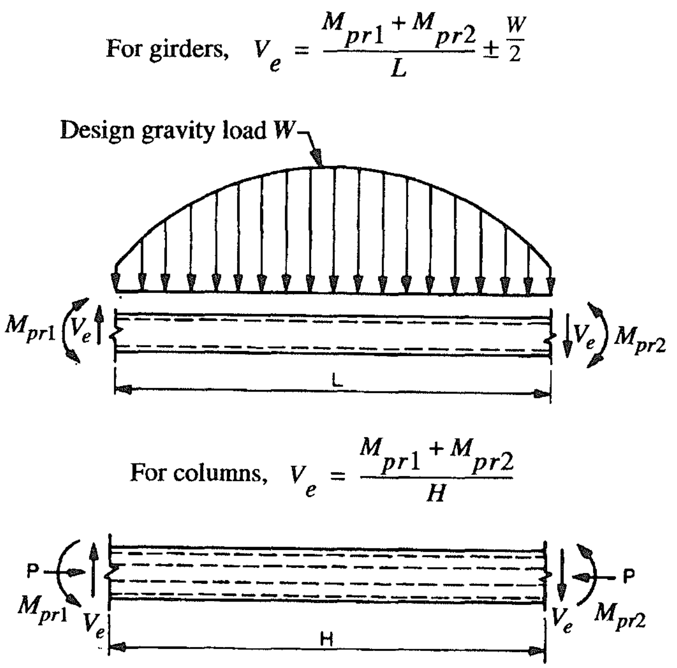
COMMENTARY

Because spalling of the concrete shell is anticipated during strong motion, especially at and near regions of flexural yielding, all web reinforcement must be provided in the form of closed hoops as defined in 21.3.3.5.

R21.3.4 — Shear strength requirements

R21.3.4.1 — Design forces

In determining the equivalent lateral forces representing earthquake effects for the type of frames considered, it is assumed that frame members will dissipate energy in the nonlinear range of response. Unless a frame member possesses a strength that is a multiple on the order of 3 or 4 of the design forces, it must be assumed that it will yield in the event of a major earthquake. The design shear force must be



For girders, $V_e = \frac{M_{pr1} + M_{pr2}}{L} \pm \frac{W}{2}$

For columns, $V_e = \frac{M_{pr1} + M_{pr2}}{H}$

Notes:

1. Direction of shear force V_e depends on relative magnitudes of gravity loads and shear generated by end moments.

2. End moments M_{pr} based on steel tensile stress = 1.25 f_y, where f_y is the specified yield strength. (Both end moments should be considered in both directions, clockwise and counter-clockwise)

3. End moment M_{pr} for columns need not be greater than moments generated by the M_{pr} of the beams framing into the beam-column joints. V_e shall never be less than that required by analysis of the structure.

Fig. R21.3.4—Design shears for girders and columns

CODE

21.3.4.2 — Transverse reinforcement

Transverse reinforcement over the lengths identified in 21.3.3.1 shall be proportioned to resist shear assuming $V_c = 0$ when both of the following conditions occur:

(1) The earthquake-induced shear force calculated in accordance with 21.3.4.1 represents one-half or more of the maximum required shear strength within those lengths.

(2) The factored axial compressive force including earthquake effects is less than $A_g f_c'/20$.

21.4 — Frame members subjected to bending and axial load

21.4.1 — Scope

The requirements of this section apply to frame members (a) resisting earthquake-induced forces, and (b) having a factored axial force exceeding $(A_g f_c'/10)$. These frame members shall also satisfy the following conditions:

21.4.1.1 — The shortest cross-sectional dimension, measured on a straight line passing through the geometric centroid, shall not be less than 12 in.

21.4.1.2 — The ratio of the shortest cross-sectional dimension to the perpendicular dimension shall not be less than 0.4.

COMMENTARY

a good approximation of the maximum shear that may develop in a member. Therefore, required shear strength for frame members is related to flexural strengths of the designed member rather than to factored shear forces indicated by lateral load analysis. The conditions described by 21.3.4.1 are illustrated in Fig. R21.3.4.

Because the actual yield strength of the longitudinal reinforcement may exceed the specified yield strength and because strain hardening of the reinforcement is likely to take place at a joint subjected to large rotations, required shear strengths are determined using a stress of at least $1.25f_y$ in the longitudinal reinforcement.

R21.3.4.2 — Transverse reinforcement

Experimental studies of reinforced concrete members subjected to cyclic loading have demonstrated that more shear reinforcement is required to ensure a flexural failure if the member is subjected to alternating nonlinear displacements than if the member is loaded in one direction only: the necessary increase of shear reinforcement being higher in the case of no axial load.[21.11, 21.12] This observation is reflected in the specifications (21.3.4.2) by eliminating the term representing the contribution of concrete to shear strength. The added conservatism on shear is deemed necessary in locations where potential flexural hinging may occur. However, this stratagem, chosen for its relative simplicity, should not be interpreted to mean that no concrete is required to resist shear. On the contrary, it may be argued that the concrete core resists all the shear with the shear (transverse) reinforcement confining and thus strengthening the concrete. The confined concrete core plays an important role in the behavior of the beam and should not be reduced to a minimum just because the design expression does not recognize it explicitly.

R21.4 — Frame members subjected to bending and axial load

R21.4.1 — Scope

This section contains rules intended primarily for columns of frames serving to resist earthquake forces. Frame members which are not columns but do not satisfy 21.3.1 are to be proportioned and detailed according to this section.

The geometric constraints in 21.4.1.1 and 21.4.1.2 follow from previous practice.[21.1]

CODE

21.4.2 — Minimum flexural strength of columns

21.4.2.1 — Flexural strength of any column proportioned to resist a factored axial compressive force exceeding **($A_g f_c'/10$)** shall satisfy 21.4.2.2 or 21.4.2.3.

Lateral strength and stiffness of columns not satisfying 21.4.2.2 shall be ignored in determining the calculated strength and stiffness of the structure but shall conform to 21.7.

21.4.2.2 — The flexural strengths of the columns shall satisfy Eq. (21-1)

$$\Sigma M_e \geq (6/5)\, \Sigma M_g \qquad (21\text{-}1)$$

ΣM_e = sum of moments, at the center of the joint, corresponding to the design flexural strength of the columns framing into that joint. Column flexural strength shall be calculated for the factored axial force, consistent with the direction of the lateral forces considered, resulting in the lowest flexural strength.

ΣM_g = sum of moments, at the center of the joint, corresponding to the design flexural strengths of the girders framing into that joint.

Flexural strengths shall be summed such that the column moments oppose the beam moments. Eq. (21-1) shall be satisfied for beam moments acting in both directions in the vertical plane of the frame considered.

21.4.2.3 — If 21.4.2.2 is not satisfied at a joint, columns supporting reactions from that joint shall be provided with transverse reinforcement as specified in 21.4.4 over their full height.

21.4.3 — Longitudinal reinforcement

21.4.3.1 — The reinforcement ratio ρ_g shall not be less than 0.01 and shall not exceed 0.06.

21.4.3.2 — Welded splices and mechanical connections shall conform to 21.2.6.1. Lap splices shall be permitted only within the center half of the member length and shall be proportioned as tension splices.

COMMENTARY

R21.4.2 — Minimum flexural strength of columns

The intent of 21.4.2.2 is to reduce the likelihood of yielding in columns.

If 21.4.2.2 cannot be satisfied at a joint, any positive contribution of the column or columns involved to the lateral strength and stiffness of the structure is to be ignored. The engineer is cautioned not to ignore any negative contributions of the presence of the column in question to building behavior. For example, ignoring the stiffness of the columns ought not be used as a justification for reducing the design base shear. If inclusion of those columns in the analytical model of the building results in an increase in torsional effects, the increase must be provided for in accordance with the requirements of the governing code.

R21.4.3 — Longitudinal reinforcement

The lower bound to the reinforcement ratio in members carrying axial forces as well as bending refers to the traditional concern for the effects of time-dependent deformations of the concrete and the desire to have a sizeable difference between the cracking and yielding moments. The upper bound reflects concern for steel congestion, load transfer from floor elements to column in low-rise construction, and the development of large shear stresses.

Spalling of the shell concrete, which is likely to occur near the ends of the column in frames of typical configuration, makes lap splices in those locations vulnerable. If lap splices are to be used at all, they must be located near the midheight where stress reversal is likely to be limited to a smaller stress range than at locations near the joints.

CODE

21.4.4 — Transverse reinforcement

21.4.4.1 — Transverse reinforcement as specified below shall be provided unless a larger amount is required by 21.4.5.

(1) The volumetric ratio of spiral or circular hoop reinforcement ρ_s shall not be less than that indicated by Eq. (21-2).

$$\rho_s = 0.12\, f_c'/f_{yh} \qquad (21\text{-}2)$$

and shall not be less than that required by Eq. (10-6).

(2) The total cross-sectional area of rectangular hoop reinforcement shall not be less than that given by Eq. (21-3) and (21-4).

$$A_{sh} = 0.3(sh_c\, f_c'/f_{yh})[(A_g/A_{ch}) - 1] \qquad (21\text{-}3)$$

$$A_{sh} = 0.09 sh_c f_c'/f_{yh} \qquad (21\text{-}4)$$

(3) Transverse reinforcement shall be provided by either single or overlapping hoops. Crossties of the same bar size and spacing as the hoops shall be permitted to be used. Each end of the crosstie shall engage a peripheral longitudinal reinforcing bar. Consecutive crossties shall be alternated end for end along the longitudinal reinforcement.

(4) If the design strength of member core satisfies the requirement of the specified loading combinations including earthquake effect, Eq. (21-3) and (10-6) need not be satisfied.

21.4.4.2 — Transverse reinforcement shall be spaced at a distance not exceeding (a) one-quarter of the minimum member dimension, and (b) 4 in.

21.4.4.3 — Crossties or legs of overlapping hoops shall not be spaced more than 14 in. on center in the direction perpendicular to the longitudinal axis of the structural member.

21.4.4.4 — Transverse reinforcement in amount specified in 21.4.4.1 through 21.4.4.3 shall be provided over a length ℓ_o from each joint face and on both sides of any section where flexural yielding is likely to occur in connection with inelastic lateral displacements of the frame. The length ℓ_o shall not be less than (a) the depth of the member at the joint face or at the

COMMENTARY

Welding and mechanical splices may occur at any level but not more than half the bars may be spliced at any one section.

R21.4.4 — Transverse reinforcement

The reason for the requirements in this section is concern for confining the concrete and providing lateral support to the reinforcement.

For axially compressed members subjected to steadily increasing load, the effect of helical (spiral) reinforcement on strength of confined concrete has been well established.[21.13] Eq. (10-6) follows from the arbitrary design concept that, under axial loading, maximum column capacity before loss of shell be equal to that at large compressive strains with the spiral reinforcement stressed to its useful limit. The toughness of the axially loaded "spiral" column is not directly relevant to its role in the earthquake-resistant frame where toughness is related to its performance under reversals of moment as well as axial load. For earthquake-resistant construction, there is no reason to modify Eq. (10-6) other than adding the varying lower bound given by Eq. (21-2) which governs for larger columns with gross cross-sectional area A_g less than approximately 1.25 times the core area.

A conservative evaluation of the available data[21.13-21.16] pertaining to the effect of rectilinear transverse reinforcement on behavior of reinforced concrete suggests that such reinforcement improves ductility consistently, but its effect on strength is difficult to express reliably in terms of the apparently critical material properties. There is no intelligible relationship for determining an explicit equivalence between spiral and rectilinear transverse reinforcement. Considering that the basis for determining the amount of

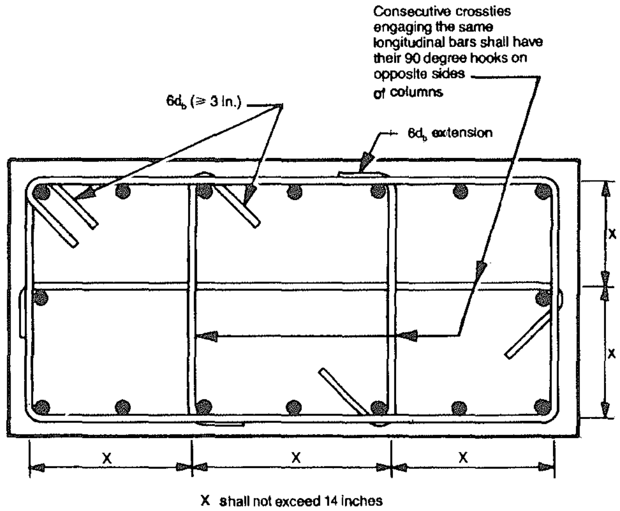

Consecutive crossties engaging the same longitudinal bars shall have their 90 degree hooks on opposite sides of columns
6d$_b$ (≥ 3 in.)
6d$_b$ extension
X
X
X
X
X
X shall not exceed 14 inches

Fig. R21.4.4—Example of transverse reinforcement in columns

CODE

section where flexural yielding is likely to occur, (b) one-sixth of the clear span of the member, and (c) 18 in.

21.4.4.5 — Columns supporting reactions from discontinued stiff members, such as walls, shall be provided with transverse reinforcement as specified in 21.4.4.1 through 21.4.4.3 over their full height beneath the level at which the discontinuity occurs if the factored axial compressive force in these members, related to earthquake effect, exceeds **($A_g f_c'/10$)**. Transverse reinforcement as specified in 21.4.4.1 through 21.4.4.3 shall extend into the discontinued member for at least the development length of the largest longitudinal reinforcement in the column in accordance with 21.5.4. If the lower end of the column terminates on a wall, transverse reinforcement as specified in 21.4.4.1 through 21.4.4.3 shall extend into the wall for at least the development length of the largest longitudinal reinforcement in the column at the point of termination. If the column terminates on a footing or mat, transverse reinforcement as specified in 21.4.4.1 through 21.4.4.3 shall extend at least 12 in. into the footing or mat.

21.4.4.6 — Where transverse reinforcement, as specified in 21.4.4.1 through 21.4.4.3, is not provided throughout the full length of the column, the remainder of the column length shall contain spiral or hoop reinforcement with center-to-center spacing not exceeding the smaller of six times the diameter of the longitudinal column bars or 6 in.

21.4.5 — Shear strength requirements

21.4.5.1 — Design forces

The design shear force V_e shall be determined from consideration of the maximum forces that can be generated at the faces of the joints at each end of the member. These joint forces shall be determined using the maximum probable moment strengths M_{pr} of the member associated with the range of factored axial loads on the member. The member shears need not exceed those determined from joint strengths based on the probable moment strength M_{pr} of the trans-

COMMENTARY

spiral reinforcement Eq. (10-6) is not directly relevant to loading conditions encountered under earthquake effects, it is possible to determine the required amount of rectilinear confining reinforcement on the general premise that, to provide confinement comparable to that of spiral reinforcement, there should be more of it. Eq. (21-3) and (21-4), which apply to rectilinear reinforcement, compare to Eq. (10-6) and (21-2), respectively, but Eq. (21-2) and (21-4) require more transverse reinforcement per unit length of member.

Eq. (21-3), which governs for large sections, is ignored if the ratio of required-to-provided strength is low.

Transverse reinforcement required by Eq. (10-6), (21-2), (21-3), and (21-4) is to be distributed over regions where inelastic action is considered to be likely (21.4.4.4).

Fig. R21.4.4 shows an example of transverse reinforcement provided by one hoop and three crossties. Fig. R21.3.3 shows examples of transverse reinforcement details for flexural elements. Crossties with a 90-deg hook are not as effective as crossties with 135-deg hooks, or hoops, in providing confinement. They have been shown to be adequate by tests to provide sufficient confinement as long as the crosstie ends with 90-deg hooks are alternated.

Dynamic response analyses and field observations indicate that columns supporting discontinued stiff members, such as walls or trusses, tend to develop considerable inelastic response. Therefore, it is required that these columns have special transverse reinforcement throughout their length. This rule covers all columns beneath the level at which the stiff member has been discontinued, unless the factored forces corresponding to earthquake effect are low (see 21.4.4.5).

R21.4.4.6 — The provisions of 21.4.4.6 were added in 1989 to provide reasonable protection and ductility to the midheight of columns between specified transverse reinforcement. Observations in earthquakes have shown significant damage to columns in the nonconfined region, and the minimum ties on spirals required should provide a more uniform toughness of the column along its length.

R21.4.5 — Shear strength requirements

R21.4.5.1 — Design forces

The provisions of 21.3.4.1 also apply to members subjected to axial loads (i.e., columns). Above the ground floor the moment at a joint may be limited by flexural strengths of the beams framing into the joint. Where beams frame into opposite sides of a joint, the combined strength may be the sum of the negative moment strength of the beam on one side of the joint and the positive moment strength of the beam on the other side of the joint. Moment strengths must be determined using a strength reduction factor of 1.0 and

CODE

verse members framing into the joint. In no case shall V_e be less than the factored shear determined by analysis of the structure.

21.4.5.2 — Transverse reinforcement over the lengths ℓ_o, identified in 21.4.4.4, shall be proportioned to resist shear assuming $V_c = 0$ when both the following conditions occur:

(1) The earthquake-induced shear force, calculated in accordance with 21.4.5.1, represents one-half or more of the maximum required shear strength within those lengths.

(2) The factored axial compressive force including earthquake effects is less than $A_g f_c'/20$.

21.5 — Joints of frames

21.5.1 — General requirements

21.5.1.1 — Forces in longitudinal beam reinforcement at the joint face shall be determined by assuming that the stress in the flexural tensile reinforcement is $1.25f_y$.

21.5.1.2 — Strength of joint shall be governed by the appropriate strength reduction factors specified in 9.3.

21.5.1.3 — Beam longitudinal reinforcement terminated in a column shall be extended to the far face of the confined column core and anchored in tension according to 21.5.4 and in compression according to Chapter 12.

21.5.1.4 — Where longitudinal beam reinforcement extends through a beam-column joint, the column dimension parallel to the beam reinforcement shall not be less than 20 times the diameter of the largest longitudinal bar for normal weight concrete. For lightweight concrete, the dimension shall be not less than 26 times the bar diameter.

COMMENTARY

reinforcing steel stress equal to at least $1.25f_y$. Distribution of the combined moment strength of the beams to the columns above and below the joint should be based on analysis. The value of M_{pr} in Fig. R21.3.4 may be computed from the flexural member strengths at the beam-column joints.

R21.5 — Joints of frames

R21.5.1 — General requirements

Development of inelastic rotations at the faces of joints of reinforced concrete frames is associated with strains in the flexural reinforcement well in excess of the yield strain. Consequently, joint shear force generated by the flexural reinforcement is calculated for a stress of $1.25f_y$ in the reinforcement (see 21.5.1.1). A detailed explanation of the reasons for the possible development of stresses in excess of the yield strength in girder tensile reinforcement is provided in Reference 21.9.

R21.5.1.4 — Various researchers[21.17-21.21] have shown that straight beam bars may slip within the beam-column joint during a series of large moment reversals. The bond stresses on these straight bars may be very large. To substantially reduce slip during the formation of adjacent beam hinging, it would be necessary to have a ratio of column dimension to bar diameter of approximately $^1/_{32}$, which would result in very large joints. On reviewing the available tests, the limit of $^1/_{20}$ of the column depth in the direction of loading for the maximum size of beam bars for normal weight concrete, and a limit of $^1/_{26}$ for lightweight concrete were chosen. Due to the lack of specific data, the modification for lightweight concrete used a factor of 1.3 from Chapter 12. Committee 318 feels that these limits provide reasonable control on the amount of potential slip of the beam bars in a beam-column joint considering the number of anticipated inelastic excursions of the building frames during a major earthquake. A thorough treatment of this topic is given in Reference 21.22.

CODE

21.5.2 — Transverse reinforcement

21.5.2.1 — Transverse hoop reinforcement, as specified in 21.4.4 shall be provided within the joint, unless the joint is confined by structural members as specified in 21.5.2.2.

21.5.2.2 — Within the depth of the shallowest framing member, transverse reinforcement equal to at least one-half the amount required by 21.4.4.1 shall be provided where members frame into all four sides of the joint and where each member width is at least three-fourths the column width. At these locations, the spacing specified in 21.4.4.2(b) shall be permitted to be increased to 6 in.

21.5.2.3 — Transverse reinforcement as required by 21.4.4 shall be provided through the joint to provide confinement for longitudinal beam reinforcement outside the column core if such confinement is not provided by a beam framing into the joint.

21.5.3 — Shear strength

21.5.3.1 — The nominal shear strength of the joint shall not be taken greater than the forces specified below for normal weight aggregate concrete.

For joints confined on all four faces............ $20\sqrt{f_c'}\,A_j$

For joints confined on three faces or on two opposite faces................................ $15\sqrt{f_c'}\,A_j$

For others.. $12\sqrt{f_c'}\,A_j$

COMMENTARY

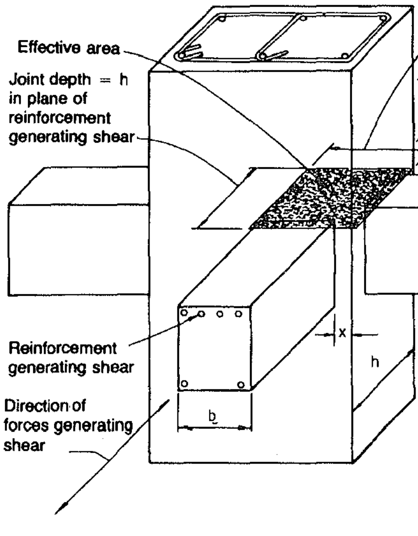
R21.5.2 — Transverse reinforcement

However low the calculated shear force in a joint of a frame resisting earthquake-induced forces, confining reinforcement (21.4.4) must be provided through the joint around the column reinforcement (21.5.2.1). As specified in 21.5.2.2, confining reinforcement may be reduced if horizontal members frame into all four sides of the joint. The 1989 code provided a maximum limit on spacing to these areas based on available data.[21.23-21.26]

Section 21.5.2.3 refers to a joint where the width of the girder exceeds the corresponding column dimension. In that case, girder reinforcement not confined by the column reinforcement must be provided lateral support either by a girder framing into the same joint or by transverse reinforcement.

R21.5.3 — Shear strength

The requirements in Chapter 21 for proportioning joints are based on Reference 21.9 in that behavioral phenomena within the joint are interpreted in terms of a nominal shear strength of the joint. Because tests of joints[21.17] and deep beams[21.10] indicated that shear strength was not as sensitive to joint (shear) reinforcement as implied by the expression developed by ACI Committee 326[21.27] for beams and adopted to apply to joints by ACI Committee 352,[21.9] Committee 318 elected to set the strength of the joint as a func-


Effective area
Joint depth = h in plane of reinforcement generating shear
Effective joint width b + h
b + 2x
x
h
b
Reinforcement generating shear
Direction of forces generating shear

Note:
Effective area of joint for forces in each direction of framing is to be considered separately. Joint illustrated does not meet conditions of Sections 21.5.2.3 and 21.5.3.1 necessary to be considered confined because the framing members do not cover at least 3/4 of each of the joints.

Fig. R21.5.3—Effective area of joint

CODE

A member that frames into a face is considered to provide confinement to the joint if at least three-quarters of the face of the joint is covered by the framing member. A joint is considered to be confined if such confining members frame into all faces of the joint.

21.5.3.2 — For lightweight aggregate concrete, the nominal shear strength of the joint shall not exceed three-quarters of the limits given in 21.5.3.1.

21.5.4 — Development length of bars in tension

21.5.4.1 — The development length ℓ_{dh} for a bar with a standard 90-deg hook in normal weight aggregate concrete shall not be less than $8d_b$, 6 in., and the length required by Eq. (21-5).

$$\ell_{dh} = f_y d_b / (65\sqrt{f_c'}) \qquad (21\text{-}5)$$

for bar sizes No. 3 through No. 11.

For lightweight aggregate concrete, the development length for a bar with a standard 90-deg hook shall not be less than $10d_b$, 7.5 in., and 1.25 times that required by Eq. (21-5).

The 90-deg hook shall be located within the confined core of a column or of a boundary element.

21.5.4.2 — For bar sizes No. 3 through No. 11, the development length ℓ_d for a straight bar shall not be less than (a) two-and-a-half (2.5) times the length required by 21.5.4.1 if the depth of the concrete cast in one lift beneath the bar does not exceed 12 in., and (b) three-and-a-half (3.5) times the length required by 21.5.4.1 if the depth of the concrete cast in one lift beneath the bar exceeds 12 in.

21.5.4.3 — Straight bars terminated at a joint shall pass through the confined core of a column or of a boundary element. Any portion of the straight embedment length not within the confined core shall be increased by a factor of 1.6.

21.5.4.4 — If epoxy-coated reinforcement is used, the development lengths in 21.5.4.1 through 21.5.4.3 shall be multiplied by the applicable factor specified in 12.2.4 or 12.5.3.6.

COMMENTARY

tion of only the compressive strength of the concrete (21.5.3) and to require a minimum amount of transverse reinforcement in the joint (21.5.2). The effective area of joint A_j is illustrated in Fig. R21.5.3. In no case is A_j greater than the column cross-sectional area.

The three level shear strength provision is based on the recommendation of ACI Committee 352.[21.9] Test data reviewed by the committee[21.25] indicate that the lower value given in 21.5.3.1 of ACI 318-83 is unconservative when applied to corner joints.

R21.5.4 — Development length of bars in tension

Minimum development length for deformed bars with standard hooks embedded in normal weight concrete is determined using Eq. (21-5). Eq. (21-5) is based on the requirements of 12.5. Because Chapter 21 stipulates that the hook is to be embedded in confined concrete, the coefficients 0.7 (for concrete cover) and 0.8 (for ties) have been incorporated in the constant used in Eq. (21-5). The development length that would be derived directly from 12.5 is increased to reflect the effect of load reversals.

The development length in tension for a reinforcing bar with a standard hook is defined as the distance, parallel to the bar, from the critical section (where the bar is to be developed) to a tangent drawn to the outside edge of the hook. The tangent is to be drawn perpendicular to the axis of the bar. (Fig. R12.5.1)

Factors such as the actual stress in the reinforcement being more than the yield force and the effective development length not necessarily starting at the face of the joint were implicitly considered in the development of the expression for basic development length which has been used as the basis for Eq. (21-5).

For lightweight aggregate concrete, the length required by Eq. (21-5) is to be increased by 25 percent to compensate for variability of bond characteristics of reinforcing bars in various types of lightweight aggregate concrete.

Section 21.5.4.2 specifies the minimum development length for straight bars as a multiple of the length indicated by 21.5.4.1. Case (b) of 21.5.4.2 refers to "top" bars.

If the required straight embedment length of a reinforcing bar extends beyond the confined volume of concrete (as defined in 21.3.3, 21.4.4, or 21.5.2), the required development length is increased on the premise that the limiting bond stress outside the confined region is less than that inside.

$$\ell_{dm} = 1.6(\ell_d - \ell_{dc}) + \ell_{dc}$$

or

$$\ell_{dm} = 1.6\ell_d - 0.6\ell_{dc}$$

where

CODE

21.6 — Structural walls, diaphragms, and trusses

21.6.1 — Scope

The requirements of this section apply to structural walls and trusses serving as parts of the earthquake force-resisting systems as well as to diaphragms, struts, ties, chords and collector elements which transmit forces induced by earthquake.

21.6.2 — Reinforcement

21.6.2.1 — The reinforcement ratio ρ_v for structural walls shall not be less than 0.0025 along the longitudinal and transverse axes. If the design shear force does not exceed $A_{cv}\sqrt{f_c'}$, the minimum reinforcement for structural walls shall be in conformance with 14.3. The minimum reinforcement ratio for structural diaphragms shall be in conformance with 7.12. Reinforcement spacing each way in structural walls and diaphragms shall not exceed 18 in. Reinforcement provided for shear strength shall be continuous and shall be distributed across the shear plane.

21.6.2.2 — At least two curtains of reinforcement shall be used in a wall if the in-plane factored shear force assigned to the wall exceeds $2A_{cv}\sqrt{f_c'}$.

21.6.2.3 — Structural truss elements, struts, ties, and collector elements with compressive stresses exceeding $0.2f_c'$ shall have special transverse reinforcement, as specified in 21.4.4, over the total length of the element. The special transverse reinforcement is allowed to be discontinued at a section where the calculated compressive stress is less than $0.15f_c'$. Stresses shall be calculated for the factored forces using a linearly elastic model and gross section properties of the elements considered.

COMMENTARY

ℓ_{dm} = required development length if bar is not entirely embedded in confined concrete

ℓ_d = required development length for straight bar embedded in confined concrete (21.5.4.3)

ℓ_{dc} = length of bar embedded in confined concrete

Lack of reference to No. 14 and No. 18 bars in 21.5.4 is due to paucity of information on anchorage of such bars subjected to load reversals simulating earthquake effects.

R21.6 — Structural walls, diaphragms, and trusses

R21.6.1 — Scope

This section contains requirements for the dimensions and details of relatively stiff structural systems including parts of roof and floor systems transmitting inertia forces, as well as walls and trusses. Stubby frame members, which constitute parts of the lateral-force resisting system, are also to be proportioned in accordance with the requirements of this section. However, it is not the intent to combine frame members and wall members to circumvent ductility requirements by developing a lateral force resisting system contrary to the general building code. Isolated elements of a framing system not conforming to 21.3 or 21.4 should be designed in conformance to 21.7.1.

R21.6.2 — Reinforcement

Reinforcement minima (21.6.2.1) follow from preceding codes of practice. The uniform distribution requirement of the shear reinforcement is related to the intent to control the width of inclined cracks. The requirement for two layers of reinforcement in walls carrying substantial design shears (21.6.2.2) is based on the observation that, under ordinary construction conditions, the probability of maintaining a single layer of reinforcement near the middle of the wall section is quite low. Furthermore, presence of reinforcement close to the surface tends to inhibit fragmentation of the concrete in the event of severe cracking during an earthquake.

Compressive stress calculated for the factored forces on a linearly elastic model based on gross section of the structural member is used as an index value to determine whether confining reinforcement is required. A calculated compressive stress of $0.2f_c'$ in a member is assumed to indicate that integrity of the entire structure is dependent on the ability of that member to resist substantial compressive force under severe cyclic loading. Therefore, transverse reinforcement, as specified in 21.4.4, is required in such members to provide confinement for the concrete and the compressed reinforcement (21.6.2.3).

CODE

21.6.2.4 — All continuous reinforcement in structural walls, diaphragms, trusses, struts, ties, chords, and collector elements shall be anchored or spliced in accordance with the provisions for reinforcement in tension as specified in 21.5.4.

21.6.3 — Design forces

The design shear force V_u shall be obtained from the lateral load analysis in accordance with the factored loads and combinations specified in 9.2.

21.6.4 — Diaphragms

21.6.4.1 — Minimum thickness of diaphragms

Concrete diaphragms and composite topping slabs serving as diaphragms used to transmit earthquake forces shall not be less than 2 in. thick.

21.6.4.2 — Cast-in-place composite topping slab diaphragms

A composite topping slab cast-in-place on a precast floor or roof system shall be permitted to be used as a diaphragm provided the topping slab is reinforced and its connections are proportioned and detailed to provide for a complete transfer of forces to chords, collector elements, and resisting elements. The surface of the previously hardened concrete on which the topping slab is placed shall be clean, free of laitance, and shall be intentionally roughened.

21.6.5 — Shear strength

21.6.5.1 — Nominal shear strength of structural walls and diaphragms shall be determined using either 21.6.5.2 or 21.6.5.3.

COMMENTARY

Because the actual forces in longitudinal reinforcing bars of stiff members may exceed the calculated forces, it is required (21.6.2.4) that all continuous reinforcement be developed fully.

R21.6.3 — Design forces

Design shears for structural walls, trusses, and diaphragms are obtained from lateral load analysis with the appropriate load factors. However, the designer should consider the possibility of yielding in components of such structures, as in the portion of a wall between two window openings, in which case the actual shear may be well in excess of the shear indicated by lateral load analysis based on factored design forces.

R21.6.4 — Diaphragms

Diaphragms as used in building construction are structural elements (such as a floor or roof system) that provide some or all of the following functions:

(a) Support for building elements (such as walls, partitions, and cladding) resisting horizontal forces but not acting as part of the building vertical lateral-force resisting system.

(b) Transfer of lateral forces from the point of application to the building vertical lateral-force resisting system.

(c) Interconnection of various components of the building vertical lateral-force resisting system with appropriate strength, stiffness, and toughness to permit the deformation and rotation of the building as unit.[21.28]

R21.6.4.1 — Minimum thickness of diaphragms

The minimum thickness of concrete diaphragms reflects current usage in joist and waffle systems and composite topping slabs on precast floor and roof systems.

R21.6.4.2 — Cast-in-place composite topping slab diaphragms

A bonded topping slab is required so that the floor or roof system can provide restraint against slab buckling. Reinforcement is required to ensure the continuity of the shear transfer across precast joints. The connection requirements are introduced to ensure that a complete system and necessary shear transfers are provided.

R21.6.5 — Shear strength

Section 21.6.5 is concerned with proportioning and detailing of structural walls and floor diaphragms which resist shear forces caused by earthquake motions. Shear strength

CODE

21.6.5.2 — Nominal shear strength V_n of structural walls and diaphragms shall be assumed not to exceed the shear force calculated from

$$V_n = A_{cv}\left(2\sqrt{f_c'} + \rho_n f_y\right) \qquad (21\text{-}6)$$

21.6.5.3 — For walls (diaphragms) and wall (diaphragm) segments having a ratio of (h_w/ℓ_w) less than 2.0, nominal shear strength of wall (diaphragm) shall be determined from Eq. (21-7)

$$V_n = A_{cv}\left(\alpha_c\sqrt{f_c'} + \rho_n f_y\right) \qquad (21\text{-}7)$$

where the coefficient α_c varies linearly from 3.0 for $(h_w/\ell_w) = 1.5$ **to 2.0** for $(h_w/\ell_w) = 2.0$.

21.6.5.4 — In 21.6.5.3, value of ratio (h_w/ℓ_w) used for determining V_n for segments of a wall or diaphragm shall be the larger of the ratios for the entire wall (diaphragm) and the segment of wall (diaphragm) considered.

21.6.5.5 — Walls (diaphragms) shall have distributed shear reinforcement providing resistance in two orthogonal directions in the plane of the wall (diaphragm). If the ratio (h_w/ℓ_w) does not exceed 2.0, reinforcement ratio ρ_v shall not be less than reinforcement ratio ρ_n.

21.6.5.6 — Nominal shear strength of all wall piers sharing a common lateral force shall not be assumed to exceed $8A_{cv}\sqrt{f_c'}$, where A_{cv} is the total cross-sectional area, and the nominal shear strength of any one of the individual wall piers shall not be assumed to exceed $10A_{cp}\sqrt{f_c'}$, where A_{cp} represents the cross-sectional area of the pier considered.

21.6.5.7 — Nominal shear strength of horizontal wall segments shall not be assumed to exceed $10A_{cp}\sqrt{f_c'}$, where A_{cp} represents the cross-sectional area of a horizontal wall segment.

COMMENTARY

requirements for walls and diaphragms are identical. All references to walls in the following discussion should be understood to include diaphragms as well.

Section 21.6.5 includes two procedures for determining shear strength of walls: a simple one (21.6.5.2), and one (21.6.5.3) which recognizes the higher shear strength of walls and wall segments with low ratios of height h_w to base length ℓ_w. If the engineer elects to use 21.6.5.2, 21.6.5.3 is to be ignored. Similarly, if 21.6.5.3 is chosen, 21.6.5.2 is to be ignored.

Eq. (21-6) in 21.6.5.2 is given in terms of the net area of the section resisting shear. For a rectangular section without openings, the term A_{cv} refers to the gross area of the cross section rather than to the product of the width and the effective depth. The definition of A_{cv} in Eq. (21-6) facilitates design calculations for walls with uniformly distributed reinforcement and walls with openings.

The only difference between 21.6.5.2 and 21.6.5.3 is in coefficient α_c of Eq. (21-7). Recognizing the higher strength of "stubby" walls or walls with high shear-to-moment ratios,[21.9, 21.27, 21.29] coefficient α_c varies from 3.0 for walls or wall segments with (h_w/ℓ_w) ratios of 1.5 or less to the value used in Eq. (21-6) for (h_w/ℓ_w) values equal to or exceeding 2.0.

The ratio (h_w/ℓ_w) may refer to overall dimensions of a wall, or of a segment of the wall bounded by two openings, or an opening and an edge. The intent of 21.6.5.4 is to make certain that any segment of a wall is not assigned a unit strength larger than that for the whole wall. However, a wall segment with a ratio of (h_w/ℓ_w) higher than that of the entire wall must be proportioned for the unit strength associated with the ratio (h_w/ℓ_w) based on the dimensions for that segment.

To restrain the inclined cracks effectively along their trajectories, reinforcement included in ρ_n and ρ_v should be appropriately distributed along the length and height of the wall (21.6.5.5). Chord reinforcement provided near wall edges in concentrated amounts for resisting bending moment is not to be included in determining ρ_n and ρ_v. Within practical limits, shear reinforcement distribution should be uniform and at a small spacing.

A wall segment refers to a part of a wall bounded by openings or by an opening and an edge. Traditionally, a "vertical" wall segment bounded, say, by two window openings has been referred to as a pier.

If the factored shear force at a given level in a structure is resisted by several walls or several piers of a perforated wall, the average unit shear strength assumed for the total available cross-sectional area is limited to $8\sqrt{f_c'}$ with the additional requirement that the unit shear strength assigned to any one pier does not exceed $10\sqrt{f_c'}$. The upper bound of strength to be assigned to any one member is imposed to limit the degree of redistribution of shear force.

CODE

21.6.6 — Boundary elements for structural walls

21.6.6.1 — Boundary elements shall be provided at boundaries and edges around openings of structural walls when the maximum extreme fiber stress, corresponding to factored forces including earthquake effect, exceeds $0.2f_c'$ unless the entire wall is reinforced to satisfy 21.4.4.1 through 21.4.4.3. The boundary element shall be permitted to be discontinued where the calculated compressive stress is less than $0.15f_c'$. Stresses shall be calculated for the factored forces using a linearly elastic model and gross section properties.

21.6.6.2 — Boundary elements, where required, shall have transverse reinforcement as specified in 21.4.4.1 through 21.4.4.3.

COMMENTARY

"Horizontal wall segment" in 21.6.5.7 refers to wall sections between two vertically aligned openings (Fig. R21.6.5.7). It is, in effect, a pier rotated through 90 deg.

R21.6.6 — Boundary elements for structural walls

Requirements in 21.5.3 of the 1989 code for both structural walls and diaphragms were separated in the 1992 code. Structural walls are subjected to both in-place bending and axial forces and the provisions are retained in 21.6.6. Diaphragms normally do not have significant in-plane axial forces and the provisions have been moved to 21.6.7. Maintenance of compressive strength is essential in structural walls, whereas development of tensile strength is most critical in diaphragms.

A simplified diagram showing the forces on the critical section A-A of a structural wall acted on by permanent loads W and the maximum shear and moment induced by earthquake in a given direction are shown in Fig. R21.6.6. Under loading conditions described, the compressed flange resists the acting gravity load plus the total tensile force generated in

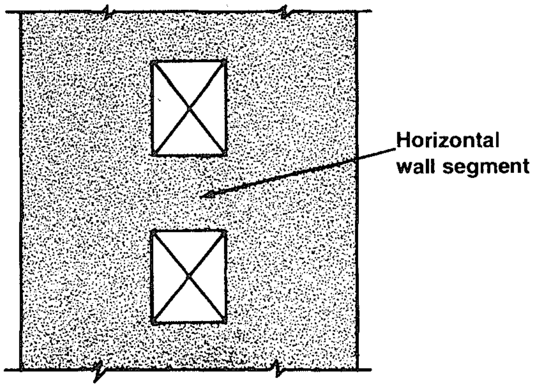


Fig. R21.6.5.7—Wall with openings

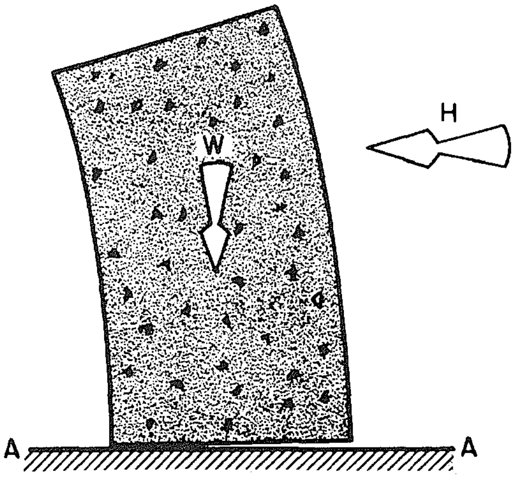


Fig. R21.6.6—Loading conditions on a structural wall

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21.6.6.3 — Boundary elements shall be proportioned to resist all factored gravity loads on the wall, including tributary loads and self-weight, as well as the vertical force required to resist overturning moment calculated from factored forces related to earthquake effect.

21.6.6.4 — Transverse reinforcement in walls with boundary elements shall be anchored within the confined core of the boundary element to develop the specified yield strength f_y of the transverse reinforcement.

21.6.6.5 — Except when V_u in the plane of the wall is less than $A_{cv}\sqrt{f_c'}$, transverse reinforcement terminating at the edges of structural walls without boundary elements shall have a standard hook engaging the edge reinforcement or the edge reinforcement shall be enclosed in U-stirrups having the same size and spacing as, and spliced to, the transverse reinforcement.

21.6.6.6 — Welded splices and mechanical connections of longitudinal reinforcement of boundary elements shall conform to 21.2.6.1.

21.6.7 — Boundary elements of structural diaphragms

21.6.7.1 — Boundary elements of structural diaphragms shall be proportioned to resist the sum of the factored axial force acting in the plane of the diaphragm and the force obtained from dividing the factored moment at the section by the distance between the boundary elements of the diaphragm at that section.

COMMENTARY

the vertical reinforcement (or compressive force associated with the bending moment at section A-A).

Recognizing that this loading condition may be repeated many times during the strong motion, it becomes essential to confine the concrete in all wall flanges where compressive forces are likely to be large as implied by the design compressive stress exceeding $0.2f_c'$ (21.6.6.1 and 21.6.6.2). The stress is to be calculated for the factored forces on the section assuming linear response of the gross concrete section. The compressive stress of $0.2f_c'$ is used as an index value and does not necessarily describe the actual state of stress that may develop at the critical section under the influence of the actual inertia forces for the anticipated earthquake intensity.

The requirement in 21.6.6.3 is based on the assumption that the boundary element may have to carry all compressive forces at the critical section at the time when maximum lateral forces are acting on the structural wall. Design requirements involve only the section properties. The cross section of the boundary element must have adequate strength (determined as an axially loaded short column with the appropriate strength reduction factors) to resist the factored axial compressive force at the critical section.

Because horizontal reinforcement in walls requiring boundary elements is likely to act as web reinforcement, it should be fully anchored in boundary elements which act as flanges (21.6.6.4). Achievement of this anchorage is difficult when large transverse cracks occur in the boundary elements. Therefore, standard 90-deg hooks or mechanical anchorage schemes are recommended in lieu of straight bar development.

The addition of hooks or U-stirrups at the ends of transverse structural wall reinforcement provides anchorage so that the reinforcement will be effective in resisting shear forces. It will also tend to inhibit the buckling of the vertical edge reinforcement. In walls with low in-plane shear, the development of the horizontal reinforcement is not necessary.

To determine whether boundary elements are required, forces in boundaries of walls are calculated using a linearly elastic model and gross section properties.

R21.6.7 — Boundary elements of structural diaphragms

For structural diaphragms, the factored flexural moments are assumed to be resisted entirely by chord forces acting at opposite edges of the diaphragm. Full development of the yield strength of edge-of-collector-element reinforcement is essential. Lap splices shall have confinement as required by 21.3.2.3. If chord reinforcement is located within a wall, the joint between the diaphragm and the wall shall be provided with adequate shear strength to transfer the shear forces.

CODE

21.6.7.2 — Splices of tensile reinforcement in the boundaries and collector elements of all diaphragms shall develop the yield strength of the reinforcement. Welded splices and mechanical connections shall conform to 21.2.6.1.

21.6.8 — Construction joints

All construction joints in walls and diaphragms shall conform to 6.4 and contact surfaces shall be roughened as specified in 11.7.9.

21.6.9 — Discontinuous walls

Columns supporting discontinuous walls shall be reinforced in accordance with 21.4.4.5.

21.7 — Frame members not proportioned to resist forces induced by earthquake motions

21.7.1 — Frame members assumed not to contribute to lateral resistance shall be detailed according to 21.7.2 or 21.7.3 depending on the magnitude of moments induced in those members when subjected to twice the lateral displacements under the factored lateral forces. When effects of lateral displacements are not explicitly checked, it shall be permitted to apply the requirements of 21.7.3.

21.7.2 — When the induced moments and shears under lateral displacements of 21.7.1 combined with the factored gravity moment and shears do not exceed the design moment and shear strength of the frame member, the following conditions shall be satisfied. For this purpose the gravity load combinations of $1.05D + 1.28L$ or $0.9D$, whichever is critical, shall be used.

21.7.2.1 — Members with factored gravity axial forces not exceeding $(A_g f_c'/10)$ shall satisfy 21.3.2.1. Stirrups shall be spaced not more than $d/2$ throughout the length of the member.

21.7.2.2 — Members with factored gravity axial forces exceeding $(A_g f_c'/10)$ shall satisfy 21.4.3, 21.4.4.1(3), 21.4.4.3, and 21.4.5. The maximum longitudinal spacing of ties shall be s_o for the full column height. The spacing s_o shall not be more than six diameters of the smallest longitudinal bar enclosed or 6 in., whichever is smaller.

21.7.2.3 — Members with factored gravity axial forces exceeding $0.35P_o$ shall satisfy 21.7.2.2 and the amount of transverse reinforcement provided shall be one-half of that required by 21.4.4.1 not to exceed a spacing s_o for the full height of the column.

21.7.3 — If the induced moment or shear under lateral displacements of 21.7.1 exceed the design moment or

COMMENTARY

R21.7 — Frame members not proportioned to resist forces induced by earthquake motions

These provisions have been modified on an interim basis from the previous edition in response to the behavior of some concrete buildings to the Northridge, Calif., earthquake of 1994. Study will be continued to determine if further modifications are required for design of economical earthquake resistant concrete construction.

The detailing requirements imposed on members that are part of the lateral-force resisting system provide that the members may undergo deformations that exceed the elastic capacity of the member without significant loss of strength. Members that are not part of the designated lateral force resisting system are not required to meet all the detailing requirements of members that are relied on to resist lateral forces, but they must be able to resist the specified gravity loads at lateral displacements corresponding to twice those calculated for the factored lateral forces.

Section 21.7 recognizes that actual displacements resulting from earthquake forces may be several times larger than the displacements calculated using the code-specified design forces and commonly used analysis models. Section 21.7.1 defines a nominal displacement for the purpose of setting detailing requirements. Actual displacements may exceed the value of 21.7.1. Section 21.7.2 defines details intended to provide a system capable of sustaining gravity loads under moderate excursions into the inelastic range. Section 21.7.3 defines details intended to provide a system capable of sustaining gravity loads under more significant inelastic displacements.

Models used to determine building deflections should be chosen to reasonably predict actual deflections including vertical, horizontal, and diaphragm systems as appropriate.

For development of the gravity load factors in 21.7.2, see R9.2.3.

CODE

shear strength of the frame member, or if induced moments are not calculated, the following conditions shall apply:

21.7.3.1 — Materials shall satisfy 21.2.4 and 21.2.5. Splices of reinforcement shall satisfy 21.2.6.

21.7.3.2 — Members with factored gravity axial forces not exceeding $(A_g f_c'/10)$ shall satisfy 21.3.2.1 and 21.3.4. Stirrups shall be spaced at not more than $d/2$ throughout the length of the member.

21.7.3.3 — Members with factored gravity axial forces exceeding $(A_g f_c'/10)$ shall satisfy 21.4.4, 21.4.5, and 21.5.2.1.

21.8 — Requirements for frames in regions of moderate seismic risk

21.8.1 — In regions of moderate seismic risk, structural frames proportioned to resist forces induced by earthquake motions shall satisfy the requirements of 21.8 in addition to those of Chapters 1 through 18.

21.8.2 — Reinforcement details in a frame member shall satisfy 21.8.4 if the factored compressive axial load for the member does not exceed $(A_g f_c'/10)$. If the factored compressive axial load is larger, frame reinforcement details shall satisfy 21.8.5 unless the member has spiral reinforcement according to Eq. (10-6). If a two-way slab system without beams is treated as part of a frame resisting earthquake effect, reinforcement details in any span resisting moments caused by lateral force shall satisfy 21.8.6.

21.8.3 — Design shear strength of beams, columns, and two-way slabs resisting earthquake effect shall not be less than either (a) the sum of the shear associated with development of nominal moment strengths of the member at each restrained end of the clear span and the shear calculated for factored gravity loads, or (b) the maximum shear obtained from design load combinations which include earthquake effect E, with E assumed to be twice that prescribed by the governing code for earthquake-resistant design.

21.8.4 — Beams

21.8.4.1 — The positive moment strength at the face of the joint shall be not less than one-third the negative moment strength provided at that face of the joint. Neither the negative nor the positive moment strength at any section along the length of the member shall be less than one-fifth the maximum moment strength provided at the face of either joint.

21.8.4.2 — At both ends of the member, stirrups shall be provided over lengths equal to twice the mem-

COMMENTARY

R21.8 — Requirements for frames in regions of moderate seismic risk

In regions of moderate seismic risk, Chapter 21 applies only to reinforced concrete frames proportioned to resist earthquake effect. There are no special requirements for walls and other structural components (Table R21.2.1). It is anticipated that reinforced concrete walls designed in accordance with the main part of the this code will possess sufficient toughness at the low drift levels which they would be likely to attain in regions of moderate seismicity. The requirements for moderate risk zones are based on the presumption that a region will be included in that zone only if it is known with reasonable confidence that the probable earthquake intensity in that region is a fraction of that in a high risk zone.

The objective of the requirements in 21.8.3 is to reduce the risk of failure in shear during an earthquake. The designer is given two options by which to determine the factored shear force.

According to option (a) of 21.8.3, the factored shear force is determined from the nominal moment strength of the member and the gravity load on it. Examples for a beam and a column are illustrated in Fig. R21.8.3.

To determine the maximum beam shear, it is assumed that its nominal moment strengths (ϕ = **1.0**) are developed simultaneously at both ends of its clear span. As indicated in Fig. R21.8.3, the shear associated with this condition $[(M_{nl} + M_{nr})/\ell_n]$ added algebraically to the effect of the factored gravity loads indicates the shear for which the beam must be designed. For this example, both the dead load w_D and the live load w_L have been assumed to be uniformly distributed.

Determination of the specified design shear for a column is also illustrated for a particular example in Fig. R21.8.3. The factored design axial load, P_u, must be chosen to develop the largest moment strength of the column.

CODE

ber depth measured from the face of the supporting member toward midspan. The first stirrup shall be located at not more than 2 in. from the face of the supporting member. Maximum stirrup spacing shall not exceed (a) $d/4$, (b) eight times the diameter of the smallest longitudinal bar enclosed, (c) 24 times the diameter of the stirrup bar, and (d) 12 in.

21.8.4.3 — Stirrups shall be placed at not more than $d/2$ throughout the length of the member.

21.8.5 — Columns

21.8.5.1 — Maximum tie spacing shall not exceed s_o over a length ℓ_o measured from the joint face. Spacing s_o shall not exceed (a) eight times the diameter of the smallest longitudinal bar enclosed, (b) 24 times the diameter of the tie bar, (c) one-half of the smallest cross-sectional dimension of the frame member, and (d) 12 in. Length ℓ_o shall not be less than (a) one-sixth of the clear span of the member, (b) maximum cross-sectional dimension of the member, and (c) 18 in.

21.8.5.2 — The first tie shall be located at not more than $s_o/2$ from the joint face.

21.8.5.3 — Joint reinforcement shall conform to 11.11.2.

21.8.5.4 — Tie spacing shall not exceed twice the spacing s_o.

21.8.6 — Two-way slabs without beams

21.8.6.1 — Factored slab moment at support related to earthquake effect shall be determined for load combinations defined by Eq. (9-2) and (9-3). All reinforcement provided to resist M_s, the portion of slab moment balanced by support moment, shall be placed within the column strip defined in 13.2.1.

21.8.6.2 — The fraction, defined by Eq. (13-1), of moment M_s shall be resisted by reinforcement placed within the effective width specified in 13.5.3.2.

21.8.6.3 — Not less than one-half of the reinforcement in the column strip at support shall be placed within the effective slab width specified in 13.5.3.2.

21.8.6.4 — Not less than one-quarter of the top reinforcement at the support in the column strip shall be continuous throughout the span.

21.8.6.5 — Continuous bottom reinforcement in the column strip shall be not less than one-third of the top reinforcement at the support in the column strip.

COMMENTARY

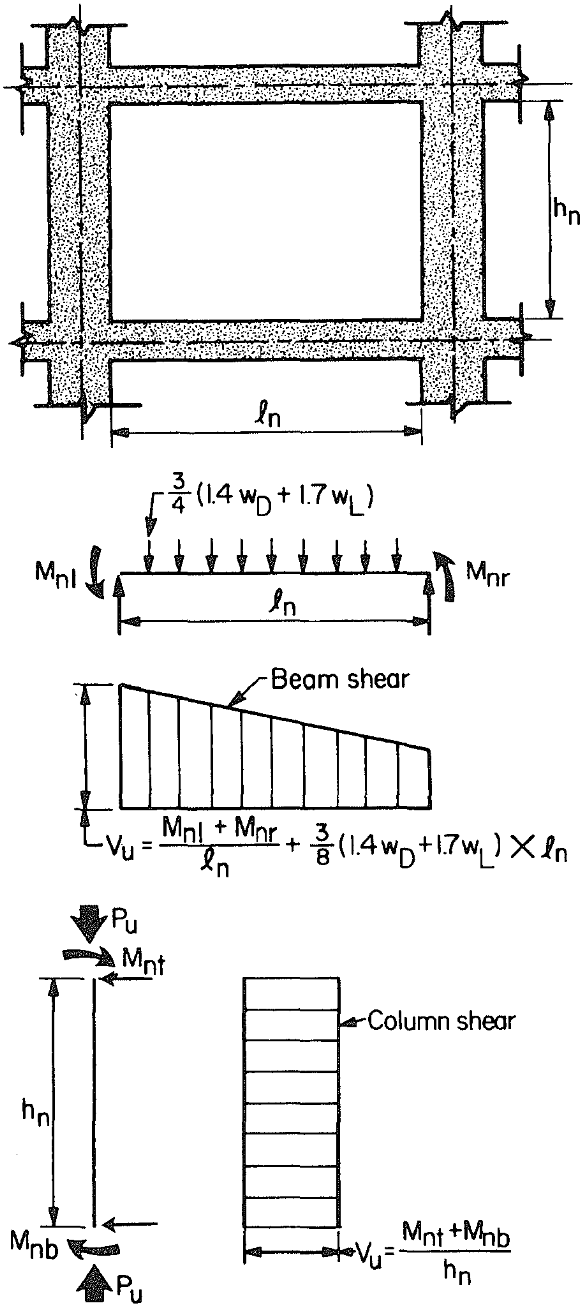

h_n
ℓ_n
$\frac{3}{4}(1.4 w_D + 1.7 w_L)$
M_{nl}
M_{nr}
ℓ_n
Beam shear
$V_u = \frac{M_{nl} + M_{nr}}{\ell_n} + \frac{3}{8}(1.4 w_D + 1.7 w_L) \times \ell_n$
P_u
M_{nt}
Column shear
h_n
M_{nb}
P_u
$V_u = \frac{M_{nt} + M_{nb}}{h_n}$

Fig. R21.8.3—Design shears for frames in regions of moderate seismic risk (21.8)

In all applications of option (a) of 21.8.3, shears must be calculated for moment, acting clockwise and counterclockwise. Fig. R21.8.3 demonstrates only one of the two conditions which must be considered for every member.

Option (b) bases V_u on the load combination including the earthquake effect, E. It should be emphasized that it is E which must be doubled. For example, the load combination defined by Eq. (9-2) would be:

$$U = 0.75\ (1.4D + 1.7L + 3.74E)$$

CODE

21.8.6.6 — Not less than one-half of all bottom reinforcement at midspan shall be continuous and shall develop its yield strength at face of support as defined in 13.6.2.5.

21.8.6.7—At discontinuous edges of the slab all top and bottom reinforcement at support shall be developed at the face of support as defined in 13.6.2.5.

COMMENTARY

where E is the value specified by the governing code.

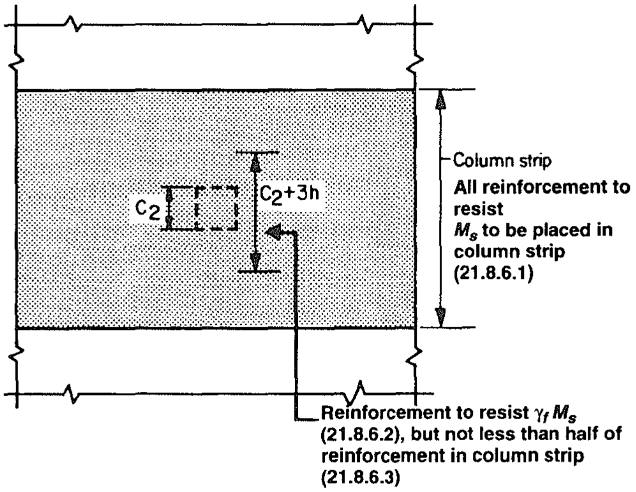
The three articles of 21.8.4 contain requirements for providing beams with a threshold level of toughness. It is expected that in most cases stirrups required by 21.8.3 for design shear force will be more than those required by 21.8.4. Requirements of 21.8.5 serve the same purpose for columns.

Section 21.8.6 is intended to apply to two-way slabs without beams (such as flat plates).

It should be noted that using load combinations defined in 9.2.3 may result in moments requiring both top and bottom reinforcement at the supports.

The moment M_s refers, for a given design load combination with E acting in one horizontal direction, to that portion of the factored slab moment which is balanced by the supporting members at a joint. It is not necessarily equal to the total design moment at support for a load combination including earthquake effect. In accordance with 13.5.3.2, only a fraction ($\gamma_f M_s$) of the moment M_s is assigned to the slab effective width.

Application of the various articles of 21.8.6 are illustrated in Fig. R21.8.6.1 and R21.8.6.2.


C_2
C_2+3h
Column strip
All reinforcement to resist M_s to be placed in column strip (21.8.6.1)
Reinforcement to resist $\gamma_f M_s$ (21.8.6.2), but not less than half of reinforcement in column strip (21.8.6.3)

Note: Applies to both top and bottom reinforcement

Fig. R21.8.6.1 — Location of reinforcement in slabs

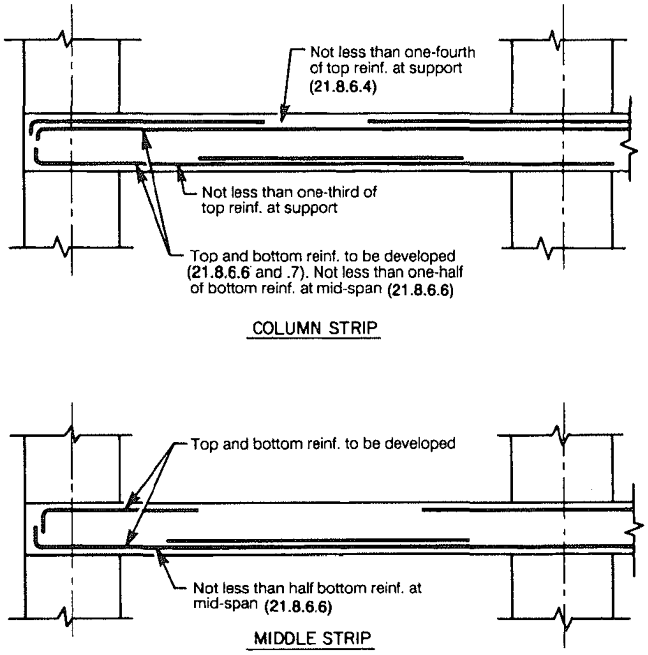

Not less than one-fourth of top reinf. at support (21.8.6.4)
Not less than one-third of top reinf. at support
Top and bottom reinf. to be developed (21.8.6.6 and .7). Not less than one-half of bottom reinf. at mid-span (21.8.6.6)
COLUMN STRIP
Top and bottom reinf. to be developed
Not less than half bottom reinf. at mid-span (21.8.6.6)
MIDDLE STRIP

Fig. R21.8.6.2—Arrangement of reinforcement in slabs

Notes

PART 7 — STRUCTURAL PLAIN CONCRETE

CHAPTER 22 — STRUCTURAL PLAIN CONCRETE

CODE

22.0 — Notation

A_g = gross area of section, in.^{2}
A_1 = loaded area, in.^{2}
A_2 = the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal, in.^{2}
b = width of member, in.
b_o = perimeter of critical section for shear in footings, in.
B_n = nominal bearing strength of loaded area
f_c' = specified compressive strength of concrete, psi. See Chapter 5
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi
f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi. See 5.1.4 and 5.1.5
h = overall thickness of member, in.
ℓ_c = vertical distance between supports, in.
M_n = nominal moment strength at section
M_u = factored moment at section
P_n = nominal strength of cross section subject to compression
P_{nw} = nominal axial load strength of wall designed by 22.6.5
P_u = factored axial load at given eccentricity
S = elastic section modulus of section
V_n = nominal shear strength at section
V_u = factored shear force at section
β_c = ratio of long side to short side of concentrated load or reaction area
ϕ = strength reduction factor. See 9.3.5

22.1 — Scope

22.1.1 — This chapter provides minimum requirements for design and construction of structural plain concrete members (cast-in-place or precast) except as specified in 22.1.1.1 and 22.1.1.2.

COMMENTARY

R22.1 — Scope

Prior to ACI 318-95, **"Building Code Requirements for Structural Concrete,"** code requirements for plain concrete were set forth in **"Building Code Requirements for Structural Plain Concrete (ACI 318.1-89) (Revised 1992)."** Requirements for plain concrete are now set forth in Chapter 22 of this code.

CODE

22.1.1.1 — Structural plain concrete basement walls shall be exempted from the requirements for special exposure conditions of 4.2.2.

22.1.1.2 — Design and construction of soil-supported slabs, such as sidewalks and slabs on grade, shall not be governed by this code unless they transmit vertical loads from other parts of the structure to the soil.

22.1.2 — For special structures, such as arches, underground utility structures, gravity walls, and shielding walls, provisions of this chapter shall govern where applicable.

22.2 — Limitations

22.2.1 — Provisions of this chapter shall apply for design of structural plain concrete members, defined as either unreinforced or containing less reinforcement than the minimum amount specified in this code for reinforced concrete. See 2.1.

22.2.2 — Use of structural plain concrete shall be limited to (a) members that are continuously supported by soil or supported by other structural members capable of providing continuous vertical support, (b) members for which arch action provides compression under all conditions of loading, or (c) walls and pedestals. See 22.6 and 22.8. The use of structural plain concrete columns shall not be permitted.

22.2.3 — This chapter does not govern design and installation of cast-in-place concrete piles and piers embedded in ground.

COMMENTARY

R22.1.1.1 — Section 22.1.1.1 exempts structural plain concrete walls from the requirements for special exposure conditions because of the successful use of large amounts of concrete with 28-day compressive strengths of 2500 and 3000 psi in the basement walls of residences and minor structures that did not meet the strength requirements of Table 4.2.2.

R22.1.1.2 — It is not within the scope of this code to provide design and construction requirements for nonstructural members of plain concrete such as soil-supported slabs (slabs on grade).

R22.2 — Limitations

R22.2.2 and R22.2.3 — Since the structural integrity of plain concrete members depends solely on the properties of the concrete, use of structural plain concrete members should be limited to: members that are primarily in a state of compression; members that can tolerate random cracks without detriment to their structural integrity; and members where ductility is not an essential feature of design. The tensile strength of concrete can be utilized in design of members when the buildup of tensile stresses due to restraint from creep, shrinkage, or temperature effects are considered and sufficiently reduced by construction techniques to avoid uncontrolled cracks, or when uncontrolled cracks due to such restraint effects can be anticipated to occur in such a manner that will not induce structural failure or collapse.

Plain concrete walls are permitted (see 22.6) without a height limitation. However, for multistory construction and other major structures, ACI Committee 318 encourages the use of walls designed as reinforced concrete members in accordance with Chapter 14 (see R22.6).

Since plain concrete lacks the necessary ductility that columns should possess and because a random crack in an unreinforced column will most likely endanger its structural integrity, the code does not permit use of plain concrete for columns. It does allow, however, its use for pedestals limited to a ratio of unsupported height to least lateral dimension of 3 or less (see 22.8.2).

CODE

22.2.4 — Minimum strength

Specified compressive strength of plain concrete to be used for structural purposes shall be not less than 2500 psi.

22.3 — Joints

22.3.1 — Contraction or isolation joints shall be provided to divide structural plain concrete members into flexurally discontinuous elements. Size of each element shall be limited to control buildup of excessive internal stresses within each element caused by restraint to movements from creep, shrinkage, and temperature effects.

22.3.2 — In determining the number and location of contraction or isolation joints, consideration shall be given to: influence of climatic conditions; selection and proportioning of materials; mixing, placing, and curing of concrete; degree of restraint to movement; stresses due to loads to which an element is subject; and construction techniques.

22.4 — Design method

22.4.1 — Structural plain concrete members shall be designed for adequate strength in accordance with provisions of this code, using load factors and design strength.

22.4.2 — Factored loads and forces shall be in such combinations as specified in 9.2 .

22.4.3 — Where required strength exceeds design strength, reinforcement shall be provided and the member designed as a reinforced concrete member in accordance with appropriate design requirements of this code.

22.4.4 — Strength design of structural plain concrete members for flexure and axial loads shall be based on a linear stress-strain relationship in both tension and compression.

COMMENTARY

Structural elements such as cast-in-place concrete piles and piers in ground or other material sufficiently stiff to provide adequate lateral support to prevent buckling are not covered by this code. Such elements are covered by the general building code.

R22.2.4 — Minimum strength

A minimum strength requirement for plain concrete construction is considered necessary because safety is based solely on strength and quality of concrete treated as a homogeneous material. Lean concrete mixtures may not produce adequately homogeneous material or well-formed surfaces.

R22.3 — Joints

Joints in plain concrete construction are an important design consideration. In reinforced concrete, reinforcement is provided to resist the stresses due to restraint of creep, shrinkage, and temperature effects. In plain concrete, joints are the only design means of controlling and thereby relieving the buildup of such tensile stresses. A plain concrete member, therefore, must be small enough, or divided into smaller elements by joints, to control the buildup of internal stresses. The joint may be a contraction joint or an isolation joint. A minimum 25 percent reduction of member thickness is considered sufficient for contraction joints to be effective. The jointing must be such that no axial tension or flexural tension can be developed across a joint after cracking, if applicable, a condition referred to by the code as flexural discontinuity. Where random cracking due to creep, shrinkage, and temperature effects will not affect the structural integrity, and is otherwise acceptable, such as transverse cracks in a continuous wall footing, transverse contraction or isolation joints are not necessary.

R22.4 — Design method

Plain concrete members are proportioned for adequate strength using factored loads and forces. When factored loads exceed the design strength for the concrete strength specified, the section must be increased and/or the specified strength of concrete increased, or the member designed as a reinforced concrete member in accordance with the requirements of this code. The designer should note, however, that an increase in concrete section may have a detrimental effect; stress due to load will decrease while stresses due to creep, shrinkage, and temperature effects may increase.

R22.4.4 — Flexural tension may be considered in design of plain concrete members to sustain loads, provided the computed stress does not exceed the permissible, and construction, contraction, or isolation joints are provided to relieve

CODE

22.4.5 — Tensile strength of concrete shall be permitted to be considered in design of plain concrete members when provisions of 22.3 have been followed.

22.4.6 — No strength shall be assigned to steel reinforcement that may be present.

22.4.7 — Tension shall not be transmitted through outside edges, construction joints, contraction joints, or isolation joints of an individual plain concrete element. No flexural continuity due to tension shall be assumed between adjacent structural plain concrete elements.

22.4.8 — In computing strength in flexure, combined flexure and axial load, and shear, the entire cross section of a member shall be considered in design, except for concrete cast against soil where overall thickness h shall be taken as 2 in. less than actual thickness.

22.5 — Strength design

22.5.1 — Design of cross sections subject to flexure shall be based on

$$\phi M_n \geq M_u \tag{22-1}$$

where M_u is factored moment and M_n is nominal moment strength* computed by

$$M_n = 5\sqrt{f_c'}S \tag{22-2}$$

where S is the elastic section modulus of the cross section.

22.5.2 — Design of cross sections subject to compression shall be based on

$$\phi P_n \geq P_u \tag{22-3}$$

where P_u is factored load and P_n is nominal compression strength computed by

COMMENTARY

the restraint and resulting tensile stresses due to creep, temperature, and shrinkage effects.

R22.4.8 — The reduced overall thickness h for concrete cast against earth is to allow for unevenness of excavation and for some contamination of the concrete adjacent to the soil.

R22.5 — Strength design

R22.5.2 — Eq. (22-4) is presented to reflect the general range of braced and restrained end conditions encountered in structural plain concrete elements. The effective length factor was omitted as a modifier of ℓ_c, the vertical distance between supports, since this is conservative for walls with assumed pin supports that are required to be braced against lateral translation as in 22.6.6.4.

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*Equations for nominal flexural and shear strengths apply for normal weight concrete; for lightweight aggregate concrete, one of the following modifications shall apply:

(a) When f_{ct} is specified and concrete is proportioned in accordance with 5.2 , $f_{ct}/6.7$ shall be substituted for $\sqrt{f_c'}$ but the value of $f_{ct}/6.7$ shall not exceed $\sqrt{f_c'}$.

(b) When f_{ct} is not specified, the value of $\sqrt{f_c'}$ shall be multiplied by 0.75 for "all-lightweight" concrete and by 0.85 for "sand-lightweight" concrete. Linear interpolation shall be permitted when partial sand replacement is used.

CODE

$$P_n = 0.60 f_c' \left[1 - \left(\frac{\ell_c}{32h} \right)^2 \right] A_1 \qquad (22\text{-}4)$$

where A_1 is the loaded area.

22.5.3 — Members subject to combined flexure and axial load in compression shall be proportioned such that on the compression face:

$$P_u / \phi P_n + M_u / \phi M_n \leq 1 \qquad (22\text{-}5)$$

and on the tension face:

$$M_u / S - P_u / A_g \leq 5\phi \sqrt{f_c'} \qquad (22\text{-}6)$$

22.5.4 — Design of rectangular cross sections subject to shear[*] shall be based on

$$\phi V_n \geq V_u \qquad (22\text{-}7)$$

where $\boldsymbol{V_u}$ is factored shear and $\boldsymbol{V_n}$ is nominal shear strength computed by

$$V_n = \frac{4}{3} \sqrt{f_c'} bh \qquad (22\text{-}8)$$

for beam action and by

$$V_n = \left[\frac{4}{3} + \frac{8}{3\beta_c} \right] \sqrt{f_c'} b_o h \leq 2.66 \sqrt{f_c'} b_o h \qquad (22\text{-}9)$$

for two-way action but not greater than $\mathbf{2.66}\sqrt{f_c'}\, \boldsymbol{b_o h}$.

COMMENTARY

R22.5.3 — Plain concrete members subject to combined flexure and axial compressive load are proportioned such that on the compression face:

$$\frac{P_u}{0.60 \phi f_c' \left[1 - \left(\frac{\ell_c}{32h} \right)^2 \right] A_1} + \frac{M_u}{0.85 \phi f_c' S} \leq 1$$

and that on the tension face:

$$\left(\begin{matrix} \text{Calculated} \\ \text{bending stress} \end{matrix} \right) - \left(\begin{matrix} \text{Calculated} \\ \text{axial stress} \end{matrix} \right) \leq 5\phi \sqrt{f_c'}$$

R22.5.4 — Proportions of plain concrete members will be controlled by tensile strength rather than shear strength for the usual plain concrete members of practical proportions. Shear stress (as a substitute for principal tensile stress) rarely will control. However, since it is difficult to foresee all possible conditions where shear may have to be investigated (e.g., shear keys), Committee 318 decided to maintain the investigation of this basic stress condition as a part of the code requirements. An experienced designer will soon recognize where shear is not critical for plain concrete members and will adjust design procedures accordingly.

The shear requirements for plain concrete assume an uncracked section. Shear failure in plain concrete will be a diagonal tension failure, occurring when the principal tensile stress near the centroidal axis becomes equal to the tensile strength of the concrete. Since the major portion of the principal tensile stress comes from the shear, the code safeguards against tensile failure by limiting the permissible shear at the centroidal axis as calculated from the equation for a section of homogeneous material:

$$v = VQ/Ib$$

where $\boldsymbol{v}$ and $\boldsymbol{V}$ are the shear stress and shear force, respectively, at the section considered, $\boldsymbol{Q}$ is the statical moment of the area outside the section being considered about centroidal axis of the gross section, $\boldsymbol{I}$ is the moment of inertia of the gross section, and $\boldsymbol{b}$ is the width where shear stress is being computed.

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[*] Equations for nominal flexural and shear strengths apply for normal weight concrete; for lightweight aggregate concrete, one of the following modifications shall apply:
(a) When $\boldsymbol{f_{ct}}$ is specified and concrete is proportioned in accordance with 5.2, $\boldsymbol{f_{ct}/6.7}$ shall be substituted for $\sqrt{f_c'}$ but the value of $\boldsymbol{f_{ct}/6.7}$ shall not exceed $\sqrt{f_c'}$.
(b) When $\boldsymbol{f_{ct}}$ is not specified, the value of $\sqrt{f_c'}$ shall be multiplied by 0.75 for "all-lightweight" concrete and by 0.85 for "sand-lightweight" concrete. Linear interpolation shall be permitted when partial sand replacement is used.

CODE

22.5.5 — Design of bearing areas subject to compression shall be based on

$$\phi B_n \geq P_u \qquad (22\text{-}10)$$

where P_u is factored bearing load and B_n is nominal bearing strength of loaded area A_1 computed by

$$B_n = 0.85 f_c' A_1 \qquad (22\text{-}11)$$

except when the supporting surface is wider on all sides than the loaded area, design bearing strength on the loaded area shall be multiplied by $\sqrt{A_2/A_1}$ but not more than 2.

22.6 — Walls

22.6.1 — Structural plain concrete walls shall be continuously supported by soil, footings, foundation walls, grade beams, or other structural members capable of providing continuous vertical support.

22.6.2 — Structural plain concrete walls shall be designed for vertical, lateral, and other loads to which they are subjected.

22.6.3 — Structural plain concrete walls shall be designed for an eccentricity corresponding to the maximum moment that can accompany the axial load but not less than $0.10h$. If the resultant of all factored loads is located within the middle-third of the overall wall thickness, the design shall be in accordance with 22.5.3 or 22.6.5. Otherwise, walls shall be designed in accordance with 22.5.3.

22.6.4 — Design for shear shall be in accordance with 22.5.4.

22.6.5 — Empirical design method

22.6.5.1 — Structural plain concrete walls of solid rectangular cross section shall be permitted to be designed by Eq. (22-12) if the resultant of all factored loads is located within the middle-third of the overall thickness of wall.

22.6.5.2 — Design of walls subject to axial loads in compression shall be based on

$$\phi P_{nw} \geq P_u \qquad (22\text{-}12)$$

where P_u is factored axial load and P_{nw} is nominal axial load strength computed by

COMMENTARY

R22.6 — Walls

Plain concrete walls are commonly used for basement wall construction for residential and light commercial buildings in low or nonseismic areas. Although the code imposes no absolute maximum height limitation on the use of plain concrete walls, designers are cautioned against extrapolating the experience with relatively minor structures and using plain concrete walls in multistory construction and other major structures where differential settlement, wind, earthquake, or other unforeseen loading conditions require the walls to possess some ductility and ability to maintain their integrity when cracked. For such conditions, ACI Committee 318 strongly encourages the use of walls designed as reinforced concrete members in accordance with Chapter 14.

The provisions for plain concrete walls are applicable only for walls laterally supported in such a manner as to prohibit relative lateral displacement at top and bottom of individual wall elements (see 22.6.6.4). This code does not cover walls where there is no horizontal support to prohibit relative displacement at top and bottom of wall elements. Such laterally unsupported walls must be designed as reinforced concrete members in accordance with this code.

R22.6.5 — Empirical design method

When the resultant load falls within the middle-third of the wall thickness ("kern" of wall section), plain concrete walls may be designed using the simplified Eq. (22-13). Eccentric loads and lateral forces are used to determine the total eccentricity of the factored load P_u. If the eccentricity does not exceed $h/6$, Eq. (22-13) may be applied, and design performed considering P_u as a concentric load. The factored axial load P_u must be less than or equal to the design axial load strength ϕP_{nw}, computed by Eq. (22-13), or $P_u \leq \phi P_{nw}$. Eq. (22-13) is presented to reflect the general range of braced and restrained end conditions encountered in wall design. The limitations of 22.6.6 apply whether the wall is proportioned by 22.5.3 or by the empirical method of 22.6.5.

CODE

$$P_{nw} = 0.45 f_c' A_g \left[1 - \left(\frac{\ell_c}{32h} \right)^2 \right] \qquad (22\text{-}13)$$

22.6.6 — Limitations

22.6.6.1 — Unless demonstrated by a detailed analysis, horizontal length of wall to be considered effective for each vertical concentrated load shall not exceed center-to-center distance between loads, nor width of bearing plus 4 times the wall thickness.

22.6.6.2 — Except as provided in 22.6.6.3, thickness of bearing walls shall be not less than $^1/_{24}$ the unsupported height or length, whichever is shorter, nor less than $5^1/_2$ in.

22.6.6.3 — Thickness of exterior basement walls and foundation walls shall be not less than $7^1/_2$ in.

22.6.6.4 — Walls shall be braced against lateral translation. See 22.3 and 22.4.7.

22.6.6.5 — Not less than two No. 5 bars shall be provided around all window and door openings. Such bars shall extend at least 24 in. beyond the corners of openings.

22.7 — Footings

22.7.1 — Structural plain concrete footings shall be designed for factored loads and induced reactions in accordance with appropriate design requirements of this code and as provided in 22.7.2 through 22.7.8.

22.7.2 — Base area of footing shall be determined from unfactored forces and moments transmitted by footing to soil and permissible soil pressure selected through principles of soil mechanics.

22.7.3 — Plain concrete shall not be used for footings on piles.

22.7.4 — Thickness of structural plain concrete footings shall be not less than 8 in. See 22.4.8.

COMMENTARY

R22.7 — Footings

R22.7.4 — Thickness of plain concrete footings will be controlled by flexural strength (extreme fiber stress in tension not greater than $5\phi\sqrt{f_c'}$ rather than shear strength for the usual proportions of plain concrete footings. Shear rarely will control (see R22.5.4). For footings cast against soil, overall thickness h used for strength computations must be taken as 2 in. less than actual thickness to allow for unevenness of excavation and contamination of the concrete adjacent to soil as required by 22.4.8. Thus, for a minimum footing thickness of 8 in., calculations for flexural and shear stresses must be based on an overall thickness h of 6 in.

CODE

22.7.5 — Maximum factored moment shall be computed at critical sections located as follows:

(a) At face of column, pedestal, or wall, for footing supporting a concrete column, pedestal, or wall.

(b) Halfway between middle and edge of wall, for footing supporting a masonry wall.

(c) Halfway between face of column and edge of steel base plate, for footing supporting a column with steel base plate.

22.7.6 — Shear in plain concrete footings

22.7.6.1 — Maximum factored shear shall be computed in accordance with 22.7.6.2, with location of critical section measured at face of column, pedestal, or wall for footing supporting a column, pedestal, or wall. For footing supporting a column with steel base plates, the critical section shall be measured at location defined in 22.7.5(c).

22.7.6.2 — Shear strength of structural plain concrete footings in the vicinity of concentrated loads or reactions shall be governed by the more severe of two conditions:

(a) Beam action for footing, with a critical section extending in a plane across the entire footing width and located at a distance h from face of concentrated load or reaction area. For this condition, the footing shall be designed in accordance with Eq. (22-8).

(b) Two-way action for footing, with a critical section perpendicular to plane of footing and located so that its perimeter b_o is a minimum, but need not approach closer than $h/2$ to perimeter of concentrated load or reaction area. For this condition, the footing shall be designed in accordance with Eq. (22-9).

22.7.7 — Circular or regular polygon shaped concrete columns or pedestals shall be permitted to be treated as square members with the same area for location of critical sections for moment and shear.

22.7.8 — Factored bearing load on concrete at contact surface between supporting and supported member shall not exceed design bearing strength for either surface as given in 22.5.5.

22.8 — Pedestals

22.8.1 — Plain concrete pedestals shall be designed for vertical, lateral, and other loads to which they are subjected.

COMMENTARY

R22.8 — Pedestals

The height-thickness limitation for plain concrete pedestals does not apply for portions of pedestals embedded in soil capable of providing lateral restraint.

CODE

22.8.2 — Ratio of unsupported height to average least lateral dimension of plain concrete pedestals shall not exceed 3.

22.8.3 — Maximum factored axial load applied to plain concrete pedestals shall not exceed design bearing strength given in 22.5.5.

22.9 — Precast members

22.9.1 — Design of precast plain concrete members shall consider all loading conditions from initial fabrication to completion of the structure, including form removal, storage, transportation, and erection.

22.9.2 — Limitations of 22.2 apply to precast members of plain concrete not only to the final condition but also during fabrication, transportation, and erection.

22.9.3 — Precast members shall be connected securely to transfer all lateral forces into a structural system capable of resisting such forces.

22.9.4 — Precast members shall be adequately braced and supported during erection to ensure proper alignment and structural integrity until permanent connections are completed.

COMMENTARY

R22.9 — Precast members

Precast structural plain concrete members are subject to all limitations and provisions for cast-in-place concrete contained in this chapter.

The approach to contraction or isolation joints is expected to be somewhat different than for cast-in-place concrete since the major portion of the internal stresses due to shrinkage takes place prior to erection. To assure stability, precast members should be connected to other members. Connection must be such that no tension will be transferred from one member to the other.

Notes

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APPENDIX A — ALTERNATE DESIGN METHOD

CODE

COMMENTARY

A.0 — Notation

Some notation definitions are modified from those in the main body of the code for specific use in the application of Appendix A.

A_g = gross area of section, in.^{2}

A_v = area of shear reinforcement within a distance s, in.^{2}

A_1 = loaded area

A_2 = maximum area of the portion of the supporting surface that is geometrically similar to and concentric with the loaded area

b_o = perimeter of critical section for slabs and footings, in.

b_w = web width, or diameter of circular section, in.

d = distance from extreme compression fiber to centroid of tension reinforcement, in.

E_c = modulus of elasticity of concrete, psi. See 8.5.1

E_s = modulus of elasticity of reinforcement, psi. See 8.5.2

f_c' = specified compressive strength of concrete, psi. See Chapter 5

$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi

f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi. See 5.1.4

f_s = permissible tensile stress in reinforcement, psi

f_y = specified yield strength of reinforcement, psi. See 3.5.3

M = design moment

n = modular ratio of elasticity

= E_s/E_c

N = design axial load normal to cross section occurring simultaneously with V; to be taken as positive for compression, negative for tension, and to include effects of tension due to creep and shrinkage

s = spacing of shear reinforcement in direction parallel to longitudinal reinforcement, in.

v = design shear stress

v_c = permissible shear stress carried by concrete, psi

v_h = permissible horizontal shear stress, psi

V = design shear force at section

α = angle between inclined stirrups and longitudinal axis of member

β_c = ratio of long side to short side of concentrated load or reaction area

CODE

ρ_w = ratio of tension reinforcement
= A_s/b_wd
ϕ = strength reduction factor. See A.2.1.

A.1 — Scope

A.1.1 — Nonprestressed reinforced concrete members shall be permitted to be designed using service loads (without load factors) and permissible service load stresses in accordance with provisions of Appendix A.

A.1.2 — For design of members not covered by Appendix A, appropriate provisions of this code shall apply.

A.1.3 — All applicable provisions of this code for nonprestressed concrete, except 8.4, shall apply to members designed by the Alternate Design Method.

COMMENTARY

RA.1 — Scope

As an alternate to the Strength Design Method of this code, the design provisions of Appendix A may be used to proportion reinforced concrete members. In the alternate method, a structural member (in flexure) is so designed that the stresses resulting from the action of service loads (without load factors) and computed by the straight-line theory for flexure do not exceed permissible service load stresses. Service load is the load, such as dead, live, and wind, which is assumed actually to occur when the structure is in service. The required service loads to be used in design are as prescribed in the general building code. The stresses computed under the action of service loads are limited to values well within the elastic range of the materials so that the straight-line relationship between stress and strain is used (see A.5).

The alternate method is similar to the "working stress design method" of previous ACI Building Codes (e.g., ACI 318-63). For members subject to flexure without axial load, the method is identical. Major differences in procedure occur in design of compression members with or without flexure (see A.6) and bond stress and development of reinforcement (see A.4). For shear, the shear strengths provided by concrete for the Strength Design Method are divided by a factor of safety and the resulting permissible service load stresses restated in Appendix A (see A.7).

In view of the simplifications permitted, the Alternate Design Method of Appendix A generally will result in more conservative designs than those designs obtained using the Strength Design Method of the code. Load factors and strength reduction factors of 1.0 are used for both design and analysis. Also, design rules for proportioning by the straight-line theory for flexure have not been updated as thoroughly as the Strength Design Method for proportioning reinforced concrete members.

RA.1.1 — Design by Appendix A does not apply to prestressed members. (Chapter 18 permits linear stress-strain assumptions for computing service load stresses and prestress transfer stresses for investigation of behavior at service conditions.)

RA.1.3 — All other provisions of this code, except those permitting moment redistribution, apply to the Alternate Design Method. These include control of deflections and

CODE

A.1.4 — Flexural members shall meet requirements for deflection control in 9.5, and requirements of 10.4 through 10.7 of this code.

A.2 — General

A.2.1 — Load factors and strength reduction factors ϕ shall be taken as unity for members designed by the Alternate Design Method.

A.2.2 — It shall be permitted to proportion members for 75 percent of capacities required by other parts of Appendix A when considering wind or earthquake forces combined with other loads, provided the resulting section is not less than that required for the combination of dead and live load.

A.2.3 — When dead load reduces effects of other loads, members shall be designed for 85 percent of dead load in combination with the other loads.

A.3 — Permissible service load stresses

A.3.1 — Stresses in concrete shall not exceed the following:

(a) Flexure
Extreme fiber stress in compression .... $0.45\ f_c'$

(b) Shear*
Beams and one-way slabs and footings:
Shear carried by concrete, v_c .... $1.1\sqrt{f_c'}$

Maximum shear carried by concrete plus
shear reinforcement .... $v_c + 4.4\sqrt{f_c'}$

COMMENTARY

distribution of flexural reinforcement, as well as all of the provisions related to slenderness effects in compression members in Chapter 10.

RA.1.4 — The general serviceability requirements of this code, such as the requirements for deflection control (see 9.5) and crack control (see 10.6), must be met regardless of whether the strength method or the alternate method is used for design.

RA.2 — General

RA.2.1 — Load factors and strength reduction factors for determining safety in the Strength Design Method are not used in the Alternate Design Method. Accordingly, load factors and strength reduction factors ϕ are set equal to 1.0 to eliminate their effect when designing by the alternate method.

When using the moment and shear equations of 8.3.3 and Chapter 13, the factored load w_u must be replaced by the service load w.

RA.2.2 — When lateral loads such as wind or earthquake combined with live and dead load govern the design, members may be proportioned for 75 percent of capacities required in Appendix A. This is similar to the working stress design provisions of previous ACI Building Codes which allowed a one-third increase in stresses for these combinations of loads.

RA.2.3 — The 15 percent reduction for dead load is required for design conditions where dead load reduces the design effects of other loads to allow for the actual dead load being less than the dead load used in design. This provision is analogous to the required strength equation [Eq. (9-3)].

RA.3 — Permissible service load stresses

For convenience, permissible service load stresses are tabulated. Compressive stress in concrete for flexure without axial load is limited to $\mathbf{0.45}f_c'$. Tensile stresses in reinforcement are limited to 20,000 psi for Grade 40 and 50 steel and 24,000 psi for Grade 60 and higher strength steel. One exception of long standing exists for one-way slabs with clear span lengths 12 ft or less and reinforced with No. 3 bars or welded wire fabric having a diameter not exceeding 3/8 in. For this design condition only, the permissible tensile stress is increased to the lesser of $\mathbf{0.5}f_y$ or 30,000 psi.

Permissible stresses for shear and bearing are percentages of the shear and bearing strengths provided for strength design. The 10 percent increase permitted for joists by 8.11 of the code is already included in the $\mathbf{1.2}\sqrt{f_c'}$ value for joists.

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* For more detailed calculation of shear stress carried by concrete v_c and shear values for lightweight aggregate concrete, see A.7.4.

CODE

Joists:*

Shear carried by concrete, v_c .... $1.2\sqrt{f_c'}$

Two-way slabs and footings:

Shear carried by concrete, v_c† .... $\left(1 + \frac{2}{\beta_c}\right)\sqrt{f_c'}$

but not greater than $2\sqrt{f_c'}$

(c) Bearing on loaded area‡ .... $0.3f_c'$

A.3.2 — Tensile stress in reinforcement f_s shall not exceed the following:

(a) Grade 40 or Grade 50 reinforcement .... 20,000 psi

(b) Grade 60 reinforcement or greater and welded wire fabric (plain or deformed) .... 24,000 psi

(c) For flexural reinforcement, $^3/_8$ in. or less in diameter, in one-way slabs of not more than 12 ft span .... $0.50f_y$ but not greater than 30,000 psi

A.4 — Development and splices of reinforcement

A.4.1 — Development and splices of reinforcement shall be as required in Chapter 12 of this code.

A.4.2 — In satisfying requirements of 12.11.3, M_n shall be taken as computed moment capacity assuming all positive moment tension reinforcement at the section to be stressed to the permissible tensile stress f_s, and V_u shall be taken as unfactored shear force at the section.

A.5 — Flexure

For investigation of stresses at service loads, straight-line theory (for flexure) shall be used with the following assumptions.

A.5.1 — Strains vary linearly as the distance from the neutral axis, except for deep flexural members with overall depth-span ratios greater than $^2/_5$ for continu-

COMMENTARY

Clarification of the use of areas A_1 and A_2 for increased bearing stress is discussed in R10.17.1.

RA.4 — Development and splices of reinforcement

In computing development lengths and splice lengths, the provisions of Chapter 12 govern both methods of design equally since, in either case, the development lengths (and splice lengths as multiples of development lengths) are based on the yield strength of the reinforcement. Where M_n and V_u are referenced in Chapter 12, M_n is the service load resisting moment capacity and V_u is the applied service load shear force (without load factors) at the section.

RA.5 — Flexure

The straight-line theory applies only to design of members in flexure without axial load. Since stresses computed under the action of service loads are well within the elastic range, the straight-line relationship between stress and strain is used with the maximum fiber stress in concrete limited to $0.45f_c'$ and the tensile stress in reinforcement limited to 24,000 psi for Grade 60 steel (see A.3.2).

Straight-line theory may be used for all sectional shapes with or without compression reinforcement when axial load is not present. Since small axial compression loads tend to increase the moment capacity of a section, small axial loads may be disregarded in most cases. When doubt exists as to whether or not the axial compression may be disregarded, the member should be investigated using A.6.

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*Designed in accordance with 8.11 of this code.

† If shear reinforcement is provided, see A.7.7.4 and A.7.7.5.

‡ When the supporting surface is wider on all sides than the loaded area, permissible bearing stress on the loaded area shall be permitted to be multiplied by $\sqrt{A_2/A_1}$ but not more than 2. When the supporting surface is sloped or stepped, A_2 shall be permitted to be taken as the area of the lower base of the largest frustum of a right pyramid or cone contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal.

CODE

ous spans and $^4/_5$ for simple spans, a nonlinear distribution of strain shall be considered. See 10.7 of this code.

A.5.2 — Stress-strain relationship of concrete is a straight line under service loads within permissible service load stresses.

A.5.3 — In reinforced concrete members, concrete resists no tension.

A.5.4 — It shall be permitted to take the modular ratio, $n = E_s/E_c$, as the nearest whole number (but not less than 6). Except in calculations for deflections, value of n for lightweight concrete shall be assumed to be the same as for normal weight concrete of the same strength.

A.5.5 — In doubly reinforced flexural members, an effective modular ratio of $2E_s/E_c$ shall be used to transform compression reinforcement for stress computations. Compressive stress in such reinforcement shall not exceed permissible tensile stress.

A.6 — Compression members with or without flexure

A.6.1 — Combined flexure and axial load capacity of compression members shall be taken as 40 percent of that computed in accordance with provisions in Chapter 10 of this code.

A.6.2 — Slenderness effects shall be included according to requirements of 10.10 through 10.13. In Eq. (10-10) and (10-19) the term P_u shall be replaced by 2.5 times the design axial load, and the factor 0.75 shall be taken equal to 1.0.

A.6.3 — Walls shall be designed in accordance with Chapter 14 of this code with flexure and axial load capacities taken as 40 percent of that computed using Chapter 14. In Eq. (14-1), ϕ shall be taken equal to 1.0.

A.7 — Shear and torsion

A.7.1 — Design shear stress v shall be computed by

$$v = \frac{V}{b_w d} \qquad \text{(A-1)}$$

COMMENTARY

Deep flexural members must be designed in accordance with 10.7 of this code.

In transforming compression reinforcement to equivalent concrete for flexural design, $2\,E_s/E_c$ must be used in locating the neutral axis and calculating moments of inertia. The lesser of twice the calculated stress in the compression reinforcement or the permissible tensile stress is then used to calculate the contribution of the compression reinforcement in computing the resisting moment at service loads.

RA.6 — Compression members with or without flexure

All compression members, with or without flexure, must be proportioned using the Strength Design Method. This departure from the 1963 and previous ACI Building Codes is to provide a more consistent factor of safety for the full range of load-moment interaction. Existing working stress design aids for columns do not satisfy requirements of Appendix A.

The permissible service load capacity is taken as 40 percent of the nominal axial load strength P_n at given eccentricity ($\phi = 1.0$) as computed by the provisions of Chapter 10, subject to appropriate reduction due to effects of slenderness. Use of 40 percent of the nominal strength is equivalent to an overall safety factor U/ϕ of 2.5.

With the Alternate Design Method, P_u/ϕ in Eq. (10-10) and (10-19) is taken as $2.5P$ when gravity loads govern and as $1.875P$ when lateral loads combined with gravity loads govern the design, where P is the design axial load in the compression member.

RA.7 — Shear and torsion

For convenience, a complete set of design provisions for shear is provided in Appendix A.

The permissible concrete stresses and limiting maximum

CODE

where V is design shear force at section considered.

A.7.2 — When the reaction, in direction of applied shear, introduces compression into the end regions of a member, sections located less than a distance d from face of support shall be permitted to be designed for the same shear v as that computed at a distance d.

A.7.3 — Whenever applicable, effects of torsion, in accordance with provisions of Chapter 11 of this code, shall be added. Shear and torsional moment strengths provided by concrete and limiting maximum strengths for torsion shall be taken as 55 percent of the values given in Chapter 11.

A.7.4 — Shear stress carried by concrete

A.7.4.1 — For members subject to shear and flexure only, shear stress carried by concrete v_c shall not exceed $1.1\sqrt{f_c'}$ unless a more detailed calculation is made in accordance with A.7.4.4.

A.7.4.2 — For members subject to axial compression, shear stress carried by concrete v_c shall not exceed $1.1\sqrt{f_c'}$ unless a more detailed calculation is made in accordance with A.7.4.5.

A.7.4.3 — For members subject to significant axial tension, shear reinforcement shall be designed to carry total shear, unless a more detailed calculation is made using

$$v_c = 1.1\left(1 + 0.004\frac{N}{A_g}\right)\sqrt{f_c'} \qquad \text{(A-2)}$$

where N is negative for tension. Quantity N/A_g shall be expressed in psi.

A.7.4.4 — For members subject to shear and flexure only, it shall be permitted to compute v_c by

$$v_c = \sqrt{f_c'} + 1300\rho_w\frac{Vd}{M} \qquad \text{(A-3)}$$

but v_c shall not exceed $1.9\sqrt{f_c'}$. Quantity Vd/M shall not be taken greater than 1.0, where M is design moment occurring simultaneously with V at section considered.

A.7.4.5 — For members subject to axial compression, it shall be permitted to compute v_c by

$$v_c = 1.1\left(1 + 0.0006\frac{N}{A_g}\right)\sqrt{f_c'} \qquad \text{(A-4)}$$

Quantity N/A_g shall be expressed in psi.

COMMENTARY

stresses for shear are 55 percent for beams, joists, walls and one-way slabs and 50 percent for two-way slabs and footings, respectively, of the shear and torsional moment strengths given in the code for the Strength Design Method.

When gravity load, wind, earthquake, or other lateral forces cause transfer of moment between slab and column, provisions of 11.12.2 must be applied with the permissible stresses on the critical section limited to those given in A.7.7.3.

CODE

COMMENTARY

A.7.4.6 — Shear stresses carried by concrete v_c apply to normal weight concrete. When lightweight aggregate concrete is used, one of the following modifications shall apply:

(a) When f_{ct} is specified and concrete is proportioned in accordance with 5.2, $f_{ct}/6.7$ shall be substituted for $\sqrt{f_c'}$ but the value of $f_{ct}/6.7$ shall not exceed $\sqrt{f_c'}$.

(b) When f_{ct} is not specified, the value of $\sqrt{f_c'}$ shall be multiplied by 0.75 for "all-lightweight" concrete and by 0.85 for "sand-lightweight" concrete. Linear interpolation shall be permitted when partial sand replacement is used.

A.7.4.7 — In determining shear stress carried by concrete v_c, whenever applicable, effects of axial tension due to creep and shrinkage in restrained members shall be included and it shall be permitted to include effects of inclined flexural compression in variable-depth members.

A.7.5 — Shear stress carried by shear reinforcement

A.7.5.1 — Types of shear reinforcement

Shear reinforcement shall consist of one of the following:

(a) Stirrups perpendicular to axis of member

(b) Welded wire fabric with wires located perpendicular to axis of member making an angle of 45 deg or more with longitudinal tension reinforcement

(c) Longitudinal reinforcement with bent portion making an angle of 30 deg or more with longitudinal tension reinforcement

(d) Combinations of stirrups and bent longitudinal reinforcement

(e) Spirals

A.7.5.2 — Design yield strength of shear reinforcement shall not exceed 60,000 psi.

A.7.5.3 — Stirrups and other bars or wires used as shear reinforcement shall extend to a distance d from extreme compression fiber and shall be anchored at both ends according to 12.13 of this code to develop design yield strength of reinforcement.

CODE

COMMENTARY

A.7.5.4 — Spacing limits for shear reinforcement

A.7.5.4.1 — Spacing of shear reinforcement placed perpendicular to axis of member shall not exceed $d/2$, nor 24 in.

A.7.5.4.2 — Inclined stirrups and bent longitudinal reinforcement shall be so spaced that every 45-deg line, extending toward the reaction from middepth of member ($d/2$) to longitudinal tension reinforcement, shall be crossed by at least one line of shear reinforcement.

A.7.5.4.3 — When $(v - v_c)$ exceeds $2\sqrt{f_c'}$, maximum spacing given in A.7.5.4.1 and A.7.5.4.2 shall be reduced by one-half.

A.7.5.5 — Minimum shear reinforcement

A.7.5.5.1 — A minimum area of shear reinforcement shall be provided in all reinforced concrete flexural members where design shear stress v is greater than one-half the permissible shear stress v_c carried by concrete, except:

(a) Slabs and footings

(b) Concrete joist construction defined by 8.11 of this code

(c) Beams with total depth not greater than 10 in., $2^1/_2$ times thickness of flange, or one-half the width of web, whichever is greatest.

A.7.5.5.2 — Minimum shear reinforcement requirements of A.7.5.5.1 shall be permitted to be waived if shown by test that required ultimate flexural and shear strength can be developed when shear reinforcement is omitted.

A.7.5.5.3 — Where shear reinforcement is required by A.7.5.5.1 or by analysis, minimum area of shear reinforcement shall be computed by

$$A_v = 50\frac{b_w s}{f_y} \qquad \text{(A-5)}$$

where b_w and s are in inches.

A.7.5.6 — Design of shear reinforcement

A.7.5.6.1 — Where design shear stress v exceeds shear stress carried by concrete v_c, shear reinforcement shall be provided in accordance with A.7.5.6.2 through A.7.5.6.8.

A.7.5.6.2 — When shear reinforcement perpen-

CODE

dicular to axis of member is used,

$$A_v = \frac{(v - v_c)\, b_w s}{f_s} \qquad \text{(A-6)}$$

A.7.5.6.3 — When inclined stirrups are used as shear reinforcement,

$$A_v = \frac{(v - v_c)\, b_w s}{f_s(\sin\alpha + \cos\alpha)} \qquad \text{(A-7)}$$

A.7.5.6.4 — When shear reinforcement consists of a single bar or a single group of parallel bars, all bent up at the same distance from the support,

$$A_v = \frac{(v - v_c)\, b_w d}{f_s \sin\alpha} \qquad \text{(A-8)}$$

where $(v - v_c)$ shall not exceed $1.6\sqrt{f_c'}$.

A.7.5.6.5 — When shear reinforcement consists of a series of parallel bent-up bars or groups of parallel bent-up bars at different distances from the support, required area shall be computed by Eq. (A-7).

A.7.5.6.6 — Only the center three-quarters of the inclined portion of any longitudinal bent bar shall be considered effective for shear reinforcement.

A.7.5.6.7 — When more than one type of shear reinforcement is used to reinforce the same portion of a member, required area shall be computed as the sum of the various types separately. In such computations, v_c shall be included only once.

A.7.5.6.8 — Value of $(v - v_c)$ shall not exceed $4.4\sqrt{f_c'}$.

A.7.6 — Shear-friction

Where it is appropriate to consider shear transfer across a given plane, such as an existing or potential crack, an interface between dissimilar materials, or an interface between two concretes cast at different times, shear-friction provisions of 11.7 of this code shall be permitted to be applied, with limiting maximum stress for shear taken as 55 percent of that given in 11.7.5. Permissible stress in shear-friction reinforcement shall be that given in A.3.2.

A.7.7 — Special provisions for slabs and footings

A.7.7.1 — Shear capacity of slabs and footings in the vicinity of concentrated loads or reactions is governed by the more severe of two conditions:

A.7.7.1.1 — Beam action for slab or footing, with a critical section extending in a plane across the entire

COMMENTARY

CODE

COMMENTARY

width and located at a distance **d** from face of concentrated load or reaction area. For this condition, the slab or footing shall be designed in accordance with A.7.1 through A.7.5.

A.7.7.1.2 — Two-way action for slab or footing, with a critical section perpendicular to plane of slab and located so that its perimeter is a minimum, but need not approach closer than **$d/2$** to perimeter of concentrated load or reaction area. For this condition, the slab or footing shall be designed in accordance with A.7.7.2 and A.7.7.3.

A.7.7.2 — Design shear stress **v** shall be computed by

$$v = \frac{V}{b_o d} \tag{A-9}$$

where **V** and **b_o** shall be taken at the critical section defined in A.7.7.1.2.

A.7.7.3 — Design shear stress **v** shall not exceed **v_c** given by Eq. (A-10) unless shear reinforcement is provided

$$v_c = \left(1 + \frac{2}{\beta_c}\right)\sqrt{f_c'} \tag{A-10}$$

but **v_c** shall not exceed **$2\sqrt{f_c'}$**. β_c is the ratio of long side to short side of concentrated load or reaction area. When lightweight aggregate concrete is used, the modifications of A.7.4.6 shall apply.

A.7.7.4 — If shear reinforcement consisting of bars or wires is provided in accordance with 11.12.3 of this code, **v_c** shall not exceed $\sqrt{f_c'}$, and **v** shall not exceed **$3\sqrt{f_c'}$**.

A.7.7.5 — If shear reinforcement consisting of steel I- or channel-shaped sections (shearheads) is provided in accordance with 11.12.4 of this code, **v** on the critical section defined in A.7.7.1.2 shall not exceed **$3.5\sqrt{f_c'}$**, and **v** on the critical section defined in 11.12.4.7 shall not exceed **$2\sqrt{f_c'}$**. In Eq. (11-39) and (11-40), design shear force **V** shall be multiplied by 2 and substituted for **V_u**.

A.7.8 — Special provisions for other members

For design of deep flexural members, brackets and corbels, and walls, the special provisions of Chapter 11 of this code shall be used, with shear strengths provided by concrete and limiting maximum strengths for

CODE

shear taken as 55 percent of the values given in Chapter 11. In 11.10.6, the design axial load shall be multiplied by 1.2 if compression and 2.0 if tension, and substituted for N_u.

A.7.9 — Composite concrete flexural members

For design of composite concrete flexural members, permissible horizontal shear stress v_h shall not exceed 55 percent of the horizontal shear strengths given in 17.5.2 of this code.

COMMENTARY

Notes

APPENDIX B — UNIFIED DESIGN PROVISIONS FOR REINFORCED AND PRESTRESSED CONCRETE FLEXURAL AND COMPRESSION MEMBERS

CODE

B.1 — Scope

Design for flexure and axial load by provisions of Appendix B shall be permitted. When Appendix B is used in design, all numbered sections in this appendix shall be used in place of the corresponding numbered sections in Chapters 8, 9, 10, and 18. If any section in this appendix is used, all sections in this appendix shall be substituted for the corresponding sections in the body of the code.

B.8.4 — Redistribution of negative moments in continuous flexural members

B.8.4.1 — Except where approximate values for moments are used, it shall be permitted to increase or decrease negative moments calculated by elastic theory at supports of continuous flexural members for any assumed loading arrangement by not more than $1000\varepsilon_t$ percent, with a maximum of 20 percent.

B.8.4.2 — The modified negative moments shall be used for calculating moments at sections within the spans.

B.8.4.3 — Redistribution of negative moments shall be made only when ε_t is equal to or greater than 0.0075 at the section at which moment is reduced.

COMMENTARY

RB.1 — Scope

This appendix to the code introduces substantial changes in design for flexure and axial loads. Reinforcement limits, strength reduction factor ϕ, and moment redistribution are affected. Designs using the provisions of this Appendix B satisfy the code, and are equally acceptable.

When this appendix is used, each section of the appendix must be substituted for the corresponding section in the body of the code. For instance, B.8.4 is substituted for 8.4, etc. through B.18.10.4 being substituted for 18.10.4. The corresponding commentary sections should also be substituted.

RB.8.4 — Redistribution of negative moments in continuous flexural members

Moment redistribution is dependent on adequate ductility in plastic hinge regions. These plastic hinge regions develop at points of maximum moment and cause a shift in the elastic moment diagram. The usual result is a reduction in the values of negative moments in the plastic hinge region and an increase in the values of positive moments from those computed by elastic analysis. Since negative moments are determined for one loading arrangement and positive moments for another, each section has a reserve capacity that is not fully utilized for any one loading condition. The plastic hinges permit the utilization of the full capacity of more cross sections of a flexural member at ultimate loads.

Using conservative values of ultimate concrete strains and lengths of plastic hinges derived from extensive tests, flexural members with small rotation capacity were analyzed for moment redistribution up to 20 percent depending on the reinforcement ratio. The results were found to be conservative (see Fig. RB.8.4). Studies by Cohn[8.2] and Mattock[8.3] support this conclusion and indicate that cracking and deflection of beams designed for moment redistribution are not significantly greater at service loads than for beams designed by the elastic theory distribution of moments. Also, these studies indicated that adequate rotation capacity for the moment redistribution allowed by the code is available if the members satisfy the code requirements.

Moment redistribution does not apply to members designed by the Alternate Design Method of Appendix A; nor may it be used for slab systems designed by the Direct Design Method (see 13.6.1.7).

Section 8.4 of the code specifies the permissible redistribution percentage in terms of reinforcement indices. This

CODE

B.9.2 — Required strength

B.9.2.1 — Required strength ***U*** to resist dead load ***D*** and live load ***L*** shall be at least equal to

$$U = 1.4D + 1.7L \qquad \text{(B.9-1)}$$

COMMENTARY

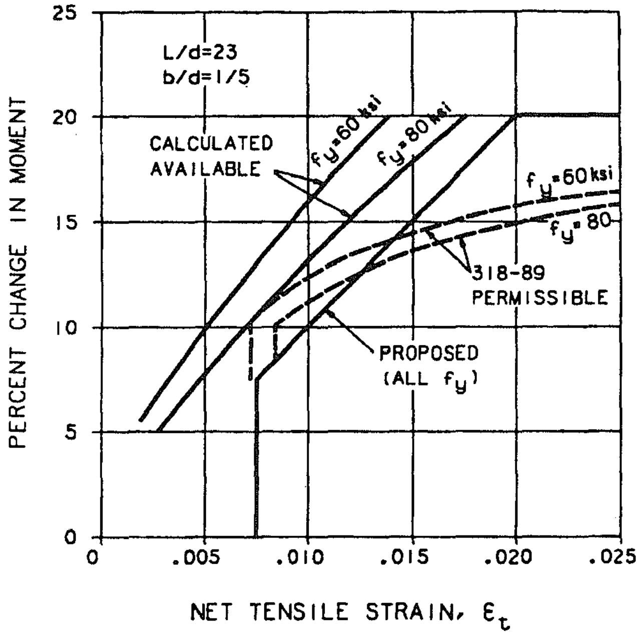

Fig. RB.8.4—Permissible moment redistribution for minimum rotation capacity

appendix specifies the permissible redistribution percentage in terms of the net tensile strain ε_t. See Reference B.1 for a comparison of these moment redistribution provisions.

The concept of net tensile strain is discussed in RB.10.3.3.

RB.9.2 — Required strength

The required strength *U* is expressed in terms of factored loads, or related internal moments and forces. Factored loads are the loads specified in the general building code multiplied by appropriate load factors.

The factor assigned to each load is influenced by the degree of accuracy to which the load effect usually can be calculated and the variation that might be expected in the load during the lifetime of the structure. Dead loads, because they are more accurately determined and less variable, are assigned a lower load factor than live loads. Load factors also account for variability in the structural analysis used to compute moments and shears.

The code gives load factors for specific combinations of loads. In assigning factors to combinations of loading, some consideration is given to the probability of simultaneous occurrence. While most of the usual combinations of loadings are included, the designer should not assume that all cases are covered.

Due regard is to be given to sign in determining *U* for combinations of loadings, as one type of loading may produce effects of opposite sense to that produced by another type.

CODE

B.9.3 — Design strength

B.9.3.1 — Design strength provided by a member, its connections to other members, and its cross sections, in terms of flexure, axial load, shear, and torsion, shall be taken as the nominal strength calculated in accordance with requirements and assumptions of this code, multiplied by a strength reduction factor ϕ.

B.9.3.2 — Strength reduction factor ϕ shall be as follows:

B.9.3.2.1 — Tension-controlled sections 0.90

B.9.3.2.2 — Compression-controlled sections:

(a) Members with spiral reinforcement conforming to 10.9.3 .... 0.75

(b) Other reinforced members....................... 0.70

For sections in which the net tensile strain in the extreme tension steel at nominal strength is between the limits for compression-controlled and tension-controlled sections, ϕ shall be linearly increased from that for compression-controlled sections to 0.90 as the net tensile strain in the extreme tension steel at nominal strength increases from the compression-controlled strain limit to 0.005. Alternatively, it shall be permitted

COMMENTARY

The load combinations with **0.9*D*** are specifically included for the case where a higher dead load reduces the effects of other loads. This loading case may also be critical for tension-controlled column sections. In such a case, a reduction in axial load and increase in moment may result in a critical load combination.

Consideration must be given to various combinations of loading to determine the most critical design condition. This is particularly true when strength is dependent on more than one load effect, such as strength for combined flexure and axial load or shear strength in members with axial load.

If special circumstances require greater reliance on the strength of particular members than encountered in usual practice, some reduction in the stipulated strength reduction factors ϕ or increase in the stipulated load factors U may be appropriate for such members.

RB.9.3 — Design strength

RB.9.3.1 — The term "design strength" of a member refers to the nominal strength calculated in accordance with the requirements stipulated in this code multiplied by a strength reduction factor ϕ, which is always less than one.

The purposes of the strength reduction factor ϕ are (1) to allow for the probability of understrength sections due to variations in material strengths and dimensions, (2) to allow for inaccuracies in the design equations, (3) to reflect the degree of ductility and required reliability of the section under the load effects being considered, and (4) to reflect the importance of the member in the structure.[9.2, 9.3]

RB.9.3.2 — Prior to the development of these provisions, the code specified the magnitude of the ϕ-factor for cases of axial load or flexure or both in terms of the type of loading. For these cases, the ϕ-factor is now determined by the strain conditions at a cross section, at nominal strength.

A lower ϕ-factor is used for compression-controlled sections than is used for tension-controlled sections because compression-controlled sections have less ductility, are more sensitive to variations in concrete strength, and generally occur in members that support larger loaded areas than members with tension-controlled sections. Members with spiral reinforcement are assigned a higher ϕ than tied columns since they have greater ductility or toughness.

For sections subjected to axial load with flexure, design strengths are determined by multiplying both P_n and M by the appropriate single value of ϕ. Compression-controlled and tension-controlled sections are defined in Chapter 2 as those which have net tensile strain in the extreme tension

CODE

to take ϕ as that for compression-controlled sections.

B.9.3.2.3 — Shear and torsion 0.85

B.9.3.2.4 — Bearing on concrete
(see also 18.13) 0.70

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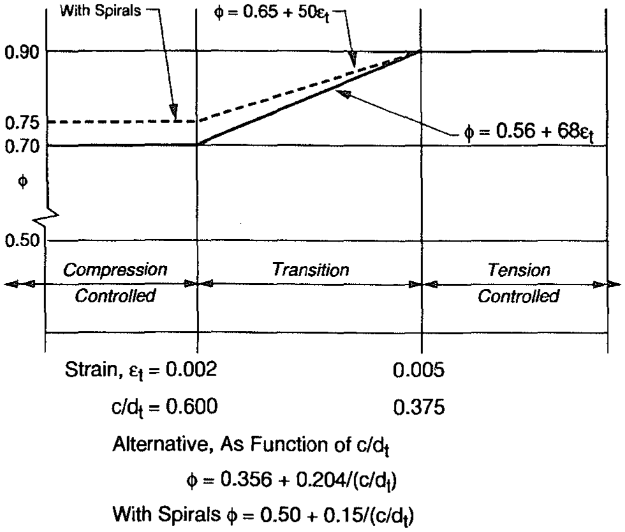

Fig. RB.9.3.2—Variation of ϕ with net tensile strain for Grade 60 reinforcement and for prestressing steel

steel at nominal strength less than or equal to the compression-controlled strain limit and equal to or greater than 0.005, respectively. For sections with net tensile strain in the extreme tension steel at nominal strength between the above limits, the value of ϕ may be determined by linear interpolation, as shown in Fig. RB.9.3.2. The concept of net tensile strain is discussed in RB.10.3.3.

Since the compressive strain in the concrete at nominal strength is assumed in 10.2.3 to be 0.003, the net tensile strain limits for compression-controlled members may also be stated in terms of the ratio c/d_t, where c is the depth of the neutral axis at nominal strength, and d_t is the distance from the extreme compression fiber to the extreme tension steel. The c/d_t limits for compression-controlled and tension-controlled sections are 0.6 and 0.375, respectively. The 0.6 limit applies to sections reinforced with Grade 60 steel and to prestressed sections. Fig. RB.9.3.2 also gives equations for ϕ as a function of c/d_t.

The net tensile strain limit for tension-controlled sections may also be stated in terms of the ρ/ρ_b ratio as defined in previous editions of the code. The net tensile strain limit of 0.005 corresponds to a ρ/ρ_b ratio of 0.63 for rectangular sections with Grade 60 reinforcement. For a comparison of these provisions with those of the body of the code, see Reference B.1.

The factor ϕ for bearing on concrete in this section does not apply to post-tensioning anchorage bearing plates (see R18.13).

B.10.3.2 — Balanced strain conditions exist at a cross section when tension reinforcement reaches the strain corresponding to its specified yield strength f_y just as concrete in compression reaches its assumed strain limit of 0.003.

The compression-controlled strain limit is the net tensile strain in the reinforcement at balanced strain conditions. For prestressed sections, it shall be permitted to use the same compression-controlled strain limit as that for reinforcement with a design yield strength f_y of 60,000 psi.

B.10.3.3 — Sections are compression-controlled when the net tensile strain in the extreme tension steel is equal to or less than the compression-controlled strain limit at the time the concrete in compression reaches its assumed strain limit of 0.003. Sections are tension-controlled when the net tensile strain in the extreme tension steel is equal to or greater than 0.005 just as the concrete in compression reaches its assumed strain limit of 0.003. Sections with net tensile strain in the extreme tension steel between the compression-controlled strain limit and 0.005 constitute a transition region between compression-controlled and tension-controlled sections.

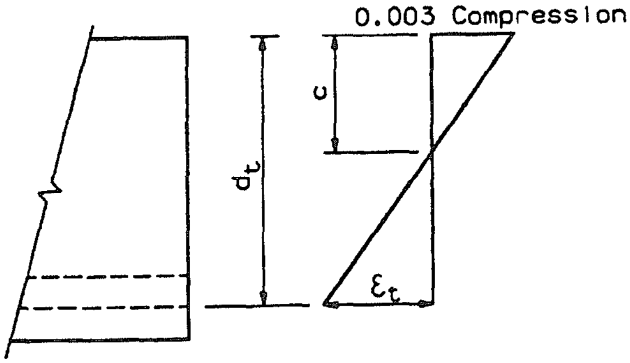

Fig. RB.10.3.3—Strain distribution and net tensile strain

RB.10.3.2 — A balanced strain condition exists at a cross section when the maximum strain at the extreme compression fiber just reaches 0.003 simultaneously with the first yield strain f_y/E_s in the tension reinforcement. The reinforcement ratio ρ_b which produces balanced conditions under flexure, depends on the shape of the cross section and the location of the reinforcement.

For Grade 60 reinforcement, the compression-controlled strain limit may be taken as a net tensile strain ε_t of 0.002. This net tensile strain may be used as the compression-controlled strain limit for prestressed sections.

RB.10.3.3 — The nominal flexural strength of a member is reached when the strain in the extreme compression fiber reaches the assumed strain limit 0.003. The net tensile strain in the extreme tension steel is determined from a linear strain distribution at nominal strength, shown in Fig. RB.10.3.3, using similar triangles.

When the net tensile strain in the extreme tension steel is sufficiently large (equal to or greater than 0.005), the section is defined as tension-controlled where ample warning of failure with extensive deflection and cracking may be expected. When the net tensile strain in the extreme tension steel is small (less than or equal to the compression-controlled strain limit), a brittle failure condition may be expected, with little warning of impending failure. Flexural members are usually tension-controlled, whereas compression members are usually compression-controlled. Some sections, such as those with small axial load and large bending moment, will have net tensile strain in the extreme tension steel between the above limits. These sections are in a transition region between compression- and tension-controlled sections. Section B.9.3.2 specifies the appropriate strength reduction factors for tension-controlled and compression-controlled sections, and for intermediate cases in the transition regions. See Reference B.1 for a comparison of these provisions to those in the body of the code.

CODE

B.18.1.3 — The following provisions of this code shall not apply to prestressed concrete, except as specifically noted: Sections 7.6.5, 8.10.2, 8.10.3, 8.10.4, 8.11, 10.5, 10.6, 10.9.1, and 10.9.2; Chapter 13; and Sections 14.3, 14.5, and 14.6.

COMMENTARY

Prior to the development of these provisions, the code defined balanced strain conditions as those existing at a cross section when tension reinforcement reaches the strain corresponding to its specified yield strength f_y just as the concrete in compression reaches its assumed strain limit of 0.003. The reinforcement ratio ρ_b was defined as the reinforcement ratio producing balanced strain conditions. The limiting tensile strain for flexural members was not stated, but was implicit in the maximum tension reinforcement ratio that was given as a fraction of ρ_b, which was dependent on the yield strength of the reinforcement. The new net tensile strain limit of 0.005 for tension-controlled sections was chosen to be a single value which applies to all types of steel (prestressed and nonprestressed) permitted by this code. Note that the net tensile strain limit of 0.005 is not an absolute limit (as was the $0.75\rho_b$ limit in earlier editions), but a point at which the capacity reduction factor begins to change. High reinforcement ratios producing net tensile strain less than 0.005 are permitted, but are not economical because of the reduced ϕ-factor. In flexural members, it is more economical to add compression reinforcement if necessary to make $\varepsilon_t \geq 0.005$.

Unless unusual amounts of ductility are required, the 0.005 limit will provide ductile behavior for most designs. One condition where greater ductile behavior is required is in design for redistribution of moments in continuous members and frames. Section B.8.4 permits redistribution of negative moments. Since moment redistribution is dependent on adequate ductility in hinge regions, moment redistribution is limited to sections that have a net tensile strain of at least 0.0075.

For beams with compression reinforcement, or T-beams, the effects of the compression reinforcement and flanges are automatically accounted for in the computation of net tensile strain ε_t.

RB.18.1.3 — Some sections of the code are excluded from use in the design of prestressed concrete for specific reasons. The following discussion provides explanation for such exclusions:

Section 7.6.5 — Section 7.6.5 of the code is excluded from application to prestressed concrete since the requirements for bonded reinforcement and unbonded tendons for cast-in-place members are provided in 18.9 and 18.12, respectively.

Sections 8.10.2, 8.10.3, and 8.10.4 — The empirical provisions of 8.10.2, 8.10.3, and 8.10.4 for T-beams were developed for conventionally reinforced concrete and if applied to prestressed concrete would exclude many standard prestressed products in satisfactory use today. Hence, proof by experience permits variations.

By excluding 8.10.2, 8.10.3, and 8.10.4, no special requirements for prestressed concrete T-beams appear in the code.

CODE

COMMENTARY

Instead, the determination of an effective width of flange is left to the experience and judgment of the engineer. Where possible, the flange widths in 8.10.2, 8.10.3, and 8.10.4 should be used unless experience has proven that variations are safe and satisfactory. It is not necessarily conservative in elastic analysis and design considerations to use the maximum flange width as permitted in 8.10.2.

Sections 8.10.1 and 8.10.5 provide general requirements for T-beams that are also applicable to prestressed concrete units. The spacing limitations for slab reinforcement are based on flange thickness, which for tapered flanges can be taken as the average thickness.

Section 8.11 — The empirical limits established for conventionally reinforced concrete joist floors are based on successful past performance of joist construction using "standard" joist forming systems. See R8.11. For prestressed joist construction, experience and judgment should be used. The provisions of 8.11 may be used as a guide.

Sections 10.5, 10.9.1, and 10.9.2 — For prestressed concrete, the limitations on reinforcement given in 10.5, 10.9.1, and 10.9.2 are replaced by those in 18.8.3, 18.9, and 18.11.2.

Section 10.6 — When originally prepared, the provisions of 10.6 for distribution of flexural reinforcement were not intended for prestressed concrete members. The behavior of a prestressed member is considerably different from that of a nonprestressed member. Experience and judgment must be used for proper distribution of reinforcement in a prestressed member.

Chapter 13 — The design of prestressed concrete slabs requires recognition of secondary moments induced by the undulating profile of the prestressing tendons. Also, volume changes due to the prestressing force can create additional loads on the structure that are not adequately covered in Chapter 13. Because of these unique properties associated with prestressing, many of the design procedures of Chapter 13 are not appropriate for prestressed concrete structures and are replaced by the provisions of 18.12.

Sections 14.5 and 14.6 — The requirements for wall design in 14.5 and 14.6 are largely empirical, utilizing considerations not intended to apply to prestressed concrete.

B.18.8 — Limits for reinforcement of flexural members

B.18.8.1 — Prestressed concrete sections shall be classified as tension-controlled and compression-controlled sections in accordance with B.10.3.3. The appropriate ϕ-factors from B.9.3.2 shall apply.

RB.18.8 — Limits for reinforcement of flexural members

RB.18.8.1 — The net tensile strain limits for compression- and tension-controlled sections given in B.10.3.3 apply to prestressed sections. These provisions take the place of the maximum reinforcement limits in the code.

CODE

B.18.8.2 — Total amount of prestressed and nonprestressed reinforcement shall be adequate to develop a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture f_r specified in 9.5.2.3, except for flexural members with shear and flexural strength at least twice that required by 9.2.

B.18.8.3 — Part or all of the bonded reinforcement consisting of bars or tendons shall be provided as close as practicable to the extreme tension fiber in all prestressed flexural members, except that in members prestressed with unbonded tendons, the minimum bonded reinforcement consisting of bars or tendons shall be as required by 18.9.

B.18.10.4 — Redistribution of negative moments in continuous prestressed flexural members

B.18.10.4.1 — Where bonded reinforcement is provided at supports in accordance with 18.9.2, it shall be permitted to increase or decrease negative moments calculated by elastic theory for any assumed loading, in accordance with B.8.4.

B.18.10.4.2 — The modified negative moments shall be used for calculating moments at sections within spans for the same loading arrangement.

COMMENTARY

The net tensile strain limit for tension-controlled sections given in B.10.3.3 may also be stated in terms of ω_p as defined in previous editions of the code. The net tensile strain limit of 0.005 corresponds to $\omega_p = 0.32\beta_1$ for prestressed rectangular sections.

RB.18.8.2 — This provision is a precaution against abrupt flexural failure developing immediately after cracking. A flexural member designed according to code provisions requires considerable additional load beyond cracking to reach its flexural strength. Thus, considerable deflection would warn that the member strength is approaching. If the flexural strength should be reached shortly after cracking, the warning deflection would not occur.

RB.18.8.3 — Some bonded steel is required to be placed near the tension face of prestressed flexural members. The purpose of this bonded steel is to control cracking under full service loads or overloads.

RB.18.10.4 — Redistribution of negative moments in continuous prestressed flexural members

The provisions for redistribution of negative moments given in B.8.4 of this code apply equally to prestressed members. See Reference B.1 for a comparison to research results and code provisions.

For the moment redistribution principles of B.18.10.4 to be applicable to beams with unbonded tendons, it is necessary that such beams contain sufficient bonded reinforcement to ensure they will act as beams after cracking and not as a series of tied arches. The minimum bonded reinforcement requirements of 18.9 will serve this purpose.

APPENDIX C — ALTERNATIVE LOAD AND STRENGTH REDUCTION FACTORS

CODE

C.1 — General

C.1.1 — It shall be permitted to proportion the concrete members of a building structure using the load-factor combinations in ASCE 7-88 in conjunction with the following strength reduction factors, if the structural framing includes primary members of other materials proportioned to satisfy the load factor combinations in Section 2.4 of ASCE 7-88.

C.1.1.1 — Flexure, without axial load 0.80

C.1.1.2 — Axial tension and axial tension with flexure 0.80

C.1.1.3 — Axial compression and axial compression with flexure:

(a) Members with spiral reinforcement conforming to 10.9.3 .... 0.70

(b) Other reinforced members 0.65

except that for low values of axial compression, it shall be permitted to increase ϕ towards the value for flexure, 0.80, using the linear interpolation provided in either 9.3.2.2 or B.9.3.2.2.

(c) In regions of high seismic risk, members resisting earthquake forces without transverse reinforcement conforming to 21.4.4 .... 0.50

C.1.1.4 — Shear and torsion 0.75

except that in regions of high seismic risk:

(a) Shear in members resisting earthquake forces if the nominal shear strength of the member is less than the nominal shear corresponding to the development of the nominal flexural strength of the member 0.55

(b) Shear in joints of building structures 0.80

C.1.1.5 — Bearing 0.65

C.1.1.6 — Plain concrete 0.55

COMMENTARY

RC.1 — General

Appendix C has been included to facilitate the proportioning of building structures that include members made of materials other than concrete. If those members are to be proportioned using the minimum design loads specified in ASCE 7,[C.1] it is convenient to execute the entire design using the same load requirements.

The strength reduction factors in Appendix C were calibrated so that if they are used in conjunction with the minimum design load combinations from Section 2.4.2 of Reference C.1, the designs, in most cases, will be comparable to those that would be obtained using the load factors and strength reduction factors specified in Chapter 9. It is unsafe to use the load factors from Reference C.1 with the strength reduction factors from Chapter 9.

Relevant sections of Chapter 2 of Reference C.1* are reproduced here:

2.2 — Symbols and Notation

D = *dead load consisting of: (a) weight of the member itself; (b) weight of all materials of construction incorporated into the building to be permanently supported by the member, including built-in partitions; and (c) weight of permanent equipment;*

E = *earthquake load;*

F = *loads due to fluids with well-defined pressures and maximum heights;*

L = *live loads due to intended use and occupancy, including loads due to movable objects and movable partitions and loads temporarily supported by the structure during maintenance.* ***L*** *includes any permissible reduction. If resistance to impact loads is taken into account in design, such effects shall be included with the live load* ***L****;*

L_r = *roof live loads;*

S = *snow loads;*

R = *rain loads, except ponding;*

H = *loads due to the weight and lateral pressure of soil and water in soil;*

P = *loads, forces, and effects due to ponding;*

T = *self-straining forces and effects arising from contraction or expansion resulting from temperature changes, shrinkage, moisture changes, creep in component materials, move-*

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CODE

COMMENTARY

ment due to differential settlement, or combinations thereof;

W = *wind load;*

2.4 — Combining Loads Using Strength Design

2.4.1 — Applicability. *The load combinations and load factors given in 2.4.2 and 2.4.3 shall be used only in those cases in which they are specifically authorized by the applicable material design standard.*

2.4.2 — Basic Combinations. *Except where applicable codes and standards provide otherwise, structures, components, and foundations shall be designed so that their design strength exceeds the effects of the factored loads in the following combinations:*

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
3. $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (0.5L \text{ or } 0.8W)$
4. $1.2D + 1.3W + 0.5L + 0.5(L_r \text{ or } S \text{ or } R)$
5. $1.2D + 1.5E + (0.5L \text{ or } 0.2S)$
6. $0.9D - (1.3W \text{ or } 1.5E)$

Exception: the load factor on **L** *in combinations (3), (4), and (5) shall equal 1.0 for garages, areas occupied as places of public assembly, and all areas where the live load is greater than 100 lb/ft*^{2} *(pounds-force per square foot).*

Each relevant strength limit state shall be considered. The most unfavorable effect may occur when one or more of the contributing loads are not acting.

2.4.3 — Other Combinations. *The structural effects of* ***F, H, P,*** *or* ***T*** *shall be considered in design as the following factored loads:* ***1.3F, 1.6H, 1.2P,*** *and* ***1.2T.***

The load and strength reduction factors in Chapter 9 of this code have evolved since the early 1960s.[C.2] There have been advances in recent years in understanding the probabilities of structural failure. Probability considerations provide a basis for assessing relative measures of structural safety if the variables affecting safety are distributed randomly and if the natures of the distributions are known. The load factors in Section 2.4.2 of ASCE 7 are said to be based on a survey of "reliabilities inherent in existing design practice."[C.1] For reinforced concrete buildings in countries where the ACI Building Code and similar codes have been used, the best and most compact survey of "reliabilities inherent in existing design practice" are the load and strength reduction factors used in the ACI Building Code. Currently, the strongest support for the strength reduction factors in Appendix C is the fact that, used with the load-factor combinations from ASCE 7, the results are generally compatible with those obtained using Chapter 9.

APPENDIX D — NOTATION

Items set in Times Roman type appear only in the commentary.

a = depth of rectangular stress block. Chapter 9

a = depth of equivalent rectangular stress block as defined in 10.2.7.1. Chapters 10 and 12

a = shear span, distance between concentrated load and face of support. Chapter 11

A = effective tension area of concrete surrounding the flexural tension reinforcement and having the same centroid as that reinforcement, divided by the number of bars or wires, in.^{2} When the flexural reinforcement consists of different bar or wire sizes the numbers of bars or wires shall be computed as the total area of reinforcement divided by the area of the largest bar or wire used. Chapter 10

A = area of that part of cross section between flexural tension face and center of gravity of gross section, in.^{2} Chapter 18

A_b = area of an individual bar, in.^{2} Chapter 12

A_c = area of core of spirally reinforced compression member measured to outside diameter of spiral, in.^{2} Chapter 10

A_c = area of concrete section resisting shear transfer, in.^{2} Chapter 11

A_c = area of concrete of assumed critical section for transfer of moment at slab-column connection, in.^{2} See Fig. R11.12.6.2. Chapter 11

A_c = area of contact surface being investigated for horizontal shear, in.^{2} Chapter 17

A_{ch} = cross-sectional area of a structural member measured out-to-out of transverse reinforcement, in.^{2} Chapter 21

A_{cp} = area enclosed by outside perimeter of concrete cross section, in.^{2} See 11.6.1. Chapter 11

A_{cp} = area of concrete section, resisting shear, of an individual pier or horizontal wall segment, in.^{2} Chapter 21

A_{cv} = net area of concrete section bounded by web thickness and length of section in the direction of shear force considered, in.^{2} Chapter 21

A_f = area of reinforcement in bracket or corbel resisting factored moment, $[V_u a + N_{uc}(h - d)]$, in.^{2} Chapter 11

A_f = base area of footing, in.^{2} Chapter 15

A_g = gross area of section, in.^{2} Chapters 9, 10, 11, 14, 15, 21, 22, and Appendix A

A_g = gross area of column, in.^{2} Chapter 16

A_h = area of shear reinforcement parallel to flexural tension reinforcement, in.^{2} Chapter 11

A_j = effective cross-sectional area within a joint, in.,^{2} see 21.5.3.1, in a plane parallel to plane of reinforcement generating shear in the joint. The joint depth shall be the overall depth of the column. Where a beam frames into a support of larger width, the effective width of the joint shall not exceed the smaller of:
(a) beam width plus the joint depth
(b) twice the smaller perpendicular distance from the longitudinal axis of the beam to the column side. See 21.5.3.1. Chapter 21

A_ℓ = total area of longitudinal reinforcement to resist torsion, in.^{2} Chapter 11

A_n = area of reinforcement in bracket or corbel resisting tensile force N_{uc}, in.^{2} Chapter 11

A_o = gross area enclosed by shear flow path, in.^{2} Chapter 11

A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement, in.^{2} Chapter 11

A_{ps} = area of prestressed reinforcement in tension zone, in.^{2} Chapters 11 and 18

A_s = area of nonprestressed tension reinforcement, in.^{2} Chapters 8, 10, 11, 12, and 18

A_s = area of tension reinforcement. Chapter 9

A_s' = area of compression reinforcement, in.^{2} Chapters 8, 9, and 18

A_{sh} = total cross-sectional area of transverse reinforcement (including crossties) within spacing s and perpendicular to dimension h_c. Chapter 21

A_{sk} = area of skin reinforcement per unit height in one side face, in.^{2}/ft. See 10.6.7. Chapter 10

$A_{s,min}$ = minimum amount of flexural reinforcement, in.^{2} See 10.5. Chapter 10

A_{st} = total area of longitudinal reinforcement, (bars or steel shapes), in.^{2} Chapter 10

A_t = area of structural steel shape, pipe, or tubing in a composite section, in.^{2} Chapter 10

A_t = area of one leg of a closed stirrup resisting torsion within a distance s, in.^{2} Chapter 11

A_{tr} = total cross-sectional area of all transverse reinforcement which is within the spacing s and which crosses the potential plane of splitting through the reinforcement being developed, in.^{2} Chapter 12

A_v = area of shear reinforcement within a distance s, or area of shear reinforcement perpendicular to flexural tension reinforcement within a distance s for deep flexural members, in.^{2} Chapter 11

A_v = area of shear reinforcement within a distance s, in.^{2} Chapter 12 and Appendix A

A_v = area of ties within a distance s, in.,^{2} Chapter 17.

A_{vf} = area of shear-friction reinforcement, in.^{2} Chapter 11

A_{vh} = area of shear reinforcement parallel to flexural tension reinforcement within a distance s_2, in.^{2} Chapter 11

A_w = area of an individual wire to be developed or spliced, in.^{2} Chapter 12

A_1 = loaded area. Chapter 10 and Appendix A

A_1 = loaded area, in.^{2} Chapter 22

A_2 = the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal. Chapter 10

A_2 = the area of the lower base of the largest frustum of a pyramid, cone, or tapered wedge contained wholly within the support and having for its upper base the loaded area, and having side slopes of 1 vertical to 2 horizontal, in.^{2} Chapter 22

A_2 = maximum area of the portion of the supporting surface that is geometrically similar to and concentric with the loaded area. Appendix A

b = width of compression face of member, in. Chapters 8, 9, 10, 11, 18

b = effective compressive flange width of a structural member, in. Chapter 21

b = width of member, in. Chapter 22

b_o = perimeter of critical section for slabs and footings, in. Chapter 11 and Appendix A

b_o = critical perimeter for shear for pile groups. Chapter 15

b_o = perimeter of critical section for shear in footings, in. Chapter 22

b_t = width of that part of cross section containing the closed stirrups resisting torsion. Chapter 11

b_v = width of cross section at contact surface being investigated for horizontal shear. Chapter 17

b_w = web width, in. Chapter 10

b_w = web width, or diameter of circular section, in. Chapters 11, 12, 21, and Appendix A

b_1 = width of the critical section defined in 11.12.1.2 measured

CODE

in the direction of the span for which moments are determined, in. Chapters 11 and 13
b_2 = width of the critical section defined in 11.12.1.2 measured in the direction perpendicular to b_1, in. Chapters 11 and 13
B_n = nominal bearing strength of loaded area. Chapter 22
c = distance from extreme compression fiber to neutral axis, in. Chapters 9 and 10
c = spacing or cover dimension, in. See 12.2.4. Chapter 12
c_{AB} = distance from centroidal axis of critical section to perimeter of
c_{CD} critical section. See Fig. R11.12.6.2. Chapter 11
c_1 = size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined, in. Chapters 11 and 13
c_2 = size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction of the span for which moments are being determined, in. Chapters 11 and 13
C = cross-sectional constant to define torsional properties. The constant C for T- or L-sections shall be permitted to be evaluated by dividing the section into separate rectangular parts and summing the values of C for each part. Chapter 13

$$= \Sigma\left(1-0.63\frac{x}{y}\right)\frac{x^3y}{3}$$

C_m = a factor relating actual moment diagram to an equivalent uniform moment diagram. Chapter 10
d = distance from extreme compression fiber to centroid of tension reinforcement, in. Chapters 7, 8, 9, 10, 12, and Appendix A
d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, but need not be less than $0.80h$ for prestressed members, in. (For circular sections, d need not be less than the distance from extreme compression fiber to centroid of tension reinforcement in opposite half of member). Chapter 11
d = effective depth of footing. Chapter 15
d = distance from extreme compression fiber to centroid of tension reinforcement for entire composite section, in. Chapter 17
d = distance from extreme compression fiber to centroid of nonprestressed tension reinforcement, in. Chapter 18
d = effective depth of section. Chapter 21
d' = distance from extreme compression fiber to centroid of compression reinforcement, in. Chapters 9 and 18
d_b = nominal diameter of bar, wire, or prestressing strand, in. Chapters 7 and 12
d_b = diameter of flexural reinforcement. Chapter 11
d_b = bar diameter, Chapter 21
d_c = thickness of concrete cover measured from extreme tension fiber to center of bar or wire located closest thereto, in. Chapter 10
d_p = diameter of pile at footing base. Chapter 15
d_p = distance from extreme compression fiber to centroid of prestressed reinforcement. Chapter 18
d_s = distance from extreme tension fiber to centroid of tension reinforcement, in. Chapter 9
d_t = distance from extreme compression fiber to extreme tension steel, in. Chapters 9 and 10
D = dead loads, or related internal moments and forces. Chapters 9, 18, and 20
D = dead loads, or related internal moments and forces. Chapter 21
D = dead load consisting of: (a) weight of the member itself; (b) weight of all materials of construction incorporated into the building to be permanently supported by the member, including built-in partitions; and (c) weight of permanent equipment. Appendix C

COMMENTARY

D_i = resolution of V_i into diagonal compression force. Chapter 11
e = eccentricity of load parallel to axis of member measured from centroid of gross section. Chapter 10
e = base of Napierian logarithms. Chapter 18
E = load effects of earthquake, or related internal moments and forces. Chapters 9 and 21
E = earthquake load. Appendix C
E_c = modulus of elasticity of concrete, psi. See 8.5.1. Chapters 8, 9, 10, 19, and Appendix A
E_{cb} = modulus of elasticity of beam concrete. Chapter 13
E_{cs} = modulus of elasticity of slab concrete. Chapter 13
EI = relative flexural stiffness of member. Chapter 8
EI = flexural stiffness of compression member. See Eq. (10-12) and (10-13). Chapter 10
E_s = modulus of elasticity of reinforcement, psi. See 8.5.2 or 8.5.3. Chapters 8, 10, and Appendix A
f_c' = specified compressive strength of concrete, psi. Chapters 4, 5, 8, 9, 10, 11, 12, 14, 18, 19, 20, 21, 22, and Appendix A
f_{cr}' = required average concrete strength, psi. Chapter 4
f_{cr}' = required average compressive strength of concrete used as the basis for selection of concrete proportions, psi. Chapter 5
$\sqrt{f_c'}$ = square root of specified compressive strength of concrete, psi. Chapters 9, 11, 12, 18, 19, 21, 22, and Appendix A
f_{ci}' = compressive strength of concrete at time of initial prestress, psi. Chapter 18
$\sqrt{f_{ci}'}$ = square root of compressive strength of concrete at time of initial prestress, psi. Chapter 18
f_{ct} = average splitting tensile strength of lightweight aggregate concrete, psi. Chapters 5, 9, 11, 12, 22, and Appendix A
f_d = stress due to unfactored dead load, at extreme fiber of section where tensile stress is caused by externally applied loads, psi. Chapter 11
f_{pc} = compressive stress in concrete (after allowance for all prestress losses) at centroid of cross section resisting externally applied loads or at junction of web and flange when the centroid lies within the flange, psi. (In a composite member, f_{pc} is resultant compressive stress at centroid of composite section, or at junction of web and flange when the centroid lies within the flange, due to both prestress and moments resisted by precast member acting alone). Chapter 11
f_{pc} = average compressive stress in concrete due to effective prestress force only (after allowance for all prestress losses), psi. Chapter 18
f_{pe} = compressive stress in concrete due to effective prestress forces only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads, psi. Chapter 11
f_{ps} = prestressing tendon stress at ultimate at section of maximum moment. Chapter 11
f_{ps} = stress in prestressed reinforcement at nominal strength. See text for units. Chapters 12 and 18
f_{pu} = specified tensile strength of prestressing tendons, psi. Chapters 11 and 18
f_{py} = specified yield strength of prestressing tendons, psi. Chapter 18
f_r = modulus of rupture of concrete, psi. Chapters 9 and 18
f_s = calculated stress in reinforcement at service loads, ksi. Chapter 10
f_s = permissible tensile stress in reinforcement, psi. Appendix A
f_{se} = effective prestressing tendon stress after all prestress losses. Chapter 11
f_{se} = effective stress in prestressed reinforcement (after allowance for all prestress losses). See text for units. Chapters 12 and 18

CODE

f_y = specified yield strength of nonprestressed reinforcement, psi. Chapters 3, 7, 8, 9, 10, 11, 12, 18, 19, 21 and Appendix A

f_y = yield strength of tension reinforcement. Chapter 20

f_{yh} = specified yield strength of transverse reinforcement, psi. Chapter 21

$f_{y\ell}$ = yield strength of longitudinal torsional reinforcement, psi. Chapter 11

f_{yt} = specified yield strength of transverse reinforcement, psi. Chapter 12

f_{yv} = yield strength of closed transverse torsional reinforcement, psi. Chapter 11

F = loads due to weight and pressures of fluids with well-defined densities and controllable maximum heights, or related internal moments and forces. Chapter 9

F = loads due to fluids with well-defined pressures and maximum heights. Appendix C

GJ = relative torsional stiffness of member. Chapter 8

h = overall thickness of member, in. Chapters 9, 10, 11, 12, 13, 14, 18, 20, and 22

h = overall thickness of composite member, in. Chapter 17

h = thickness of shell or folded plate, in. Chapter 19

h_c = cross-sectional dimension of column core measured center-to-center of confining reinforcement. Chapter 21

h_v = total depth of shearhead cross section, in. Chapter 11

h_w = total height of wall from base to top, in. Chapter 11

h_w = height of entire wall (diaphragm) or of the segment of wall (diaphragm) considered. Chapter 21

H = loads due to weight and pressure of soil, water in soil, or other materials, or related internal moments and forces. Chapter 9

H = loads due to the weight and lateral pressure of soil and water in soil. Appendix C

I = moment of inertia of section resisting externally applied factored loads. Chapter 11

I_b = moment of inertia about centroidal axis of gross section of beam as defined in 13.2.4. Chapter 13

I_{cr} = moment of inertia of cracked section transformed to concrete. Chapter 9

I_e = effective moment of inertia for computation of deflection. Chapter 9

I_g = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement. Chapters 9 and 10

I_s = moment of inertia about centroidal axis of gross section of slab. Chapter 13

= $h^3/12$ times width of slab defined in notations α and β_t

I_{se} = moment of inertia of reinforcement about centroidal axis of member cross section. Chapter 10

I_t = moment of inertia of structural steel shape, pipe, or tubing about centroidal axis of composite member cross section. Chapter 10

jd = moment arm at a section, in. Chapter 12

J_c = property of assumed critical section analogous to polar moment of inertia. See Fig. 11.12.6.2. Chapter 11

k = effective length factor for compression members. Chapter 10

k = effective length factor. Chapter 14

K = wobble friction coefficient per foot of prestressing tendon. Chapter 18

K_t = torsional stiffness of torsional member; moment per unit rotation. See R13.7.5. Chapter 13

K_{tr} = transverse reinforcement index. Chapter 12

= $\dfrac{A_{tr} f_{yt}}{1500\,sn}$ (constant 1500 carries the unit lb/in.^{2})

K_1 = factor to determine portion of shear strength provided by concrete at a section. Chapter 11

ℓ = span length of beam or one-way slab, as defined in 8.7; clear projection of cantilever, in. Chapter 9

COMMENTARY

ℓ = span length of flexural member measured center-to-center of joints. Chapter 10

ℓ = span length of member. Chapter 11

ℓ = clear span, in. Chapter 16

ℓ = length of span of two-way flat plates in direction parallel to that of the reinforcement being determined, in. See Eq. (18-8). Chapter 18

ℓ_a = additional embedment length at support or at point of inflection, in. Chapter 12

ℓ_c = length of a compression member in a frame, measured from center to center of the joints in the frame. Chapter 10

ℓ_c = vertical distance between supports, in. Chapters 14 and 22

ℓ_d = development length, in. Chapters 7 and 19

ℓ_d = development length, in. Chapter 12

= ℓ_{db} x applicable modification factors

ℓ_d = development length for a straight bar. Chapter 21

ℓ_d = required development length for straight bar embedded in confined concrete (Section 21.5.4.3). Chapter 21

ℓ_{db} = basic development length, in. Chapter 12

ℓ_{dc} = length of bar embedded in confined concrete. Chapter 21

ℓ_{dh} = development length of standard hook in tension, measured from critical section to outside end of hook (straight embedment length between critical section and start of hook [point of tangency] plus radius of bend and one bar diameter), in. Chapter 12

= ℓ_{hb} x applicable modification factors

ℓ_{dh} = development length for a bar with a standard hook as defined in Eq. (21-5). Chapter 21

ℓ_{dm} = required development length if bar is not entirely embedded in confined concrete. Chapter 21

ℓ_{hb} = basic development length of standard hook in tension, in. Chapter 12

ℓ_n = clear span for positive moment or shear and average of adjacent clear spans for negative moment. Chapter 8

ℓ_n = clear span measured face-to-face of supports. Chapter 11

ℓ_n = length of clear span in long direction of two-way construction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases. Chapter 9

ℓ_n = length of clear span in direction that moments are being determined, measured face-to-face of supports. Chapter 13

ℓ_n = beam clear span. Chapter 21

ℓ_o = minimum length, measured from joint face along axis of structural member, over which transverse reinforcement must be provided, in. Chapter 21

ℓ_t = span of member under load test, in. (The shorter span for two-way slab systems.) Span is the smaller of (a) distance between centers of supports, and (b) clear distance between supports plus thickness, h, of member. In Eq. (20-1), span for a cantilever shall be taken as twice the distance from support to cantilever end, in. Chapter 20

ℓ_u = unsupported length of compression member. Chapter 10

ℓ_v = length of shearhead arm from centroid of concentrated load or reaction, in. Chapter 11

ℓ_w = horizontal length of wall, in. Chapter 11

ℓ_w = length of entire wall (diaphragm) or a segment of wall (diaphragm) considered in direction of shear force. Chapter 21

ℓ_x = length of prestressing tendon element from jacking end to any point x ft. See Eq. (18-1) and (18-2). Chapter 18

ℓ_1 = length of span in direction that moments are being determined, measured center-to-center of supports. Chapter 13

ℓ_2 = length of span transverse to ℓ_1, measured center-to-center of supports. See also 13.6.2.3 and 13.6.2.4. Chapter 13

L = live loads or related internal moments and forces. Chapters 9, 18, and 20

L = live loads, or related internal moments and forces. Chapter 21

CODE

L = live loads due to intended use and occupancy, including loads due to movable objects and movable partitions and loads temporarily supported by the structure during maintenance. L includes any permissible reduction. If resistance to impact loads is taken into account in design, such effects shall be included with the live load L. Appendix C
L_r = roof live loads. Appendix C
M = design moment. Appendix A
M_a = maximum moment in member at stage deflection is computed. Chapter 9
M_c = factored moment to be used for design of compression member. Chapter 10
M_{cr} = cracking moment. See 9.5.2.3. Chapter 9
M_{cr} = moment causing flexural cracking at section due to externally applied loads. See 11.4.2.1. Chapter 11
M_{ct} = total moment including dead load to cause cracking at extreme fiber in tension. Chapter 11
M_d = service dead load moment. Chapter 9
M_ℓ = service live load moment. Chapter 9
M_m = modified moment. Chapter 11
M_{max} = maximum factored moment at section due to externally applied loads. Chapter 11
M_n = nominal moment strength at section. Chapter 22
M_n = nominal moment strength. Chapters 9, 11, and 18
M_n = nominal moment strength at section, in.-lb. Chapter 12
= $A_s f_y (d - a/2)$
$M_{n\ell}$ = nominal beam moment, left. Chapter 21
M_{nr} = nominal beam moment, right. Chapter 21
M_{ns} = unmagnified nonsway moment at each end of each column. Chapter 10
M_o = total factored static moment. Chapter 13
M_p = required plastic moment strength of shearhead cross section. Chapter 11
M_{pr} = probable flexural moment strength of members, with or without axial load, determined using the properties of the member at the joint faces assuming a tensile strength in the longitudinal bars of at least 1.25 f_y and a strength reduction factor ϕ of 1.0. Chapter 21
M_s = moment due to loads causing appreciable sway. Chapter 10
M_s = portion of slab moment balanced by support moment. Chapter 21
M_u = required moment strength. Chapter 9
M_u = factored moment at section. Chapters 10, 11, 13, and 22
M_v = moment resistance contributed by shearhead reinforcement. Chapter 11
M_1 = smaller factored end moment on a compression member, positive if member is bent in single curvature, negative if bent in double curvature. Chapter 10
M_{1ns} = factored end moment on a compression member at the end at which M_1 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis. Chapter 10
M_{1s} = factored end moment on compression member at the end at which M_1 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis. Chapter 10
M_2 = larger factored end moment on compression member, always positive. Chapter 10
$M_{2,min}$ = minimum value of M_2. Chapter 10
M_{2ns} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis. Chapter 10
M_{2s} = factored end moment on compression member at the end at which M_2 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis. Chapter 10

COMMENTARY

n = number of consecutive strength tests. Chapter 5
n = number of bars or wires being spliced or developed along the plane of splitting. Chapter 12
n = modular ratio of elasticity. Appendix A
= E_s/E_c
n_1, n_2 = number of tests in each test record respectively. Chapter 5
N = design axial load normal to cross section occurring simultaneously with V; to be taken as positive for compression, negative for tension, and to include effects of tension due to creep and shrinkage. Appendix A
N_c = tensile force in concrete due to unfactored dead load plus live load $(D + L)$. Chapter 18
N_i = resolution of V_i into axial tension force. Chapter 11
N_u = factored axial load normal to cross section occurring simultaneously with V_u; to be taken as positive for compression, negative for tension, and to include effects of tension due to creep and shrinkage. Chapter 11
N_{uc} = factored tensile force applied at top of bracket or corbel acting simultaneously with V_u, to be taken as positive for tension. Chapter 11
p_{cp} = outside perimeter of the concrete cross section, in. See 11.6.1. Chapter 11
p_h = perimeter of centerline of outermost closed transverse torsional reinforcement, in. Chapter 11
P = design axial loads. Appendix A
P = loads, forces, and effects due to ponding. Appendix C
P_b = nominal axial load strength at balanced strain conditions. See 10.3.2. Chapters 9 and 10
P_c = critical load. See Eq. (10-11). Chapter 10
P_n = nominal axial load strength at given eccentricity. Chapters 9 and 10
P_n = nominal strength of cross section subject to compression. Chapter 22
P_n = nominal axial load strength. Appendix A
P_{ni} = nominal axial load strength at given eccentricity along both axes. Chapter 10
P_{nw} = nominal axial load strength of wall designed by 14.4. Chapter 14
P_{nw} = nominal axial load strength of wall designed by 22.6.5. Chapter 22
P_{nx} = nominal axial load strength at given eccentricity along x-axis. Chapter 10
P_{ny} = nominal axial load strength at given eccentricity along y-axis. Chapter 10
P_o = nominal axial load strength at zero eccentricity. Chapter 10
P_s = prestressing tendon force at jacking end. Chapter 18
P_u = required axial load strength. Chapters 9 and 14
P_u = factored axial load at given eccentricity $\leq \phi P_n$. Chapter 10
P_u = factored design axial load. Chapter 21
P_u = factored axial load at given eccentricity. Chapter 22
P_x = prestressing tendon force at any point x. Chapter 18
q = shear flow. Chapter 11
q_s = soil reaction due to factored loading. Chapter 15
Q = stability index for a story. See 10.11.4. Chapter 10
r = radius of gyration of cross section of a compression member. Chapter 10
R = rain loads, except ponding. Appendix C
s = standard deviation, psi. Chapter 5
s = spacing of shear or torsion reinforcement in direction parallel to longitudinal reinforcement, in. Chapter 11
s = maximum spacing of transverse reinforcement within ℓ_d center-to-center, in. Chapter 12
s = spacing of ties measured along the longitudinal axis of the member, in. Chapter 17
s = spacing of transverse reinforcement measured along the longitudinal axis of the structural member, in. Chapter 21
s = spacing of shear reinforcement in direction parallel to longitudinal reinforcement, in. Appendix A

CODE

COMMENTARY

s_o = maximum spacing of transverse reinforcement, in. Chapter 21
s_w = spacing of wire to be developed or spliced, in. Chapter 12
s_1, s_2 = standard deviations calculated from two test records, 1 and 2, respectively. Chapter 5
s_1 = spacing of vertical reinforcement in wall, in. Chapter 11
s_2 = spacing of shear or torsion reinforcement in direction perpendicular to longitudinal reinforcement — or spacing of horizontal reinforcement in wall, in. Chapter 11
$\bar{s}$ = statistical average standard deviation where two test records are used to estimate the standard deviation. Chapter 5
S = elastic section modulus of section. Chapter 22
S = snow loads. Appendix C
t = thickness of a wall of a hollow section, in. Chapter 11
T = cumulative effect of temperature, creep, shrinkage, differential settlement, and shrinkage-compensating concrete. Chapter 9
T = torsional moment on a member. Chapter 11
T = self-straining forces and effects arising from contraction or expansion resulting from temperature changes, shrinkage, moisture changes, creep in component materials, movement due to differential settlement, or combinations thereof. Appendix C
T_{cr} = torque or torsion on a member causing first crack. Chapter 11
T_n = nominal torsional moment strength. Chapter 11
T_u = factored torsional moment at section. Chapter 11
u = service load bond stress, psi. Chapter 12
U = required strength to resist factored loads or related internal moments and forces. Chapter 9
U = factored concentric load on footing. Chapter 15
v = design shear stress. Appendix A
v_c = shear stress provided by the concrete at a section, psi. Chapter 11
v_c = permissible shear stress carried by concrete, psi. Appendix A
v_h = permissible horizontal shear stress, psi. Appendix A
v_n = nominal shear stress, psi. See 11.12.6.2, Chapter 11
v_u = factored shear stress. Chapter 11
V = shear required to cause a flexural crack at the section in question. Chapter 11
V = service load shear. Chapter 12
V = design shear force at section. Appendix A
V_c = nominal shear strength provided by concrete. Chapters 8, 11, and 21
V_c = nominal shear strength provided by concrete. See 11.12.2.1. Chapter 13
V_{ci} = nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment. Chapter 11
V_{cw} = nominal shear strength provided by concrete when diagonal cracking results from excessive principal tensile stress in web. Chapter 11
V_d = shear force at section due to unfactored dead load. Chapter 11
V_e = design shear force determined from 21.3.4.1 or 21.4.5.1. Chapter 21
V_i = one of the shear forces V_1 to V_4. Chapter 11
V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}. Chapter 11
V_n = nominal shear strength. Chapters 9 and 11
V_n = nominal shear strength. Chapters 11 and 21
V_n = nominal shear strength at section. Chapter 22
V_{nh} = nominal horizontal shear strength. Chapter 17
V_p = vertical component of effective prestress force at section. Chapter 11
V_s = nominal shear strength provided by shear reinforcement. Chapter 11
V_u = required shear strength. Chapter 9
V_u = factored horizontal shear in a story. Chapter 10
V_u = factored shear force at section. Chapters 11, 12, 13, 17, 21, and 22
V_1, V_2, V_3, V_4 = resolution of shear flow into shear forces on sides of tube or space truss. Chapter 11
w = crack width, in. Chapter 10
w = service load per unit length or per unit area. Appendix A
w_c = weight of concrete, lb/ft.^{3} Chapters 8 and 9
w_d = factored dead load per unit area. Chapter 13
w_D = dead load per unit length or per unit area. Chapter 21 and Appendix A
w_ℓ = factored live load per unit area. Chapter 13
w_L = live load per unit length or per unit area. Chapter 21 and Appendix A
w_u = factored load per unit length of beam or per unit area of slab. Chapter 8
w_u = factored load per unit area. Chapter 13
w_u = factored load per unit length or per unit area. Appendix A
W = wind load, or related internal moments and forces. Chapter 9
W = wind load. Appendix C
x = distance from section being investigated to the support. Chapter 11
x = distance between adjacent spliced bars. Chapter 12
x = shorter overall dimension of rectangular part of cross section. Chapter 13
X_i = individual strength tests as defined in 5.6.1.4. Chapter 5
$\bar{X}$ = average of n strength test results. Chapter 5
y = longer overall dimension of rectangular part of cross section. Chapter 13
y_t = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension. Chapters 9 and 11
z = quantity limiting distribution of flexural reinforcement. See 10.6. Chapter 10
α (alpha) = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of the beam. Chapters 9 and 13

$$= \frac{E_{cb} I_b}{E_{cs} I_s}$$

α = angle between inclined stirrups and longitudinal axis of member. Chapter 11 and Appendix A
α = reinforcement location factor. See 12.2.4. Chapter 12
α = total angular change of prestressing tendon profile in radians from tendon jacking end to any point x. Chapter 18
α_c = coefficient defining the relative contribution of concrete strength to wall strength. See Eq. (21-7) Chapter 21
α_f = angle between shear-friction reinforcement and shear plane. Chapter 11
α_m = average value of α for all beams on edges of a panel. Chapter 9
α_s = constant used to compute V_c in slabs and footings. Chapter 11
α_v = ratio of stiffness of shearhead arm to surrounding composite slab section. See 11.12.4.5. Chapter 11
α_1 = α in direction of ℓ_1. Chapter 13
α_2 = α in direction of ℓ_2. Chapter 13
β (beta) = ratio of clear spans in long to short direction of two-way slabs. Chapter 9
β = ratio of distances to neutral axis from extreme tension fiber and from centroid of flexural tension reinforcement. Chapter 10
β = coating factor. See 12.2.4. Chapter 12
β = ratio of long side to short side of footing. Chapter 15
β_b = ratio of area of reinforcement cut off to total area of tension reinforcement at section. Chapter 12
β_c = ratio of long side to short side of concentrated load or reaction area. Chapters 11, 22, and Appendix A
β_d = (a) for non-sway frames, β_d is the ratio of the maximum

CODE

factored axial dead load to the total factored axial load

(b) for sway frames, except as required in (c), β_d is the ratio of the maximum factored sustained shear within a story to the total factored shear in that story

(c) for stability checks of sway frames carried out in accordance with 10.13.6, β_d is the ratio of the maximum factored sustained axial load to the total factored axial load. Chapter 10

β_p = constant used to compute V_c in prestressed slabs. Chapter 11

β_t = ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports. Chapter 13

= $\dfrac{E_{cb}C}{2E_{cs}I_s}$

β_1 = factor defined in 10.2.7.3. Chapters 8, 10, and 18

γ **(gamma)** = reinforcement size factor. See 12.2.4. Chapter 12

γ_f = fraction of unbalanced moment transferred by flexure at slab-column connections. See 13.5.3.2. Chapters 11 and 13

γ_f = fraction of M_s assigned to slab effective width. Chapter 21

γ_p = factor for type of prestressing tendon. Chapter 18

= 0.55 for f_{py}/f_{pu} not less than 0.80

= 0.40 for f_{py}/f_{pu} not less than 0.85

= 0.28 for f_{py}/f_{pu} not less than 0.90

γ_v = fraction of unbalanced moment transferred by eccentricity of shear at slab-column connections. See 11.12.6.1. Chapters 11 and 13

= $1-\gamma_f$

δ_b = moment magnification factor for frames braced against sidesway. Chapter 10

δ_{ns} = moment magnification factor for frames braced against sidesway, to reflect effects of member curvature between ends of compression member. Chapter 10

δ_s = moment magnification factor for frames not braced against sidesway, to reflect lateral drift resulting from lateral and gravity loads. Chapter 10

Δf_p **(delta)** = difference between f_{ps} and prestressing tendon stress at ultimate at section being considered. Chapter 11

Δ_{max} = measured maximum deflection in. See Eq. (20-1). Chapter 20

Δ_{rmax} = measured residual deflection, in. See Eq. (20-2) and (20-3). Chapter 20

Δ_{fmax} = maximum deflection measured during the second test relative to the position of the structure at the beginning of the second test, in. See Eq. (20-3). Chapter 20

Δ_o = relative lateral deflection between the top and bottom of a story due to V_u computed using a first-order elastic frame analysis and stiffness values satisfying 10.11.1. Chapter 10

ε_s **(epsilon)** = strain in reinforcement corresponding to calculated stress. Chapter 10

ε_t = net tensile strain in extreme tension steel at nominal strength. Chapters 9 and 10

ε_y = yield strain of reinforcement. Chapter 10

η**(eta)**= number of identical arms of shearhead. Chapter 11

θ **(theta)** = angle of compression diagonals in truss analogy for torsion. Chapter 11

λ **(lambda)** = multiplier for additional long-term deflection as defined in 9.5.2.5. Chapter 9

λ = correction factor related to unit weight of concrete. Chapters 11 and 17

λ = lightweight aggregate concrete factor. See 12.2.4. Chapter 12

μ **(mu)** = coefficient of friction. See 11.7.4.3. Chapter 11

COMMENTARY

μ = curvature friction coefficient. Chapter 18

ξ **(xi)** = time-dependent factor for sustained load. See 9.5.2.5. Chapter 9

ρ **(rho)** = ratio of nonprestressed tension reinforcement. Chapters 8, 9, 10, 11, 13, 18, and 21

= A_s/bd

ρ = ratio of tension reinforcement. Chapter 20

ρ' = ratio of nonprestressed compression reinforcement. Chapter 8

= A_s'/bd

ρ' = reinforcement ratio for nonprestressed compression reinforcement, A_s'/bd. Chapter 9

ρ' = ratio of compression reinforcement. Chapter 18 = A_s'/bd.

ρ_b = reinforcement ratio producing balanced strain conditions. See 10.3.2. Chapters 8, 10, and 13

ρ_b = reinforcement ratio producing balanced strain conditions. See B10.3.2. Chapter 9

ρ_g = ratio of total reinforcement area to cross-sectional area of column. Chapter 21

ρ_h = ratio of horizontal shear reinforcement area to gross concrete area of vertical section. Chapter 11

ρ_n = ratio of vertical shear reinforcement area to gross concrete area of horizontal section. Chapter 11

ρ_n = ratio of distributed shear reinforcement on a plane perpendicular to plane of A_{cv}. Chapter 21

ρ_p = ratio of prestressed reinforcement. Chapter 18

= A_{ps}/bd_p

ρ_s = ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of a spirally reinforced compression member. Chapter 10

ρ_s = ratio of volume of spiral reinforcement to the core volume confined by the spiral reinforcement (measured out-to-out). Chapter 21

ρ_v = ratio of the tie reinforcement area to area of contact surface. Chapter 17

= $A_v/b_v s$

ρ_v = A_{sv}/A_{cv}; where A_{sv} is the projection on A_{cv} of area of distributed shear reinforcement crossing the plane of A_{cv}. Chapter 21

ρ_w = $A_s/b_w d$. Chapter 11

ρ_w = ratio of tension reinforcement. Appendix A

= $A_s/b_w d$

Σ_o **(sigma)** = perimeter of bar, in. Chapter 12

τ **(tau)** = shear stress. Chapter 11

ϕ **(phi)** = strength reduction factor. See 9.3. Chapters 8, 9, 10, 11, 13, 14, 17, 18, 19, and 21

ϕ = strength reduction factor. See 9.3.5. Chapter 22

ϕ = strength reduction factor. See A.2.1. Appendix A

ϕ_K = stiffness reduction factor. See R10.12.3. Chapter 10

ψ **(psi)** = ratio of sum of stiffnesses of compression members to sum of stiffnesses of flexural members at one end of a compression member. Chapter 10

ψ_{min} = smaller of ψ-values at two ends of a compression member. Chapter 10

ψ_m = average of ψ-values at two ends of a compression member. Chapter 10

ω **(omega)** = $\rho f_y/f_c'$. Chapter 18

ω' = $\rho' f_y/f_c'$. Chapter 18

ω_p = $\rho_p f_{ps}/f_c'$. Chapter 18

$\omega_w, \omega_{pw}, \omega'_w$

= reinforcement indices for flanged sections computed as for ω, ω_p, and ω' except that b shall be the web width, and reinforcement area shall be that required to develop compressive strength of web only. Chapter 18

APPENDIX E — STEEL REINFORCEMENT INFORMATION

As an aid to users of the ACI Building Code, information on sizes, areas, and weights of various steel reinforcement is presented.

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

ASTM STANDARD PRESTRESSING TENDONS

Type*	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
Seven-wire strand (Grade 250)	1/4 (0.250)	0.036	0.122
	5/16 (0.313)	0.058	0.197
	3/8 (0.375)	0.080	0.272
	7/16 (0.438)	0.108	0.367
	1/2 (0.500)	0.144	0.490
	(0.600)	0.216	0.737
Seven-wire strand (Grade 270)	3/8 (0.375)	0.085	0.290
	7/16 (0.438)	0.115	0.390
	1/2 (0.500)	0.153	0.520
	(0.600)	0.217	0.740
Prestressing wire	0.192	0.029	0.098
	0.196	0.030	0.100
	0.250	0.049	0.170
	0.276	0.060	0.200
Prestressing bars (plain)	3/4	0.44	1.50
	7/8	0.60	2.04
	1	0.78	2.67
	1-1/8	0.99	3.38
	1-1/4	1.23	4.17
	1-3/8	1.48	5.05
Prestressing bars (deformed)	5/8	0.28	0.98
	3/4	0.42	1.49
	1	0.85	3.01
	1-1/4	1.25	4.39
	1-3/8	1.58	5.56

* Availability of some tendon sizes should be investigated in advance.

ASTM STANDARD WIRE REINFORCEMENT

W & D size		Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft	Area, in.^{2}/ft of width for various spacings						
					Center-to-center spacing, in.						
Plain	Deformed				2	3	4	6	8	10	12
W31	D31	0.628	0.310	1.054	1.86	1.24	0.93	0.62	0.465	0.372	0.31
W30	D30	0.618	0.300	1.020	1.80	1.20	0.90	0.60	0.45	0.366	0.30
W28	D28	0.597	0.280	0.952	1.68	1.12	0.84	0.56	0.42	0.336	0.28
W26	D26	0.575	0.260	0.934	1.56	1.04	0.78	0.52	0.39	0.312	0.26
W24	D24	0.553	0.240	0.816	1.44	0.96	0.72	0.48	0.36	0.288	0.24
W22	D22	0.529	0.220	0.748	1.32	0.88	0.66	0.44	0.33	0.264	0.22
W20	D20	0.504	0.200	0.680	1.20	0.80	0.60	0.40	0.30	0.24	0.20
W18	D18	0.478	0.180	0.612	1.08	0.72	0.54	0.36	0.27	0.216	0.18
W16	D16	0.451	0.160	0.544	0.96	0.64	0.48	0.32	0.24	0.192	0.16
W14	D14	0.422	0.140	0.476	0.84	0.56	0.42	0.28	0.21	0.168	0.14
W12	D12	0.390	0.120	0.408	0.72	0.48	0.36	0.24	0.18	0.144	0.12
W11	D11	0.374	0.110	0.374	0.66	0.44	0.33	0.22	0.165	0.132	0.11
W10.5		0.366	0.105	0.357	0.63	0.42	0.315	0.21	0.157	0.126	0.105
W10	D10	0.356	0.100	0.340	0.60	0.40	0.30	0.20	0.15	0.12	0.10
W9.5		0.348	0.095	0.323	0.57	0.38	0.285	0.19	0.142	0.114	0.095
W9	D9	0.338	0.090	0.306	0.54	0.36	0.27	0.18	0.135	0.108	0.09
W8.5		0.329	0.085	0.289	0.51	0.34	0.255	0.17	0.127	0.102	0.085
W8	D8	0.319	0.080	0.272	0.48	0.32	0.24	0.16	0.12	0.096	0.08
W7.5		0.309	0.075	0.255	0.45	0.30	0.225	0.15	0.112	0.09	0.075
W7	D7	0.298	0.070	0.238	0.42	0.28	0.21	0.14	0.105	0.084	0.07
W6.5		0.288	0.065	0.221	0.39	0.26	0.195	0.13	0.097	0.078	0.065
W6	D6	0.276	0.060	0.204	0.36	0.24	0.18	0.12	0.09	0.072	0.06
W5.5		0.264	0.055	0.187	0.33	0.22	0.165	0.11	0.082	0.066	0.055
W5	D5	0.252	0.050	0.170	0.30	0.20	0.15	0.10	0.075	0.06	0.05
W4.5		0.240	0.045	0.153	0.27	0.18	0.135	0.09	0.067	0.054	0.045
W4	D4	0.225	0.040	0.136	0.24	0.16	0.12	0.08	0.06	0.048	0.04
W3.5		0.211	0.035	0.119	0.21	0.14	0.105	0.07	0.052	0.042	0.035
W3		0.195	0.030	0.102	0.18	0.12	0.09	0.06	0.045	0.036	0.03
W2.9		0.192	0.029	0.098	0.174	0.116	0.087	0.058	0.043	0.035	0.029
W2.5		0.178	0.025	0.085	0.15	0.10	0.075	0.05	0.037	0.03	0.025
W2		0.159	0.020	0.068	0.12	0.08	0.06	0.04	0.03	0.024	0.02
W1.4		0.135	0.014	0.049	0.084	0.056	0.042	0.028	0.021	0.017	0.014

INDEX

Acceptance of concrete, 5.6
Admixtures, 3.6
  -Accelerating, 3.6
  -Air-entraining, 3.6
  -Definition, 2.1
  -Retarding, 3.6
  -Water-reducing, 3.6
Aggregates, 3.3
  -Definition, 2.1
  -Lightweight—Definition, 2.1
  -Nominal maximum size, 3.3
Air-entraining admixtures, 3.6
Aluminum conduits or pipes, 6.3
American Society for Testing and Materials—See ASTM
American Welding Society—See AWS
Analysis methods, 8.3
Anchorage—Definition, 2.1
Anchorage—Mechanical—Development, 12.6
Anchorages—Post-tensioning, 18.19
Anchorage zones—Prestressed tendons, 18.13
Alternate design method, A.1, A.2
  -Compression members, A.6
  -Development and splices of reinforcement, A.4
  -Flexure, A.5
  -Permissible service load stresses, A.3
  -Shear and torsion, A.7
Alternative load and strength reduction factors, C.1
ASCE (American Society of Civil Engineers) standard cited in this code, 3.8
ASTM (American Society for Testing and Materials) standards cited in this code, 3.8
AWS (American Welding Society) standards cited in this code, 3.8
Axial load
  -Design assumptions, 10.2
  -Principles and requirements, 10.3
Axially loaded members—Slab system support, 10.14

Base of structure—Definition, 21.1
Beam
  -Deflections—Minimum thickness, 9.5
  -Distribution of flexural reinforcement, 10.6
  -Grade—Walls—Design, 14.3
Bearing strength, 10.17
Bearing walls
  -Design, 14.2
  -Precast, 16.5
Bending, 7.3
Bends—Reinforcement, 7.2
Bonded reinforcement—Minimum—Prestressed concrete, 18.9
Bonded tendon—Definition, 2.1
Boundary members—Definition, 21.1
Brackets—Shear provision, 11.9
Building official—Definition, 1.2
Bundled bars
  -Development, 12.4
  -Spacing limits, 7.6

Calculations, 1.2
Cement, 3.2
Cementitious materials—Definition, 2.1
Chloride—Admixtures, 3.6
Cold weather concreting, 5.12
Collector elements—Definition, 21.1
Columns
  -Definition, 2.1
  -Design, 8.8
  -Equivalent—Slab design, 13.7
  -Moment transfer, 11.11
  -Reinforcement splices, 12.17
  -Special reinforcement details, 7.8
  -Steel cores, 7.8
Column loads—Transmission through floor system, 10.15
Composite compression members—Axial load, 10.16
Composite construction—Deflections, 9.5
Composite flexural members, 17.1, 17.2
  -Definition, 2.1
  -Horizontal shear strength, 17.5
  -Shoring, 17.3
  -Ties for horizontal shear, 17.6
  -Vertical shear strength, 17.4
Compression-controlled section—Definition, 2.1, B9.3
Compression-controlled strain limit—Definition, 2.1
Compression members
  -Alternate design method, A.6
  -Design dimensions, 10.8
  -Effective length, 10.11
  -Limits for reinforcement, 10.9
  -Prestressed concrete, 18.11
  -Slenderness effects, 10.10, 10.11
Computer programs, 1.2
Concrete
  -Conveying, 5.9
  -Curing, 5.11
  -Definition, 2.1
  -Depositing, 5.10
  -Evaluation and acceptance, 5.6
  -Proportioning, 5.2, 5.3, 5.4
  -Mixing, 5.8
Conduits, embedded, 6.3
Connections
  -Reinforcement, 7.9
Construction joints, 6.4
Continuous construction—Prestressed concrete, 18.10
Contraction joint—Definition, 2.1
Conveying concrete, 5.9
Corbels—Shear provisions, 11.9
Corrosion
  -Protection of reinforcement, 4.4
  -Protection of unbonded prestressing tendons, 18.14
Couplers—Post-tensioning, 18.19
Cover, 7.7
Creep—Required strength, 9.2
Crosstie—Definition, 21.1
Curing, 5.11
  -Accelerated, 5.11
Curvature friction—Definition, 2.1, 18.6
Cylinders—Testing, 5.6

Dead load—See Load, dead
Deep flexural members, 10.7
  -Special provisions for shear, 11.8
Definitions, 2.1, 13.2, 19.1, 21.1
Deflection
  -Composite construction, 9.5
  -Control, 9.5
  -Maximum, 9.5
  -Prestressed concrete construction, 9.5
Deformed bars, 12.4
  -Compression—Splices, 12.16
  -Tension—Splices, 12.15

Deformed reinforcement—Definition, 21.1
Depositing concrete, 5.10
Design load combinations—Definition, 21.1
Design methods, 8.1
  -Structural plain concrete, 22.4
Design strength, 9.3
  -Reinforced and prestressed flexural and compression members, B9.3
  -Reinforcement, 9.4
  -See also Strength, design
Development
  -Bundled bars, 12.4
  -Deformed bars in compression, 12.3
  -Deformed reinforcement in tension, 12.2
  -Flexural reinforcement, 12.10
  -Footing reinforcement, 15.6
  -Hooks, 12.5
  -Mechanical anchorages, 12.6
  -Mechanical connectors for reinforcement, 12.15
  -Negative moment reinforcement, 12.12
  -Positive moment reinforcement, 12.11
  -Prestressing strand, 12.9
  -Reinforcement, 12.1
  -Reinforcement—Alternate design method, A.4
  -Splices, 12.14
  -Splices in column reinforcement, 12.17
  -Web reinforcement, 12.13
  -Welded deformed wire fabric, 12.7
  -Welded deformed wire fabric in tension, 12.7
  -Welded plain wire fabric, 12.8
  -Welded plain wire fabric in tension, 12.8
Development length—Definition, 2.1
Development length for a bar with a standard hook—Definition, 21.1
Direct design method—Slabs, 13.6
Drawings, 1.2
Drop panel—Two-way slab reinforcement, 13.3
Ducts
  -Post-tensioning, 18.15
  -Spacing limits, 7.6

Earth pressure, 9.2
Earthquake loads, 8.2, 9.2
Effective depth of section (*d*)—Definition, 2.1
Effective prestress—Definition, 2.1
Embedded conduits and pipes, 6.3
Embedment—Development of reinforcement, 12.13
Embedment length—Definition, 2.1
Equivalent frame method—Slabs, 13.7
Evaluation and acceptance of concrete, 5.6
Expansive cement, 3.2
Exposure
  -Cover requirements, 7.7
  -Special requirements, 4.1, 4.2, 4.3
Extreme tension steel—Definition, 2.1

Factored load and forces—Definition, 21.1
Factored load—See Load, factored
Field-cured specimens—Tests, 5.6
Flexural members—Limits for reinforcement, 10.5, 18.8, B18.8
Flexural reinforcement
  -Development, 12.10
  -Principles and requirements, 10.3
Floor finish, separate, 8.12
Floors—Transmission of column loads, 10.15
Fly ash, 3.6
Folded plates—Definition, 19.1
Footings, 15.1
  -Combined, 15.10
  -Loads and reactions, 15.2
  -Minimum depth, 15.7
  -Moments, 15.4
  -Reinforcement development, 15.6
  -Shear, 11.12, 15.5
  -Sloped or stepped, 15.9
  -Structural plain concrete, 22.7
  -Supporting circular or polygon columns, 15.3
  -Transfer of force at base of column or pedestal, 15.8
Formwork
  -Design of, 6.1
  -Prestressed concrete, 6.1
  -Removal, 6.2
Frames—Prestressed concrete, 18.10

Grade beam—Walls—Design, 14.7
Grout—Bonded prestressing, 18.16

Haunches—Effect on stiffness, 8.6
Hooks
  -Development, 12.13
  -Standard, 7.1
Hoop—Definition, 21.1
Hot weather concreting, 5.13

Impact, 9.2
Isolation joint—Definition, 2.1
Inspection, 1.3
Isolated beams, 8.10

Jacking force—Definition, 2.1
Joints—Structural plain concrete, 22.3
Joist construction, 8.11

Laboratory-cured specimens—Tests, 5.6
Lap splices—Development of reinforcement, 12.14, 12.15, 12.16
Lateral-force resisting system—Definition, 21.1
Lateral reinforcement
  -Compressed members, 7.10
  -Flexural members, 7.11
Lateral supports—Distance between for flexural members, 10.4
Lightweight aggregate, 3.3
Lightweight concrete
  -Shear strength, 11.2
  -Splitting tensile strength, 5.1
  -Structural—Definition, 2.1
Liquid pressure, lateral, 9.2
Live load—See Load, live
Load
  -Dead—Definition, 2.1
  -Factored—Definition, 2.1
  -Live—Arrangement, 8.9
  -Live—Definition, 2.1
  -Service, 8.2
  -Service—Definition, 2.1
Loading, 8.2
Load tests, 20.3
  -Loading criteria, 20.4
Loss of prestress, 18.6
Low-strength concrete, 5.6

Magnified moments, 10.11
  -Non-sway frames, 10.12
  -Sway frames, 10.13
Materials storage, 3.7
Materials, tests, 3.1

Mats—Combined, 15.10
Mechanical connection—Reinforcement development, 12.14
Minimum reinforcement—Flexural members, 10.5
Mixing and placing equipment, 5.7
Mixing concrete, 5.8
Mix proportioning, 5.2, 5.3, 5.4
Modal analysis—Shells, 19.2
Modulus of elasticity, 8.5
  -Definition, 2.1
Moment magnification—Slenderness effects—Compression members, 10.11
Moment magnifier
  -Biaxial bending, 10.11
  -Unbraced frames, 10.11
Moments
  -Designs, 8.3
  -Footings, 15.4
  -Negative—Redistribution, 8.4, 18.10
  -Negative—Reinforcement—Development, 12.12
  -Positive—Reinforcement—Development, 12.11
  -Slab design, 13.6
Moment transfer—Columns, 11.11

Net tensile strain—Definition, 2.1
Nominal strength—See Strength, nominal
Non-sway frames—Magnified moments, 10.12
Notation, Appendix D

Offset bars—Reinforcement details for columns, 7.8
Openings
  -Slabs, 11.12
  -Two-way slabs, 13.4
Pedestal
  -Definition, 2.1
  -Structural plain concrete, 22.8
Piles and piers, 1.1
Pipes
  -Embedded, 6.3
  -Steel—Reinforcement, 3.5
Placing
  -Preparation of place of deposit, 5.7
  -Rate—Formwork, 6.1
Placing equipment, 5.7
Plain concrete
  -Definition, 2.1
  -Structural, 21.1, 21.2
Plain reinforcement—Definition, 2.1
Post-tensioning—Definition, 2.1
Pozzolans, 3.6
Precast concrete
  -Bearing design, 16.6
  -Definition, 2.1
  -Design, 16.4
  -Distribution of forces, 16.3
  -Handling, 16.9
  -Strength evaluation, 16.10
  -Structural integrity, 16.5
Precast members—Structural plain concrete, 22.9
Prestressed concrete, 18.1, 18.2
  -Application of prestressing force, 18.18
  -Compression members, 18.11
  -Corrosion protection for unbonded tendons, 18.14
  -Definition, 2.1
  -Deflection, 9.5
  -Design assumptions, 18.3
  -Flexural members—Limits of reinforcement, 18.8
  -Flexural strength, 18.7
  -Frames and continuous construction, 18.10
  -Grout for bonded tendons, 18.16
  -Loss of prestress, 18.6
  -Measurement of prestressing force, 18.18
  -Minimum bonded reinforcement, 18.9
  -Permissible stresses—Flexural members, 18.4
  -Permissible stresses in tendons, 18.5
  -Post-tensioning anchorages and couplers, 18.19
  -Post-tensioning ducts, 18.15
  -Protection for tendons, 18.17
  -Shear, 11.4
  -Slab systems, 18.12
  -Statically indeterminate structures, 18.10
  -Tendon anchorage zones, 18.13
  -Torsion, 11.6
Prestressing strand—Development, 12.9
Prestressing tendons, 3.5
  -Spacing limits, 7.6
  -Surface conditions, 7.4
Pretensioning—Definition, 2.1
PTI (Post-Tensioning Institute)
  standard cited in this code, 3.8

Quality of concrete, 5.1

Radius of gyration—Compression members—Slenderness effects, 10.11
Reinforced concrete—Definition, 2.1
Reinforcement
  -Bending of, 7.3
  -Bend tests, 3.5
  -Bundled bars—Development, 12.4
  -Bundled bars—Spacing limits, 7.6
  -Columns—Splices, 12.17
  -Connections, 7.9
  -Corrosion protection for unbonded prestressing tendons, 18.14
  -Definition, 2.1
  -Deformed, 3.5
  -Deformed—Compression—Splices, 12.16
  -Deformed—Definition, 2.1
  -Deformed—Development in compression, 12.3
  -Deformed—Development in tension, 12.2
  -Deformed—Tension—Splices, 12.15
  -Design strength, 9.4
  -Development, 12.1
  -Development and splices—Alternate design method, A.4
  -Flexural—Development, 12.10
  -Flexural—Distribution in beams and one-way slabs, 10.6
  -Footings—Development, 15.6
  -Hooks—Development, 12.5
  -Lateral for compression members, 7.10
  -Lateral for flexural members, 7.11
  -Limits in compression members, 10.9
  -Limits—Prestressed flexural members, 18.8
  -Mats, 3.5
  -Mechanical anchorage—Development, 12.6
  -Minimum—Flexural members, 10.5
  -Minimum bonded—Prestressed concrete, 18.9
  -Negative moment—Development, 12.12
  -Placing, 7.5
  -Placing—Welding, 7.5
  -Plain, 3.5
  -Plain—Definition, 2.1
  -Plain welded wire fabric—Splices, 12.19
  -Plain wire fabric, 12.8
  -Positive moment—Development, 12.11
  -Prestressing strand—Development, 12.9
  -Prestressing tendons—Protection, 18.17
  -Shear—Alternate design method, A.7
  -Shear—Minimum, 11.5
  -Shear—Requirements, 11.5

  -Shells, 19.4
  -Shrinkage, 7.12
  -Slab, 13.3
  -Spacing limits, 7.6
  -Special details for columns, 7.8
  -Splices, 12.14
  -Steel pipe, 3.5
  -Structural integrity, 7.13, 13.3.8.5, 13.3.8.6, 16.5, 18.12.6
  -Structural steel, 3.5
  -Surface conditions, 7.4
  -Temperature, 7.12
  -Transverse, 8.10
  -Tubing, 3.5
  -Two-way slabs, 13.3
  -Web—Development, 12.13
  -Welded deformed wire fabric, 12.7
  -Welded deformed wire fabric—Development, 12.7
Required strength—See Strength, required
Reshores—Definition, 2.1
  -Formwork—Removal, 6.2
Retarding admixtures, 3.6
Retempered concrete, 5.10

Safety—Strength evaluation, 20.7
Sampling, 5.6
Scope of code, 1.1
Seismic design
  -Definitions, 21.1
  -Flexural members of frames, 21.3
  -Frame members, 21.4, 21.7, 21.8
  -General requirements, 21.2
  -Joints of frames, 21.5
  -Shear strength requirements, 21.4, 21.5, 26
  -Structural walls, diaphragms, and trusses, 21.6
Seismic hook—Definition, 21.1
Service loads—See Load, service
Service load stresses—Permissible—Alternate design method, A.3
Settlement—Required strength, 9.2
Shear
  -Alternate design method, A.7
  -Brackets, 11.9
  -Corbels, 11.9
  -Deep flexural members, 11.8
  -Footings, 11.12, 15.5
  -Horizontal—Ties—Composite flexural members, 17.6
  -Slabs, 11.12, 13.6
  -Walls, 11.10
Shear-friction, 11.7
  -Alternate design method, A-7
Shear strength, 11.1
  -Concrete—Nonprestressed members, 11.3
  -Concrete—Prestressed members, 11.4
  -Horizontal—Composite flexural members, 17.5
  -Lightweight concrete, 11.2
  -Vertical—Composite flexural members, 17.4
Shell concrete—Definition, 21.1
Shells
  -Construction, 19.5
  -Definitions, 19.1
  -Reinforcement, 19.4
  -Strength of materials, 19.3
Shored construction, 9.5
Shores—Definition, 2.1
Shoring—Formwork—Removal, 6.2
Shrinkage—Required strength, 9.2
Shrinkage reinforcement, 7.12
Slabs
  -Moment transfer to columns, 11.11
  -One-way—Deflections—Minimum thickness, 9.5
  -One-way—Distribution of flexural reinforcement, 10.6
  -Shear provisions, 11.12
  -Two-way—Definitions, 13.2
  -Two-way—Design, 13.1
  -Two-way—Design procedures, 13.5
  -Two-way—Direct design method, 13.6
  -Two-way—Equivalent frame method, 13.7
  -Two-way—Openings, 13.4
  -Two-way—Reinforcement, 13.3
Slab support—Axially loaded members, 10.14
Slab systems—Prestressed concrete, 18.12
Slenderness effects
  -Compression members, 10.10
  -Evaluation, 10.11, 10.12, 10.13
Spacing—Reinforcement—Limits, 7.6
Span length, 8.7
  -Definition, 8.7
Special structures, 1.1
Special systems of design or construction, 1.4
Specified compressive strength of concrete (f_c')—Definition, 2.1
Specified lateral forces—Definition, 21.1
Spiral reinforcement
  -Definition, 2.1
  -Structural steel core, 10.16
Spirals, 7.10
Splices, 12.14
  -Alternate design method, A.4
  -Columns, 12.17
  -End bearing, 12.16
  -Lap, 12.14, 12.15, 12.16
  -Smooth wire fabric, 12.19
  -Welded deformed wire fabric, 12.18
Splitting tensile strength (f_{ct})—Definition, 2.1
Standards cited in this code, 3.8
Steam curing, 5.11
Steel reinforcement, 3.5
Stiffness, 8.6
Stirrup
  -Definition, 2.1
  -Development, 12.13
  -Shear reinforcement requirements, 11.5
Storage—Materials, 3.7
Strength, design, 9.1, 9.3
  -Definition, 2.1
  -Reinforcement, 9.4
  -Structural plain concrete, 22.5
Strength evaluation, 16.10, 20.1
  -Acceptance criteria, 20.5
  -Analytical evaluation, 20.1
  -Load tests, 20.3
  -Load criteria, 20.4
  -Lower load rating, 20.6
  -Safety, 20.7
Strength, nominal—Definition, 2.1
Strength reduction, 5.5
Strength, required, 9.2
  -Definition, 2.1
Strain—Reinforcement, 10.2
Stress
  -Definition, 2.1
  -Permissible—Prestressed flexural members, 18.4
  -Permissible—Prestressed tendons, 18.5
  -Reinforcement, 10.2
  -Service load—Permissible—Alternate design method, A.3
Structural concrete—Definition, 2.1
Structural diaphragms—Definition, 2.1

Structural integrity
  -Requirements, 7.13, 13.3.8.5, 13.3.8.6, 16.5, and 18.12.6
Structural plain concrete
  -Design method, 22.4
  -Footings, 22.7
  -Joints, 22.3
  -Limitations, 22.2
  -Pedestals, 22.8
  -Precast members, 22.9
  -Strength design, 22.5
  -Walls, 22.6
Structural steel—Reinforcement, 3.5
Structural steel core—Concrete encased, 10.16
Structural trusses—Definition, 21.1
Structural walls—Definition, 21.1
Strut—Definition, 21.1
Sulfate exposures, 4.2
Sway frames—Magnified moments, 10.13

T-beams, 8.10
  -Flanges in tension—Tension reinforcement, 10.6
Temperature reinforcement, 7.12
Tendon—Prestressing, 3.5
  -Anchorage zones, 18.13
  -Definition, 2.1
  -Protection, 18.17
Tensile strength—Concrete, 10.2
Tension-controlled section—Definition, 2.1, B9.3
Testing for acceptance of concrete, 5.6
Tests, materials, 3.1
Thickness, minimum—Deflection—Nonprestressed beams or one-way slabs, 9.5
Thin shells—Definition, 19.1
Tie—Definition, 2.1
Tie elements—Definition, 21.1
Ties, 7.10
  -Definition, 2.1
  -Horizontal shear—Composite flexural members, 17.6
  -Steel core encased in concrete, 10.16
Tolerances—Placing reinforcement, 7.5
Torsion
  -Alternate design method, A.7
  -Design, 11.6
Torsional members—Slab design, 13.7
Torsional moment strength, 11.6
Torsion reinforcement requirements, 11.6
Transfer—Definition, 2.1
Tubing—Reinforcement, 3.5
Two-way construction—Deflections, 9.5

Unshored construction, 9.5

Wall
  -Definition, 2.1
  -Empirical design, 14.5
  -Grade beams—Design, 14.7
  -Shear provisions, 11.10
  -Structural design, 14.1
  -Structural plain concrete, 22.6
Water, 3.4
Water-cementitious materials ratio, 4.1, 5.4
Water-reducing admixtures, 3.6
Web reinforcement—Development, 12.13
Welded splices—Tension—Reinforcement, 12.14, 12.16, 12.17
Welded wire fabric, 3.5
  -Bends, 7.2
  -Deformed—Development, 12.7
  -Deformed—Splices, 12.18
  -Placing, 7.5
  -Plain—Development, 12.8
  -Plain—Splices, 12.19
Welding—Reinforcement—Placing, 7.5
Wind loads, 8.2
Wobble friction—Definition, 2.1, 18.6

Yield strength—Definition, 2.1