Indian Roads Congress
Special Publication
No. 23

VERTICAL CURVES FOR HIGHWAYS

NEW DELHI 1993

Indian Roads Congress
Special Publication 23

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Published by
The Indian Roads Congress

Copies can be had
from the Secretary,
Indian Roads Congress,
Jamnagar House,
Shahjahan Road,
New Delhi - 110 011

New Delhi 1993

Price Rs. 140/-
(Plus packing & postage)

First Published	:	December, 1983
Reprinted	:	September, 1989
Reprinted	:	September, 1993
Reprinted	:	March, 1999
Reprinted	:	July, 2001
Reprinted	:	August, 2005
Reprinted	:	March, 2007
Reprinted	:	March, 2010

Printed at : I. G. Printers Pvt. Ltd., New Delhi
(500 copies)

PREFACE

Vertical curves constitute an important component in the design of the longitudinal profile of a road. For providing guidance to the designers in this regard, the Specifications and Standards Committee of the Indian Roads Congress had published a detailed Paper on the subject in the year 1952 (Vertical Curves for Highways, Paper No. 156, Journal of the Indian Roads Congress, Vol. XVI-1). For many years, this Paper served as a useful guide in the design of highways in the country.

In the meantime, a lot of changes have taken place in the geometric design standards as also in the design concepts. The revised geometric design standards have been published separately by the Indian Roads Congress, vide IRC: 73-1980 for Rural (Non-Urban) Highways and IRC: 86-1983 for Urban Roads. For providing guidance on the design of vertical curves in the light of the current geometric design standards and concepts, the original Paper No. 156 was modified and rewritten by Shri K. Arunachalam, Deputy Secretary (Research), Indian Roads Congress.

It is hoped that this Publication will be useful in the design of proper vertical curves for roads in the country besides being a guide to engineering students.

New Delhi
December, 1983

NINAN KOSHI
Secretary
Indian Roads Congress

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VERTICAL CURVES FOR HIGHWAYS

1. INTRODUCTION

1.1. In the alignment of a highway, it is a generally accepted practice to follow, as closely as possible, the natural lie of the land. This practice, while satisfying the aesthetic principles of road location, lends itself to economical road construction. As the natural ground is rarely level, the road located therein according to these principles will also have a series of grades, often changing to suit the ground level. For the economical and safe operation of vehicular traffic, however, certain other important considerations set definite limits to the grades and also define the way the changes in grades are to be effected by the introduction of vertical curves in the longitudinal profile of the road.

1.2. Not so long ago, the average speed of motor vehicles on the main roads in this country was about 35 km/h. In recent years there has been a rapid advance in design. Motor vehicles, with low centres of gravity and equipped with powerful brakes, are now built to travel at high speeds. To provide full advantage to these improved vehicles and thereby to increase the speed of road transport, it is necessary to improve the design of the road itself by applying proper geometric standards and by other means.

1.3. To attain the primary objectives of safety and comfort in travelling over different grades, the design of vertical curves has to be given due attention. This publication gives a rational conception of the principles governing the design of vertical curves on roads.

1.4. The design of vertical curves on highways is not entirely a matter of mathematical analysis. Factors such as the "personal equation" of the driver of the vehicle have to be taken into account and subjected to extensive research. On many such factors, research work and field observations carried out in other countries, particularly the United States of America, the United Kingdom, and Australia have supplied valuable material. Not all the data used in this publication have been supported by adequate experimental observations. As further investigations are made some of the conclusions may need revision in the course of time.

2. GRADIENTS

2.1. The rate of rise or fall with respect to the horizontal along the length of a road, expressed as a ratio or a percentage, is termed the "**Gradient**". It is customary to express a gradient in terms of the natural tangent of the angle of its inclination to the horizontal. This may also be stated as a ratio, e.g., 1 in 20, 1 in 25, etc. In the U.S.A. and some other countries the grade is more often expressed as a percentage as 5 per cent or 4 per cent, etc.

2.2. When an angle is small, its tangent is approximately equal to its circular measure. For this purpose all angles within the practical range of gradients on roads may be treated as small.

2.3. In this publication n_1 and n_2 are used to denote the natural tangents as well as circular measures of the angles of inclination to the horizontal of the two intersecting grade lines. Thus in Fig. 1, n_1 is the tangent (or circular measure) of the angle BAE and n_2 that of the angle BCF. Signs + and − are used to denote ascending and descending gradients respectively, in the line of travel, which, by convention, is generally left to right in figures.

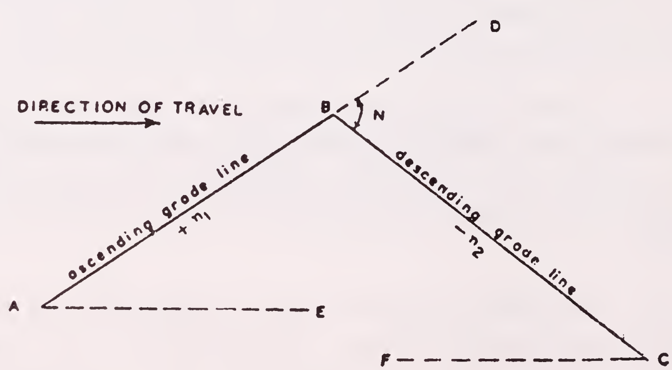


Fig. 1.

2.4. The angle which measures the change of direction in the path of motion at the intersection of two grade lines is called the deviation angle. Thus in Fig. 1, angle DBC is the total deviation angle. In this publication 'N' will denote the natural tangent or the circular measure of the deviation angle.

The deviation angle 'N' is given by the algebraic difference between the two grade angles.

Thus $N = n_1 - n_2$

Example :

$$\text{Let } n_1 = +\frac{1}{20} \text{ or } +\frac{5}{100} \text{ or } +5 \text{ per cent}$$

$$n_2 = -\frac{1}{25} \text{ or } -\frac{4}{100} \text{ or } -4 \text{ per cent}$$

$$\text{Then } N = \frac{1}{20} + \frac{1}{25}$$

$$= +0.09$$

2.5. Gradients must be fixed before a vertical curve can be designed. The designer has always to keep an eye on economy in selecting the alignment and suggesting the longitudinal profile of a road. The choice of the alignment of a projected road is influenced by many considerations, gradients being one of the most important. The necessity of securing easy grades sometimes compels a long and expensive alignment. On many an existing road, grades can be improved only by abandoning the present alignment and re-locating it. Thus, for road projects it is necessary for the designer to know what gradients are to be aimed at. With this knowledge the designer is in a position to achieve a balance between the economy of design and its utility to the road user.

On motor roads in hilly country the gradients should be such that they can be negotiated with the least changing of gears by the heavier vehicles (there is not much animal-drawn traffic on such roads). This saves time and operation costs. The problem is somewhat complicated in the plains where roads are used by the slow moving bullock cart on the one hand, and the fast modern motor vehicle on the other. For many years to come the bullock cart will remain a dominating element in the agricultural economy of this country. Gradients adopted on roads in the plains should, therefore not be such as to have an adverse effect on bullock cart traffic. There are many varieties of bullock carts, differing in design and capacity, and in the strength, and number of bullocks used to pull. Taking all these factors into account, the Indian Roads Congress has laid down standards for gradients to be adopted in different terrains, vide Table 1. Terrain is classified by the general slope of the country across the highway alignment, for which the following criteria should be followed :

Terrain classification		*Per cent cross slope of the country*
Plain	...	0—10
Rolling	...	>10—25
Mountainous	...	>25—60
Steep	...	>60

While classifying a terrain, short isolated stretches of varying terrain should not be taken into consideration.

TABLE 1. GRADIENTS FOR ROADS IN DIFFERENT TERRAINS

S. No.	Terrain	Ruling gradient	Limiting gradient	Exceptional gradient
1.	Plain or rolling	3.3 per cent (1 in 30)	5 per cent (1 in 20)	6.7 per cent (1 in 15)
2.	Mountainous terrain, and steep terrain having elevation more than 3,000 m above the mean sea level	5 per cent (1 in 20)	6 per cent (1 in 16.7)	7 per cent (1 in 14.3)
3.	Steep terrain upto 3,000 m height above mean sea level	6 per cent (1 in 16.7)	7 per cent (1 in 14.3)	8 per cent (1 in 12.5)

Gradients upto the 'ruling gradient' may be used as a matter of course in design. However in special situations such as isolated ever-bridges in flat country or roads carrying a large volume of slow moving traffic, it will be desirable to adopt a flatter gradient of 2 per cent from the angle of aesthetics, traffic operation, and safety.

The 'limiting gradients' may be used where the topography of a place compels this course or where the adoption of gentler gradients would add enormously to the cost. In such cases, the length of continuous grade steeper than the ruling gradient should be as short as possible.

'Exceptional gradients' are meant to be adopted only in very difficult situations and for short lengths not exceeding 100 m at a stretch. In mountainous and steep terrain, successive stretches of exceptional gradients must be separated by a minimum length of 100 m having gentler gradient (i.e. limiting gradient or flatter).

The rise in elevation over a length of 2 km shall not exceed 100 m in mountainous terrain and 120 m in steep terrain.

2.6. Compensation in Grade for Horizontal Curves

2.6.1. When a vehicle driven by the rear wheels travels on a curve there is some loss in the tractive force as is explained below.

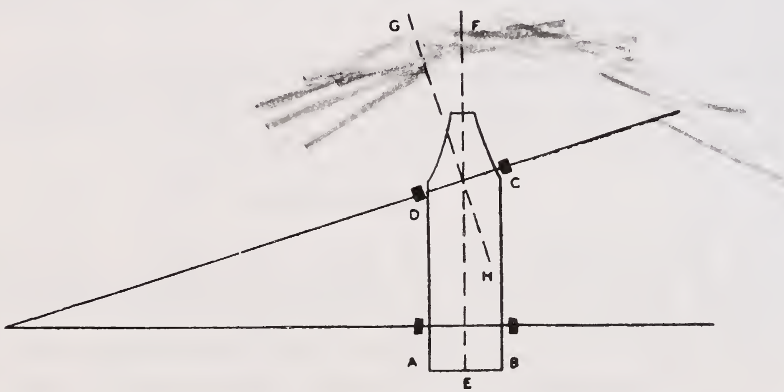


Fig. 2.

Let A and B be the rear driving wheels and C and D the front wheels of a motor vehicle in plan travelling on a curve (Fig. 2). It is seen that the tractive force acts in the direction EF while the front wheels should move in the direction GH so as to follow the curve. As GH is inclined to EF there will be a tendency for the vehicle to slide in the direction EF. This tendency is resisted by the friction between the wheels and the road surface, but in effect there will be a sliding movement when the curve is sharp. This action takes up some of the tractive force driving the vehicle forward. In the case of a vehicle with driving wheels in the front this action would be absent. To maintain the same speed on curve as on a straight, more of the tractive effort of the vehicle has therefore to be mobilised in case of vehicles driven by rear wheels. The effect of the curve on the tractive effort is, therefore, the same as that of a grade. If, in addition to the curve, there occurs also a sharp grade the total effect of the curve and grade should not exceed that of the limiting gradient specified.

2.6.2. The amount by which the gradients should be eased to offset the extra tractive effort involved at horizontal curves is known as 'grade compensation'. This should be calculated by the following formula :

$$\text{Grade compensation (per cent)} = \frac{30+R}{R}$$

Subject to a maximum of $75/R$ where R is the radius of the curve in metres.

Since grade compensation is not necessary for gradients flatter than 4 per cent, when applying grade compensation correction, the gradients need not be eased beyond 4 per cent.

3. DESIGN SPEEDS

3.1. As stated earlier in para 1, the purpose of designing proper vertical curves is to achieve a safe and sustained speed of travel on a road. The designer must therefore know what maximum speed is to be sustained on each class of the roads. The analytical treatment of vertical curves centres round the one dominating factor "speed".

3.2. The design speeds laid down by the Indian Roads Congress for the various classes of roads are given in Tables 2 and 3. While Table 2 pertains to rural (non-urban) highways, Table 3 is for urban roads in plains.

4. THE PURPOSE OF VERTICAL CURVES

4.1. It is a well known fact that considerable forces are involved when a change takes place in the direction of motion of a body. When a motor vehicle travelling along one grade is to

TABLE 2. DESIGN SPEEDS FOR RURAL (NON-URBAN) HIGHWAYS

S. No.	Road classification	Design speed, km/h							
		Plain terrain		Rolling terrain		Mountainous terrain		Steep terrian	
		Ruling design speed	Minimum design speed	Ruling design speed	Minimum design speed	Ruling design speed	Minimum design speed	Ruling design speed	Minimum design speed
1.	National and State Highways	100	80	80	65	50	40	40	30
2.	Major District Roads	80	65	65	50	40	30	30	20
3.	Other District Roads	65	50	50	40	30	25	25	20
4.	Village Roads	50	40	40	35	25	20	25	20

TABLE 3. DESIGN SPEEDS FOR URBAN ROADS IN PLAINS

S. No.	Road classification	(Design speed (km/h)
1.	Arterials	80
2.	Sub-arterials	60
3.	Collector streets	50
4.	Local streets	30

move on to another grade, a change of direction of motion in the vertical plane is involved. If this change is not effected gradually the vehicle will be subjected to shock and the occupants of the vehicle will experience discomfort. Therefore vertica curves are required to ease off the changes in gradients.

4.2. Vertical curves can be classed into two types viz., (1) Summit curves to ease off intersections convex upwards, and (2) Valley curves to ease off intersections concave upwards.

4.3. When a vehicle approaches a summit curve, the view of the road is cut off beyond the summit. Therefore to secure the required sight distance, the intersection of the two grades should be eased off by interposing a properly designed vertical curve. For valley curves, visibility is not a problem during day time. However, for night travel, the design must ensure that the roadway ahead is illuminated by vehicle headlights to a sufficient length enabling the vehicle to brake to a stop if necessary.

4.4. **Sight Distance Considerations**

4.4.1. Three types of sight distance (see IRC : 66-1976 for more details) are relevant for the design of summit curves. These are Stopping Sight Distance, Overtaking Sight Distance, and Intermediate Sight Distance. Sight distance values for different design

TABLE 4. SIGHT DISTANCE FOR VARIOUS SPEEDS

Speed km/h	Sight distance (metre)		
	Stopping	Intermediate	Overtaking
20	20	40	
25	25	50	
30	30	60	
35	40	80	
40	45	90	165
50	60	120	235
60	80	160	300
65	90	180	340
80	120	240	470
100	180	360	640

TABLE 5. CRITERIA FOR MEASURING SIGHT DISTANCE

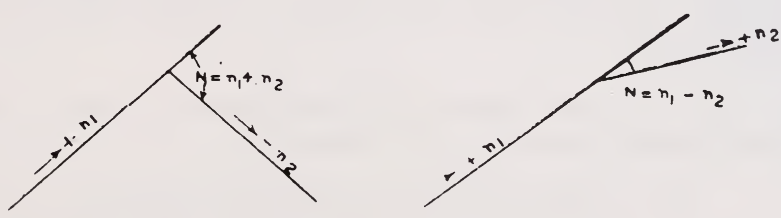
S. No.	Sight distance	Driver's eye height	Height of object
1.	Safe stopping sight distance	1.2 m	0.15 m
2.	Intermediate sight distance	1.2 m	1.2 m
3.	Overtaking sight distance	1.2 m	1.2 m

speeds are given in Table 4. The criteria for measurement of the sight distance are indicated in Table 5.

4.4.2. For valley curves, the design is governed by night visibility which is reckoned in terms of Headlight Sight Distance. This distance should at least equal the safe stopping sight distance given in Table 4.

5. SUMMIT CURVES

5.1. A curve with convexity upwards is called a Summit Curve. Figure 3 illustrates cases where Summit Curves have to be used.



(a) ascending grade meeting a descending grade

(b) ascending grade meeting another ascending grade to interfere with visibility. The dynamics of movement over an ordinary summit curve is of little consequence. This can be inferred from two considerations : (1) The centrifugal force generated by the movement of the vehicle along the curve acts practically in opposition to the force of gravity and is, therefore, beneficial in so far as it relieves the pressure on the tyres and springs of the vehicle; (2) Vertical deviation angles on roads are so small because the summit curves prescribed by the sight distance are so long and easy that "shock" is automatically rendered imperceptible to the travellers.

5.3. It, therefore, follows that on summit curves transitions are not essential and simple circular arcs are good enough. Since a circular arc has a constant radius of curvature throughout its length, it gives a constant sight distance all along. From this viewpoint the alternative of a curve fully transitional and symmetrical about the intersection is unsuitable, as the radius of the curve decreases towards its apex and the visibility on a vertical transition curve varies from point to point and is smallest across its apex. At a given intersection of gradients a transition curve will have to be much longer than a circular arc for equal visibility across the apex. Because of this disadvantage a transition curve is not recommended.

5.4. In actual practice a simple parabolic curve is used instead of the circular arc. The reasons are :

(i) A simple parabola is nearly congruent with a circular arc between the same tangent points, because on road work the vertical deviation angles are very small and lengths of curves are very great.

(ii) A parabola is very easy of arithmetical manipulation for computing ordinates.

5.5. Summit Curve Formulae

In Fig. 4, let AD and DC be the two grade lines intersecting at D and inclined at $+n_1$ and $-n_2$ to the horizontal. Let ABC be a parabolic curve between the tangent points A and C. With A as origin, measure x horizontally and take y as the vertical intercept between the curve and the grade line ADE. Let the equation of the curve be

$$y = \frac{x^2}{a} \qquad [1]$$

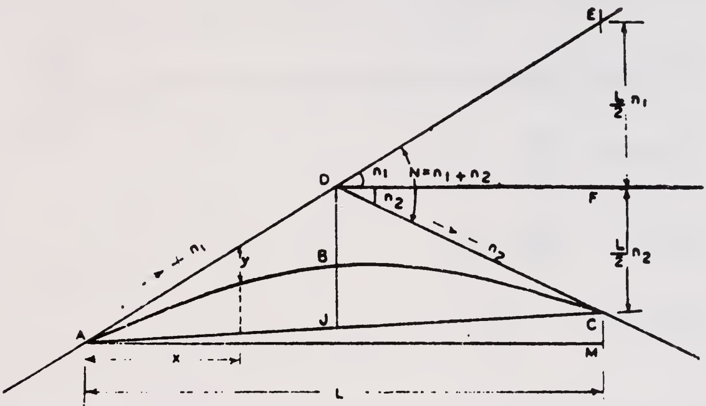


Fig. 4.

From the geometry of the figure it is clear that

$$EC = EF + FC$$

$$= \frac{Ln_1}{2} + \frac{Ln_2}{2}$$

$$= \frac{L}{2}(n_1 + n_2) = \frac{LN}{2}$$

Since summit curves are long and flat, the length of the curve ABC and its horizontal projection AM are for all practical purposes equal, each $= L$.

At the point C on the curve, $x = L$ and $y = EC = \dfrac{LN}{2}$

Putting these values in the equation of the curve.

$$\frac{LN}{2} = \frac{L^2}{a}$$

Therefore $a = \dfrac{2L}{N}$ .... [2]

5.6. Radius of Curvature of Summit Curve

Let (x, y) be the cartesian coordinates of any point on any curve. Let R be the radius of curvature at that point. If the curve is flat, $\frac{1}{R} = \frac{d^2y}{dx^2}$ .... [3]

The equation to the summit parabolic curve $y = \frac{x^2}{a}$

Therefore $\frac{dy}{dx} = \frac{2x}{a}$

and $\frac{d^2y}{dx^2} = \frac{2}{a}$

that is $R = \frac{a}{2}$ .... [4]

But in equation (2), we have shown that

$$a = \frac{2L}{N}$$

Hence $R = \frac{L}{N}$ .... [5]

5.7. Formulae for Length of Summit Curves

5.7.1. The length of a summit curve depends on (i) the deviation angle, (N), and (ii) the required sight distance (S), which may be either the overtaking sight distance or the intermediate sight distance. or the minimum sight distance which is equal to the safe stopping distance.

5.7.2. The gradients on both sides of the intersection are selected on the principles already discussed in para. 2.5. The deviation angle and the chainage of the point of intersection of the gradients can then be measured and recorded.

5.7.3. In calculating the length of the curve two cases have to be considered:

For Overtaking Sight Distances*

Case I: **When the length of the curve exceeds the required sight distance, that is, L is greater than S.** In Fig. 5, ABC is a parabolic curve; A and C are tangent points, $E_1E_2=S$, the required sight distance; H, the height of the driver's eye above the road level; N, the deviation angle ODC; and L, the horizontal projection (AM) of the curve ABC.

From the geometry of the Figure it is obvious that

$$OC = OF + FC$$

$$= \frac{L}{2}(n_1 + n_2)$$

$$= \frac{L}{2}N.$$

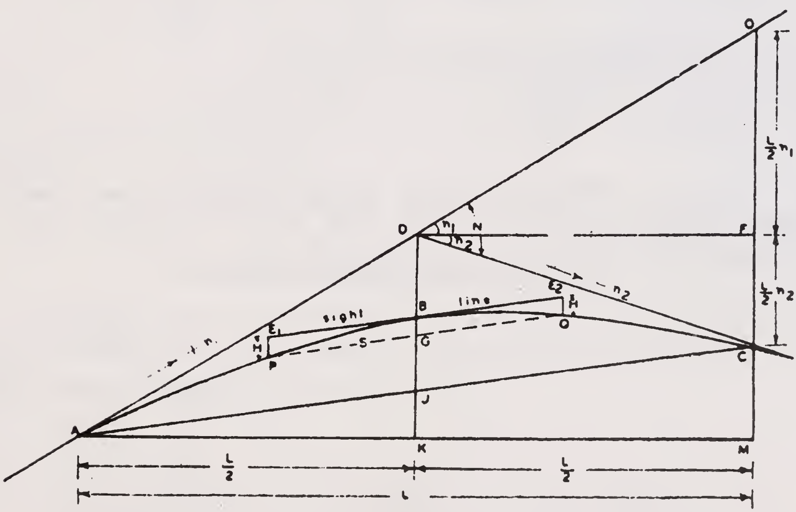


Fig. 5.

Since grade angles are always small, the lengths of the curve ABC and the lines AC and AM can each be taken as very nearly equal to L.

*This also applies to Intermediate Sight Distance as the criteria for measurement are the same.

From the properties of the parabola—

$$\frac{BG}{BJ} = \frac{(PQ)^2}{(AC)^2}$$

Putting $BG = H$

$$BJ = \tfrac{1}{2}DJ$$

$$= \tfrac{1}{4} \times \frac{LN}{2} = \frac{LN}{8}$$

$$PQ = S$$

and $AC = L,$

We get $\dfrac{8H}{LN} = \dfrac{(S)^2}{(L)^2}$

Or $$L = \frac{N.S^2}{8H} \qquad \text{[6]}$$

As indicated in Table 5, $H = 1.2$ m

Hence $$L = \frac{N.S^2}{9.6} \qquad \text{[7]}$$

Case II : **When the length of the curve is less than the required sight distance, that is L is less than S.** In Fig. 6, ABC is the parabolic curve and E_1E_2 is the sight distance S.

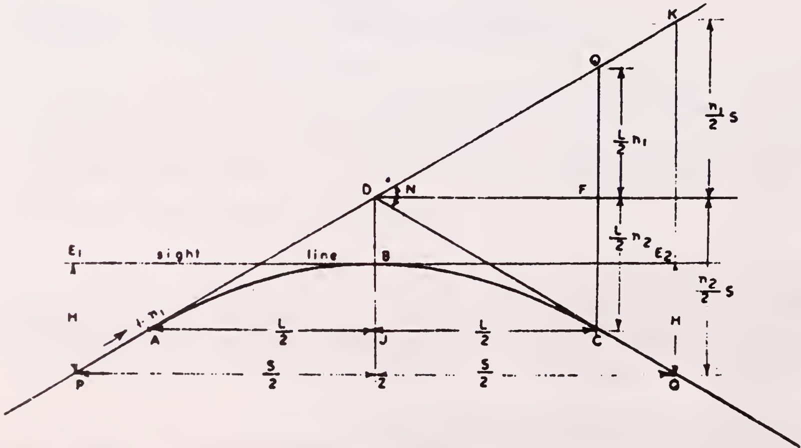


Fig. 6.

Now $DB = \frac{1}{4} DJ$

$$= \frac{1}{4} \times \frac{L}{2}(n_1 + n_2)$$

$$= \frac{1}{8} LN$$

Also from the geometry of the figure,

$$BZ = H, \text{ and } DZ = DB + BZ$$

Therefore $DZ = \frac{LN}{8} + H$

but $DZ = \frac{1}{2} KQ$

$$= \frac{1}{2} S \frac{(n_1 + n_2)}{2}$$

$$= \frac{S.N}{4}$$

Equating the two values of DZ,

$$\frac{LN}{8} + H = \frac{SN}{4}$$

Hence $L = 2S - \frac{8H}{N}$ .... [8]

Putting $H = 1.2$ m

$L = 2S - \frac{9.6}{N}$ .... [9]

For Safe Stopping Sight Distance

In this case, the situation corresponds to that shown in Fig. 7. The driver of the vehicle sights the top of an object 0.15 m high lying beyond the apex of the curve.

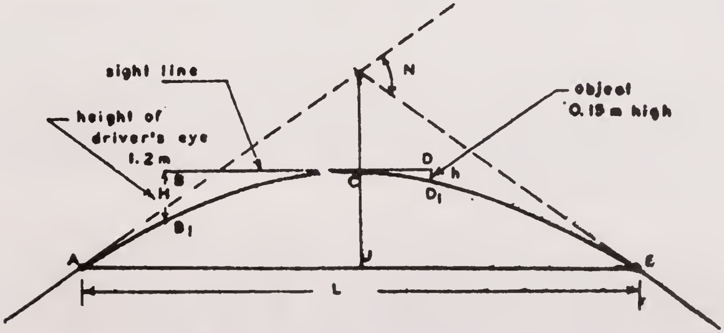


Fig. 7.

Case I : **When the length of the curve exceeds the required sight distance, that is, L is greater than S.** From geometry of the parabola,

$$\frac{(BC)^2}{(CD)^2} = \frac{H}{h} = \frac{1.2}{0.15} = 8$$

$$\therefore\ CD = \frac{BC}{\sqrt{8}} = 0.354\ BC$$

$$S = BC + CD = 1.354\ BC \text{ or}$$

$$BC = 0.738\ S$$

$$\frac{BB_1}{CJ} = \frac{BC^2}{AJ^2}$$

But $$CJ = \frac{LN}{8} \text{ and } BB_1 = H$$

$$\therefore\ \frac{8H}{LN} = \frac{(0.738\ S)^2}{\left(\frac{L}{2}\right)^2} = \frac{2.18\ S^2}{L^2}$$

$$L = \frac{2.18\ S^2}{8H} = \frac{2.18\ NS^2}{8 \times 1.2} = \frac{NS^2}{4.4} \quad \text{..............} \quad [10]$$

Case II : **When the length of the curve is less than the required sight distance, that is, L is less than S.** The formula works out to—

$$L = 2S - \frac{4.4}{N} \quad \text{..} \quad [11]$$

5.7.4. The length of summit curve for various cases mentioned above can be read from Plates 1, 2 and 3. In these Plates, value of the ordinate 'M' to the curve from the intersection point of the grade lines is also shown.

5.7.5. For quick comparison of the length of summit curve for the three types of sight distance for the case when L is greater than S, the lengths are shown in terms of the grade difference in Table 6.

TABLE 6. LENGTH OF VERTICAL CURVES FOR DIFFERENT SPEEDS WHEN LENGTH OF CURVE IS GREATER THAN SIGHT DISTANCE

Design speed (km/h)	Length of summit curve (metre) for			Length of valley curve (metre) for headlight distance
	Stopping sight distance	Intermediate sight distance	Overtaking sight distance	
20	0.9A	1.7A		1.8A
25	1.4A	2.6A		2.6A
30	2.0A	3.8A		3.5A
35	3.6A	6.7A		5.5A
40	4.6A	8.4A	28.4A	6.6A
50	8.2A	15.0A	57.5A	10.0A
60	14.5A	26.7A	93.7A	15.0A
65	18.4A	33.8A	120.4A	17.4A
80	32.6A	60.0A	230.1A	25.3A
100	73.6A	135.0A	426.7A	41.5A

Notes : 1. 'A' in the above Table is the algebraic difference in grades expressed as percentage.

2. The length of curves should be subject to minimum values given in Table 7.

5.8. Minimum Length of Vertical Curve

From equations (6) to (9), it is seen that the length L of a vertical curve decreases as N and/or S decreases. Therefore, in some cases the length of the curve needed for providing the required sight distance would be very small. Further in flat grades no vertical curve may be necessary for visibility; but for comfort in driving and to avoid shock, it is necessary to introduce a vertical curve except perhaps in very flat grades. The minimum length of the curve should be as indicated in Table 7. This Table also shows the maximum grade change not requiring a vertical curve.

5.9. Calculating Ordinates of Summit Curves

For the purposes of plotting and laying out a curve, its length is divided into a number of equal chords and the ordinates to the curve calculated at the ends of these chords.

Table 7. Minimum Length of Vertical Curves

Design speed (km/h)	Maximum grade change (per cent) not requiring a vertical curve	Minimum length of vertical curve (metre)
Upto 35	1.5	15
40	1.2	20
50	1.0	30
65	0.8	40
80	0.6	50
100	0.5	60

Ordinates y_1, y_2, y_3,..... y_r at stations 1, 2, 3........ r (Fig. 8) are calculated as under :

Since $y = \frac{x^2}{a}$

$$y_1 = \frac{u^2}{a}, \text{ (where } u \text{ is the chosen length of the chord)}$$

$$y_2 = \frac{(2u)^2}{a} \text{ or } y_1 \times 2^2$$

$$y_3 = y_1 \times 3^2$$

$$y_r = y_1 \times r^2$$

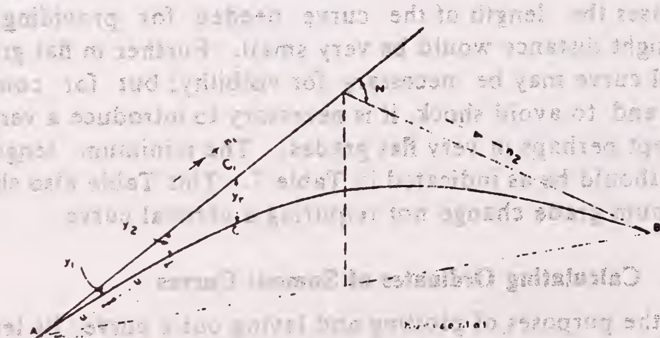


Fig. 8.

Let C be the point on the road surface curve at the end of the r^{th} sub-chord. . Let C_1 be the point on the grade line vertically above C. Let the reduced level of the tangent point A be 100.00.

$$\text{Then R. L. of } C_1 = 100.00 + r\,(u \times n_1)$$
$$\text{R. L. of } C = \text{R. L. of } C_1 - y_r$$

Similarly, the *R. Ls.* of other points on the curve should be worked out.

5.10. Highest Point on Summit Curve

It is sometimes important to know the position of the highest point on a vertical curve for the purpose of layout of drainage appurtenances and for ascertaining vertical clearances in restricted locations as road under bridges, etc.

When the two grades are equal the curve would be symmetrical about the vertical bisector of the intersecting angle and the highest point would also lie on this bisector. When the two grades are unequal the curve would be tilted and the highest point of the curve would lie on the side of the flatter gradient.

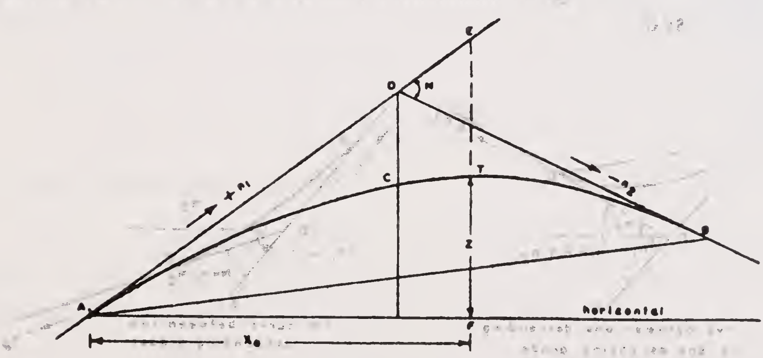


Fig. 9.

In Fig. 9 let T be the highest point distant x_o from the origin A.

The equation of the curve is $y = \dfrac{x^2}{a}$

Therefore $ET = \dfrac{x_0^2}{a}$

Also $EF = n_1 x_0$

Hence $\quad Z = FT = EF - ET = n_1 x_0 - \dfrac{x_0^2}{a}$

T will be the highest point when Z is maximum.

i.e., when $\dfrac{dz}{dx} = 0$

That is, when $n_1 - \dfrac{2x_0}{a} = 0$.

Or $x_0 = \dfrac{a}{2} n_1$

But $a = \dfrac{2L}{N} = \dfrac{2L}{n_1 + n_2}$

Hence $x_0 = \dfrac{n_1}{n_1 + n_2} L$.

6. VALLEY CURVES

6.1. A vertical curve concave upwards is known as a valley curve, dip or sag. Fig. 10 illustrates two cases where valley curves have to be used.

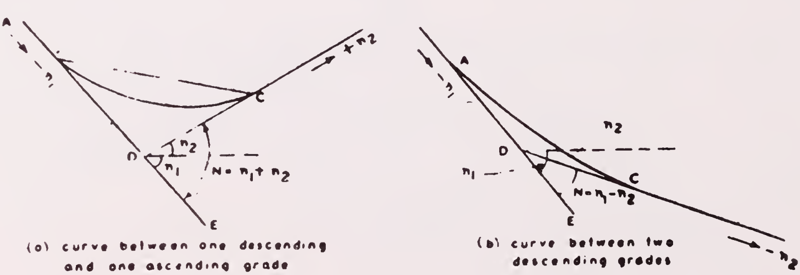


Fig. 10. Valley curve

Fig. 10 (a) is the case of a descending gradient intersecting an ascending gradient, whereas in Fig. 10 (b) a descending gradient intersects another descending gradient. In between these two cases would be the case of a descending gradient meeting a horizontal.

6.2. Deviation angle on Valley Curves

According to the general rule already stated, the deviation angle is the algebraic difference of the two grade angles. Thus, in Fig. 10 (a)

$N = \text{angle } CDE = (-n_1) - (+n_2) = -(n_1 + n_2)$ and in Fig 10 (b)

$N = \text{angle } CDE = (-n_1) - (n_2) = (n_1 - n_2)$

6.3. Length of Valley Curves

6.3.1. Valley curves should have the shape of square parabola similar to summit curves. A number of criteria are available for establishing the lengths of valley curves. Most commonly used among these are (i) headlight sight distance which is recommended in this publication and (ii) rider comfort.

6.3.2. When a vehicle traverses a valley curve at night, the portion of road lighted ahead depends on the height of the headlights above the road surface and the direction of the light beam. The valley curve should be long enough so that the distance ahead lighted by the headlights is at least equal to the safe stopping sight distance. For determining the length of valley curves based on the above considerations the following criteria apply :

(i) Height of headlight above road surface is 0.75 m

(ii) The useful beam of headlight is upto one degree upwards from the grade of the road; and

(iii) The height of object is nil.

6.3.3. The design criteria for determining the length of valley curves are depicted in Fig. 11. From the geometry, equations for calculating the length are as follows :

Case (i) When the length of the curve exceeds the required sight distance, i.e. L is greater than S

$$L = \frac{NS^2}{1.50 + 0.035\ S}$$

Case (ii) When the length of the curve is less than the required sight distance, i.e. L is less than S

$$L = 2S - \frac{1.50 + 0.035\ S}{N}$$

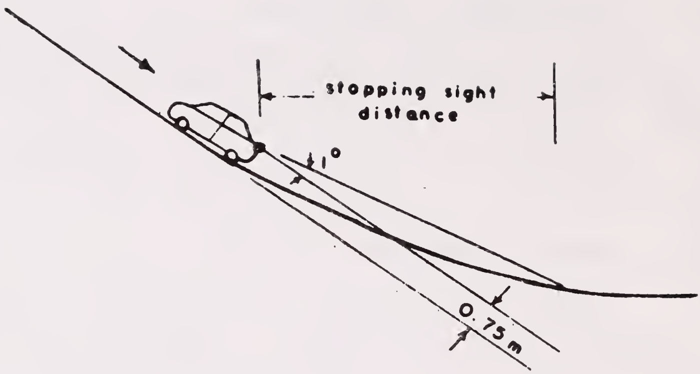


Fig. 11.

In both cases

N = deviation angle, i.e. the algebraic difference between the two grades

L = length of parabolic vertical curve in metres

S = stopping sight distance in metres

Length of valley curve for various grade differences is given in Table 6, and in graphical form in Plate 4. These are only minimum values, and longer lengths should be provided wherever feasible.

6.3.4. On valley curves, the gravitational and centrifugal forces act combinedly resulting in extra pressure on the tyres and springs of the vehicle The effect of this on travel comfort depends on several factors such as the vehicle body suspension, tyre flexibility, weight carried, etc. The broad conclusions from limited observations show that for riding comfort on valley curves, the radial acceleration should not exceed 0.3 metre per second per second. The length of vertical curve required to satisfy this comfort factor is only about 75 per cent of that required to satisfy the headlight sight distance requirement. It is, therefore, recommended that the length of valley curves for design should be based on the considerations discussed in para 6.3.2. The values as derived should, however, be subject to the minimum lengths indicated in Table 7.

6.3.5. Drainage considerations become important, for valley curves between a descending grade and an ascending grade as in Fig. 10 (a). For drainage purposes, it is desirable that the curve has a minimum gradient of 0.5 per cent if the side drains are lined and 1.0 per cent if these are unlined.

6.4 Finding the Lowest point on a Valley Curve

When a valley curve is included between descending and ascending grades, it is necessary to know the lowest point on the curve for fixing the positions of culverts, drain outlets, etc. When the two grades are unequal, the lowest point occurs on the side of the flatter gradient.

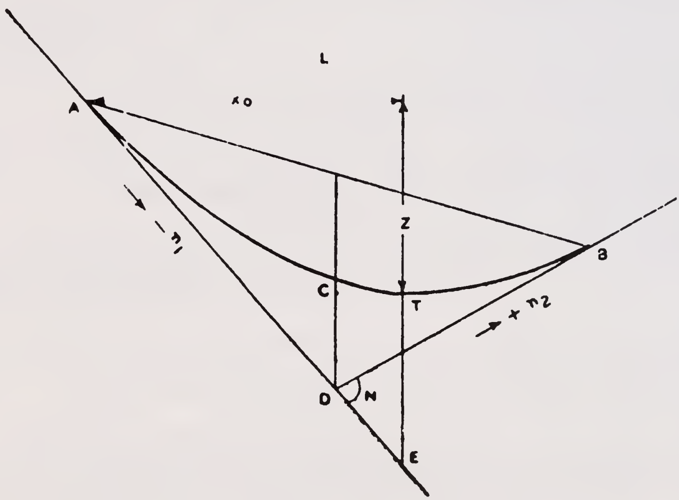


Fig. 12.

In Fig. 12, let the lowest point be distant x_o from A. From derivations similar to those for summit curves given in para 5.10, it can be shown that the lowest point is at a distance of $\frac{n_1}{n_1+n_2} L$ from point A.

6.5. Computing Ordinates of Valley Curves

Since valley curve is also in the shape of a square parabola, the ordinates can be calculated similar to summit curves described in para 5.9.

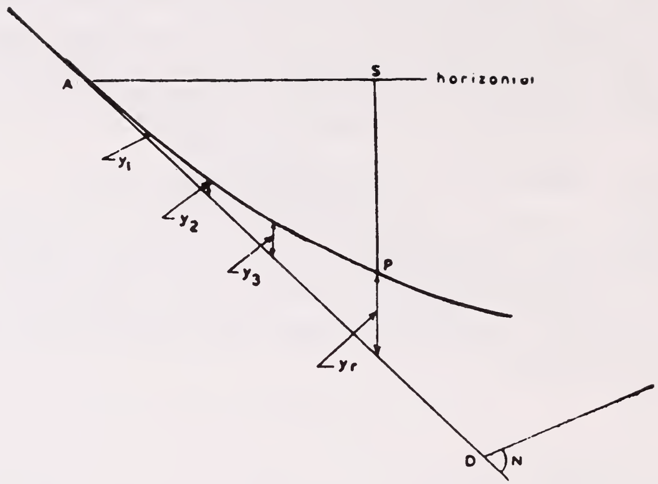


Fig. 13.

Referring to Fig. 13, the ordinates y_1, y_2, y_3.... ....y_r at stations 1, 2, 3.................r work out to :

$$y_1 = \frac{u^2}{a} \text{ (where } u \text{ is the chosen length of the chord)}$$

$$y_2 = y_1 \times 2^2$$

$$y_3 = y_1 \times 3^2$$

$$y_r = y_1 \times r^2$$

Let C be the point on the road surface at the end or the r^{th} sub-chord. Let C_1 be the point on the grade line vertically below C. Let the reduced level of the tangent point A be 100.

$$\text{Then R.L. of } C_1 = 100 - r\,(u \times n_1)$$

$$\text{R.L. of } C = \text{R.L. of } C_1 + y_r$$

Similarly, R.Ls of other points on the curve can be worked out.

7. PRACTICAL DESIGN OF VERTICAL CURVES ON HIGHWAYS

7.1. General

In the application to actual problems of the principles enunciated in the previous Sections, the following points deserve to be borne in mind.

The vertical curvature of roads should be bold in design and long easy curves should take in all minor changes in ground levels. As far as possible, numerous changes in gradients joined together with short vertical curves should be avoided, except in mountainous country where the adoption of long and easy curves might become very costly. The economic aspect of vehicle operation is very important in the choice of grades since the greater consumption of fuel and the heavier wear and tear of tyres and brakes of vehicles in traversing a wide range of vertical rises and falls would add heavily to operation costs.

In the design of the grade line of a road and its co-ordination with the horizontal alignment. the following points of guidance will be helpful :

(i) The vertical alignment should provide for a smooth longitudinal profile consistent with category of the the road and lie of the terrain. Grade changes should not be too frequent as to cause kinks and visual discontinuities in the profile. Desirably, there should be no change in grade within a distance of 150 m.

(ii) A short valley curve within an otherwise continuous profile is undesirable since this tends to distort the perspective view and can be hazardous.

(iii) Broken-back grade lines, i.e. two vertical curves in the same direction separated by a short tangent, should be avoided due to poor appearance, and preferably replaced by a single long curve.

(iv) Decks of small cross-drainage structures, (i.e. culverts and minor bridges) should follow the same profile as the flanking road section, without any break in the grade line.

(v) For small bridges upto 30 m span and having horizontal deck, it would be preferable to combine the flanking sections into a single vertical curve.

(vi) The overall appearance of a highway can be enhanced considerably by judicious combination of the horizontal and vertical alignments. Plan and profile of the road should not be designed independently but in unison so as to produce an appropriate three-dimensional effect. Proper co-ordination in this respect will ensure safety, improve utility of the highway and contribute to overall aesthetics.

(vii) The degree of curvature should be in proper balance with the gradients. Straight alignment or flat horizontal curves at the expense of steep or long grades, or excessive curvature in a road with flat grades, do not constitute balanced designs and should be avoided.

(viii) Vertical curvature superimposed upon horizontal curvature gives a pleasing effect. As such the vertical and horizontal curves should coincide as far as possible and their length should be more or less equal. If this is difficult for any reason, the horizontal curve should be somewhat longer than the vertical curve.

(ix) Sharp horizontal curves should be avoided at or near the apex of pronounced summit/sag vertical curves from safety considerations.

7.2. Design of Summit Curves

The design of summit curves follows the procedure given below.

7.2.1. On a longitudinal section of the road drawn to scale are fixed the economical gradients, selected by taking into consideration the amount of earthwork and other incidental works involved. The value of two gradients meeting at a point being known, the deviation angle N is known (see para. 2.4).

7.2.2. The sight distance applicable to the section of the road is selected, taking into account the classification of the road, the topography of the country and whether the section lies in an overtaking zone or non-overtaking zone (see IRC 66-1976).

7.2.3. The value of N and S being thus known, the appropriate length of the summit curve, L, corresponding to these values is read off from Plate 1, 2 or 3, as applicable.

7.2.4. The value of L as read from the graph is then rounded off so that the modified value is divisible into a number of equal chords of a reasonable length not exceeding $R/200$, where R is the radius of the curve at the apex given by $R = L/N$.

7.2.5. By reading the value of M for the length designed from the graph, the depth of cutting required for constructing the curve is obtained. This depth may be checked to see if the cutting would be excessive.

7.2.6. The constant "a" is calculated from equation [2] and the first ordinate y_1, obtained. The other ordinates and reduced levels of the various station points on the curve are then calculated and tabulated for facility in setting out in the field.

7.3. Design of Valley Curves

7.3.1. The gradient lines are marked on the longitudinal section of the road and the deviation angle, N, calculated as explained in para 7.2.1.

7.3.2. The design speed, V, appropriate to the class of road and the topography of the country, is noted down.

7.3.3. By using the graphs in Plate 4 is obtained the length of the curve for the corresponding values of N and V. This length, L, is rounded off so as to be divisible into a number of equal chords of a convenient length not exceeding $R/200$.

7.3.4. From equation [2] the constant "a" and the first ordinate y_1 (para. 6.5) are obtained and the other ordinates and reduced levels of the station points are calculated from these and tabulated for setting out in the field.

7.4. Shock-free Curves at Humps

7.4.1. It is desirable that the deck or top level of culverts should be fixed in line with the grade line of the flanking sections of the road so that no hump occurs. This may not, however, be possible on an existing road where culverts occur with deck levels higher than the general road levels, but the height of hump not sufficient enough to obstruct the sight line. For such cases, the approaches on either side should be provided with smooth vertical curves (summit and valley curves). The length of these curves should not be less than the minimum lengths indicated in Table 7.

7.5. Measurement of Vertical Sight Distance

7.5.1. As discussed in earlier paras, one of the important pourposes of providing a vertical curve is to ensure the necessary visibility or the sight distance along the grade line. Provision of sight distance must therefore receive attention right from early stages where the alignment is still flexible and subject to adjustments. Quick appraisals are best had by graphical means. By determining graphically the available sight distances from the longitudinal sections and recording them at convenient intervals, deficiencies in visibility become evident well before detailed design is already under way. Perusal of such records will enable the designer to decide on what modifications to make in profile for the required visibility, and to otherwise create a more balanced and effective design. For existing roads under improvements too, such a study will be highly useful in determining the visibility deficiencies and making the necessary improvements to the grade line.

7.5.2. As recommended in IRC : SP : 19 "Manual for Survey, Investigation and Preparation of Road Projects", the longitudinal sections are plotted to the following scales :.

(i) Built-up areas and stretches in hilly terrain—1 : 1000 for horizontal scale and 1 : 100 for vertical scale.

(ii) Plain and rolling terrain—1 : 2500 for horizontal scale and 1 : 250 for vertical scale.

If L is the length of the vertical curve required for gradient N, its radius R is equal to $\frac{L}{N}$. The radius 'r' for purposes of plotting is then $R \times \frac{V}{H^2}$ where V is the vertical scale of the drawing and H the horizontal. Having known the value of 'r', the vertical curve is easily drawn on the profile with the aid of spline or railway curves. For the recommended scales of plotting having vertical : horizontal scale ratio of 1:10, the error in measurement of sight distance will not be more than about 5 per cent.

7.5.3. Measurement of vertical sight distance at summit curves may be done from plotted profiles of the highway by the method illustrated in Fig. 14. A transparent straight edge with parallel edges 1.2 m apart and a dotted line 0.15 m from the upper edge, as per the vertical scale of the profile, is the tool

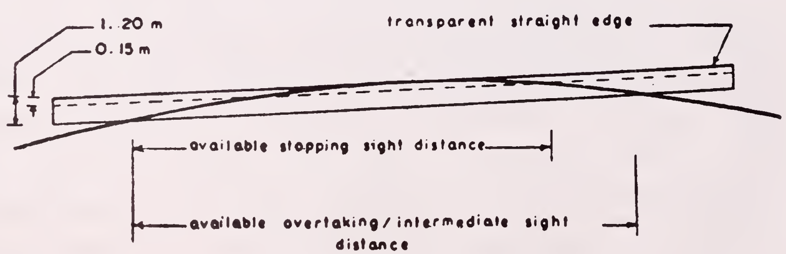


Fig. 14.

employed for these measurements. The transparent strip is placed on the profile with the lower edge at the station for which the available sight distance is desired and the strip revolved about this point until the upper edge touches the profile. Stopping sight distance available is then the distance between the first station and the point of intesection of the 0.15 m line with the profile. Overtaking/intermediate sight distance, in similar manner, is the distance between the initial station and the point where lower edge of the strip meets the profile. If overhead obstructions to visibility like underbridges, etc. have also been marked on the profile, then the graphical method explained above will unveil visibility deficiencies caused by these.

7.5.4. Availability of headlight sight distance along valley curves can also be checked in a similar way except that the template for checking will be differ as explained in Fig. 15. At the

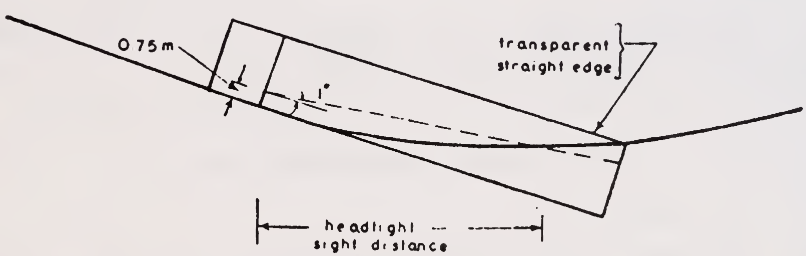


Fig. 15.

point where the sight distance is to be checked, the lower edge of the tamplate is kept tangential to the road surface, and the headlight distance is the distance between starting station and the point of intetersection between the road profile and light ray line (marked 1° upward from the headlight)

8. EXAMPLES

Problem No. 1

Design of a Vertical Summit Curve on a National Highway in Plain Terrain

Data (i) *Gradients :* $n_1 = +1/25$ or 4 per cent
and $n_2 = -1/30$ or 3.33 per cent

(ii) Class of Road—National Highway

(iii) Design Speed—100 km/h

(iv) The existing features near the locality permit the adoption of only the minimum sight distances. Case I will show the design for a curve providing non-overtaking sight distance and Case II that for over-taking sight distance.

Solution

(a) Deviation angle $N = +\frac{1}{25} - \left(-\frac{1}{30}\right)$

$$= 0.040 + 0.033$$

$$= 0.073$$

(b) Sight distance

Referring to Table 4, the minimum non-overtaking sight distance for a speed of 100 km/h is 180 m and the corresponding overtaking sight distance is 640 m for an undivided carriageway.

Case I : **Non-overtaking sight distance**

(c) Length of curve

From Plate I corresponding to $N = 0.073$ and $S = 180$ m, the length of the curve is 540 m. Divide the curve into 18 equal chords of 30 m each.

The radius of curvature $L/N = 7360$ m

(d) Calculation of the design chart :

$$a = \frac{2L}{N} \quad \ldots\ldots \text{ Equation [2] (Para. 5.5.)}$$

$$= \frac{2 \times 540}{0.073} = 14795$$

From equation $y_1 = \frac{x^2}{a}$ (Para. 5.9)

First ordinate $y_1 = \frac{30 \times 30}{14795} = 0.061$

The design is then worked out assuming the beginning of the vertical curve (B. V. C.) to be *RL* 100.00 as in Table *E*—1.

(e) Highest point on the curve

From equation in para 5.10

$$x_0 = \frac{n_1}{n_1+n_2} \times L$$

$$= \frac{0.04 \times 540}{0.04+0.033} = 295.89$$

The reduced level of this point is worked as under :

$$y \text{ max} = \left(\frac{295.89}{30}\right)^2 \times y_1 = 5.915$$

R.L. of the point along the 1/25 gradient corresponding to the highest point on the curve

$$= 100 + \frac{295.89}{100} \times 4.00$$

$$= 111.84$$

$$R.L. \text{ of the highest point} = 111.84 - 5.915$$

$$= 105.925$$

In the field it is essential to mark the highest point also. The solution is indicated diagrammatically in Fig. 16.

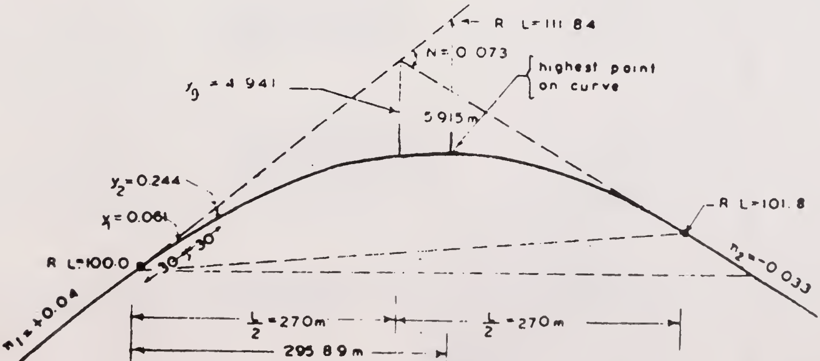


Fig. 16.

TABLE E-1. DESIGN TABLE FOR PROBLEM NO. 1, CASE 1
(Safe stopping sight distance)

-0.033

First half of curve (1 in 25)					Second half of curve (1 in 30)				
No. of station from B.V.C.	Chainage from B.V.C	R.L. of points on grade line	Ordinates between curve and grade line	R.L. of station on curve	No. of station from B.V.C.	Chainage from B.V.C.	R.L. of points on grade line	Ordinates between curve and grade line**	R.L. of station on curve
Read downwards									
0 (B.V.C.)	0	100.000	0.000	100.00	18 (E.V.C.)	540	101.800	0.000	101.800
1	30	101.200	$y_1=0.061$	101.139	17	510	102.800	0.061	102.739
2	60	102.400	$2^2y_1=0.244$	102.156	16	480	103.800	0.244	103.556
3	90	103.600	$3^2y_1=0.549$	103.051	15	450	104.800	0.549	104.241
4	120	104.800	$4^2y_1=0.976$	103.824	14	420	105.800	0.976	104.824
5	150	106.000	$5^2y_1=1.525$	104.475	13	390	106.800	1.525	105.275
6	180	107.200	$6^2y_1=2.196$	105 004	.12	360	107.800	2.196	105.604
7	210	108.400	$7^2y_1=2.989$	105.411	11	330	108.800	2.989	105.811
8	240	109.600	$8^2y_1=3.904$	105.796	10	300	109.800	3.904	105.896
9	270	110.800	$9^2y_1=4.941$	105.859	9	270	110.800	4.941	105.859
					Read upwards				

Notes : B.V.C. = Beginning of the vertical curve
E.V.C. = End of the vertical curve
** Same values as in opposite station of first half of the curve

Case II: **Overtaking sight distance**

(i) From Plate 3 corresponding to $N = 0.073$ and $V = 100$ km/h the intersection point giving the length of the curve for the given conditions is outside the charts. Therefore using equation [7],

$$L = \frac{NS^2}{9.6} = \frac{0.073 \times 640 \times 640}{9.6} = 3115\ m$$

Round off the length to 3200 *m*. The radius of curvature of the curve is 43,836 *m*. Divide the curve into 32 stations of 100 *m* each.

(ii) Calculation of the design chart : From equation [2] para 5.5.

$$a = \frac{2L}{N}$$

$$= \frac{3200 \times 2}{0.073} = 87671$$

$$y_1 = \frac{u^2}{a} = \frac{100 \times 100}{87671}$$

$$= 0.114$$

(iii) Highest point of the curve occurs at x_0. From equation (vide para 5.10.)

$$x_0 = \frac{n_1}{n_1 + n_2} \times L$$

$$= \frac{.04}{.073} \times 3200 = 1753.4$$

$$y \max = 17.53^2 \times 0.114$$

$$= 35.03\ m$$

R. L. along the 4 per cent grade $= 100.00 + 17.53 \times 4$

$= 170.12$

∴ R. L. of the highest point $= 170.12 - 35.03$

$= 135.09$

The solution is diagrammatically indicated in Fig. 17.

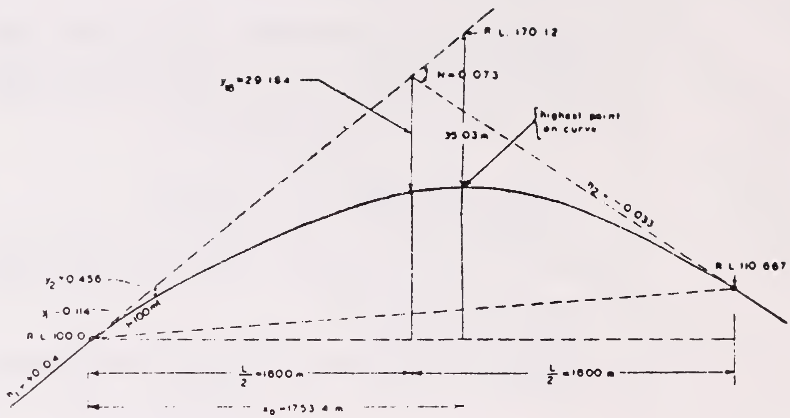


Fig. 17.

PROBLEM NO. 2

For the case in problem 1, site conditions do not permit the provision of a summit curve exceeding 300 m in length. For stopping sight distance considerations, find (i) the adjustments required in the grades for permitting the design speed of 100 km/h, and (ii) the safe speed if the grades are not changed.

(i) Adjusting the approach gradients

For the speed of 100 km/h, $S = 180\ m$

L as given $= 300\ m$

$$N = \frac{4.4L}{S^2} = \frac{4.4 \times 300}{180 \times 180} = 0.040$$

The gradients should be so adjusted that their algebraic difference is 0.04.

(ii) Limiting the safe speed

$$L = \frac{NS^2}{4.4}$$

$$S^2 = \frac{4.4 \times L}{N} = \frac{4.4 \times 300}{0.073}$$

$$S = 134.5\ m$$

The safe speed corresponding to stopping sight distance of 134.5 m = 85 km/h.

TABLE E-2. DESIGN TABLE FOR PROBLEM NO. 1, CASE 2 (OVERTAKING SIGHT DISTANCE)

First half of curve (1 in 25)					Second half of curve (1 in 30)				
No. of station from B.V.C.	Chainage from B.V.C.	R.L. of points on grade line	Ordinates between curve and grade line	R.L. of station on curve	No. of station from B.V.C.	Chainage from B.V.C.	R.L. of points on grade line	Ordinates between curve and grade line**	R.L. of station on curve
Read downwards									
0 (B.V.C.)	0	100.000	0.000	100.00	32	3200	110.667	0.000	110.667
1	100	104.000	$y_1 =$ 0.114	103.886	31	3100	114.000	0.114	113.886
2	200	108.000	$2^2y_1 =$ 0.456	107.544	30	3000	117.334	0.456	116.878
3	300	112.000	$3^2y_1 =$ 1.026	110.974	29	2900	120.667	1.026	119.641
4	400	116.000	$4^2y_1 =$ 1.824	114.176	28	2800	124.000	1.824	122.176
5	500	120.000	$5^2y_1 =$ 2.850	117.150	27	2700	127.334	2.850	124.484
6	600	124.000	$6^2y_1 =$ 4.104	119.896	26	2600	130.667	4.104	126.563
—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—
11	1100	144.000	$11^2y_1 =$13.794	130.206	21	2100	147.334	13.794	133.540
12	1200	148.000	$12^2y_1 =$16.416	131.584	20	2000	150.667	16.416	134.251
13	1300	152.000	$13^2y_1 =$19.266	132.734	19	1900	154.000	19.266	134.734
14	1400	156.000	$14^2y_1 =$22.344	133.656	18	1800	157.334	22.344	134.990
15	1500	160.000	$15^2y_1 =$25.650	134.350	17	1700	160.667	25.650	135.017
16	1600	164.000	$16^2y_1 =$29.184	134.816	16	1600	164.000	29.184	134.816
					Read upwards				

**Same values as in opposite station of first half of the curve

PROBLEM NO. 3

Design the approach to a long bridge on a National Highway in plain terrain. The deck of the bridge is 5.5 *m* above the general road level. Provide for intermediate sight distance.

Design speed = 100 km/h

S (intermediate) = 360 *m*

From Fig. 18, ignoring the straightline grade CD,

$\frac{1}{2}(Ls+Lv) \times N = 5.5\ m$

Try $N = 0.025$ (i.e. 1 in 40)

Ls (from Plate 2) = 337.5 *m* or 340 *m*

Lv (from Plate 4) = 70 *m*

$\therefore \frac{1}{2}(340 + 70) \times 0.025 = 5.125\ m$

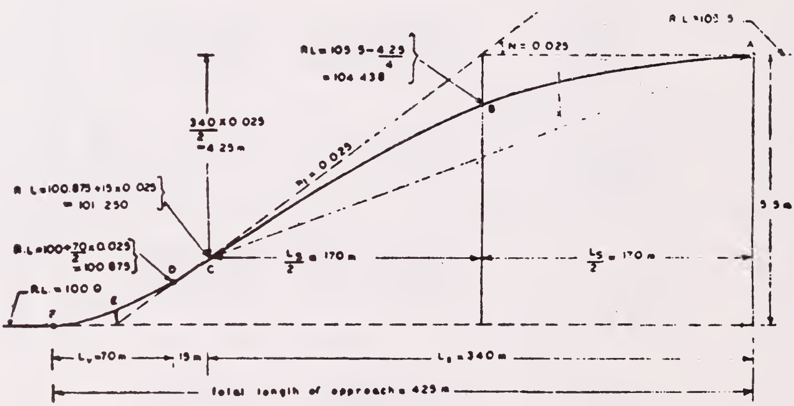


Fig. 18

This is near enough, and the balance fall (5.5 -- 5.125) or 0.375 *m* will be covered by straight grade portion CD

$$\text{Length of } CD = \frac{0.375}{0.025} = 15\ m$$

(i) Design of summit curve

Divide the length of the curve in chords 30 *m* each. The design chart may be calculated as under :

$$\text{Constant } a = \frac{2L}{N} = \frac{2 \times 340}{0.025} = 27{,}200$$

$$\text{First ordinate } y_1 = \frac{30 \times 30}{27200} = 0.033$$

The other ordinates can be calculated and the levels on the curve worked out similar to Problem No. 1.

(ii) Design of valley curve

Divide the length of the curve in chords 10 m each. The design chart may be prepared as under :

$$\text{Constant } a = \frac{2L}{N} = \frac{2 \times 70}{0.025} = 5600$$

$$\text{First ordinate } y_1 = \frac{10 \times 10}{5600} = 0.0179$$

Since valley curve is also of square parabola, the other ordinates can be calculated similar to summit curves.

The profile is shown in Fig. 18. The total length of one side bridge approach works out to 425 m.

PROBLEM NO. 4

For the case in Problem 3, work out the profile for a gradient of 1 in 50 (2 per cent) and compare with the profile obtained with the gradient of 1 in 40 (Problem No. 3).

$S = 360\ m$

$N = 0.02$

Ls (from Plate 2) $= 240\ m$

Lv (minimum) $= 60\ m$

Assuming the general level of the road to be R.L. 100.0, Referring to Fig. 18 for symbols,

R.L. of $F = 100.0$

-do- $D = 100 + \frac{1}{2} \times 60 \times 0.02 = 100.6$

-do- $E = 100 + \frac{0.6}{4} = 100.15$

-do- $C = 105.5 - \frac{1}{2} \times 240 \times 0.02 = 103.1$

-do- $B = 105.5 - \frac{2.4}{4} = 104.9$

Difference in level between C and D = 103.1 − 100.6 = 2.5 m
Length of $CD = 2.5 \times 50 = 125\ m$

Total length of approach = 240+60+125 = 425 m which is the same as that obtained in problem No. 3 with 1 in 40 gradient.

PROBLEM NO. 5

An urban arterial having divided carriageway is to cross a railway line over a bridge 25 m span. The difference in deck level of the bridge and the general road level is 6.0 m. Design suitable profile for the approaches.

As the location is in urban area where a lot of slow moving traffic is expected, it is preferable to adopt a flat gradient of 1 in 40 to 1 in 50. For the present case, adopt a gradient of 1 in 50, or 0.02.

Stopping sight distance will apply for divided carriageways. For the design speed of 80 km/h (vide IRC : 86-1983),

$$S = 120 \text{ m}$$

(i) **Design of summit curve**

As the bridge span is short, less than 30 m, it will be preferable to provide a single summit curve encompassing the bridge deck as well.

$$N = 0.02 + 0.02 = 0.04$$

$$L(\text{from Plate 1}) = 131 \text{ m, or adopt } 150 \text{ m}$$

$$R = \frac{L}{N} = \frac{150}{0.04} = 3750 \text{ m}$$

The maximum difference in level between the horizontal and the curve at the centre of bridge (see Fig. 19)

$$= \frac{l^2}{8R} = \frac{25 \times 25}{8 \times 3750} = 0.021 \text{ m or } 2.1 \text{ cm}$$

This can be accommodated in the wearing course.

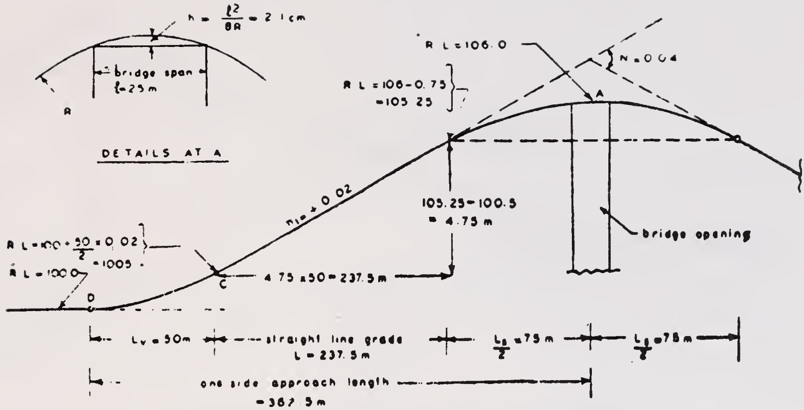


Fig. 19.

(ii) **Valley curve**

For the gradient of $N = 0.02$ and design speed of 80 km/h, the minimum lengths given Table 7 will apply. That is, the length of valley curve on either side will be 50 m.

The proposed profile is shown in Fig. 19.

PROBLEM NO. 6

The deck of a slab culvert on a National Highway is 0.6 m above the general road level on the flanks. Design suitable shock-free curves for the culvert approaches.

As the hump caused by the culvert will not obstruct the visibility, the minimum lengths from riding comfort considerations (see Table 7) will govern the design.

Referring to Fig. 20, let slope of the grade line be N.

$$\tfrac{1}{2}\,(Ls+Lv) \times N = 0.6$$

$$N = \frac{0.6 \times 2}{60+60} = 0.01$$

Let R.L of the road at A = 100.00

$$\text{R.L of } C = 100 + \frac{60}{2} \times 0.01 = 100.30$$

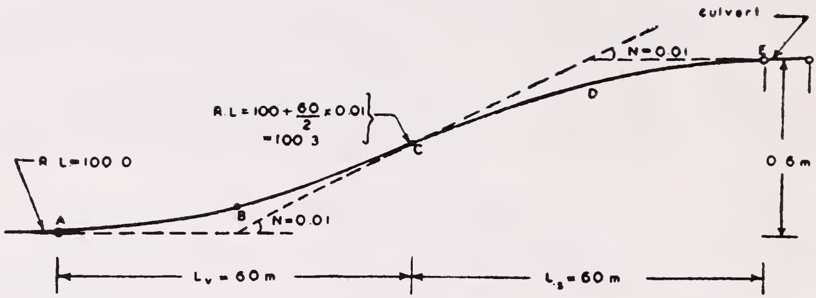


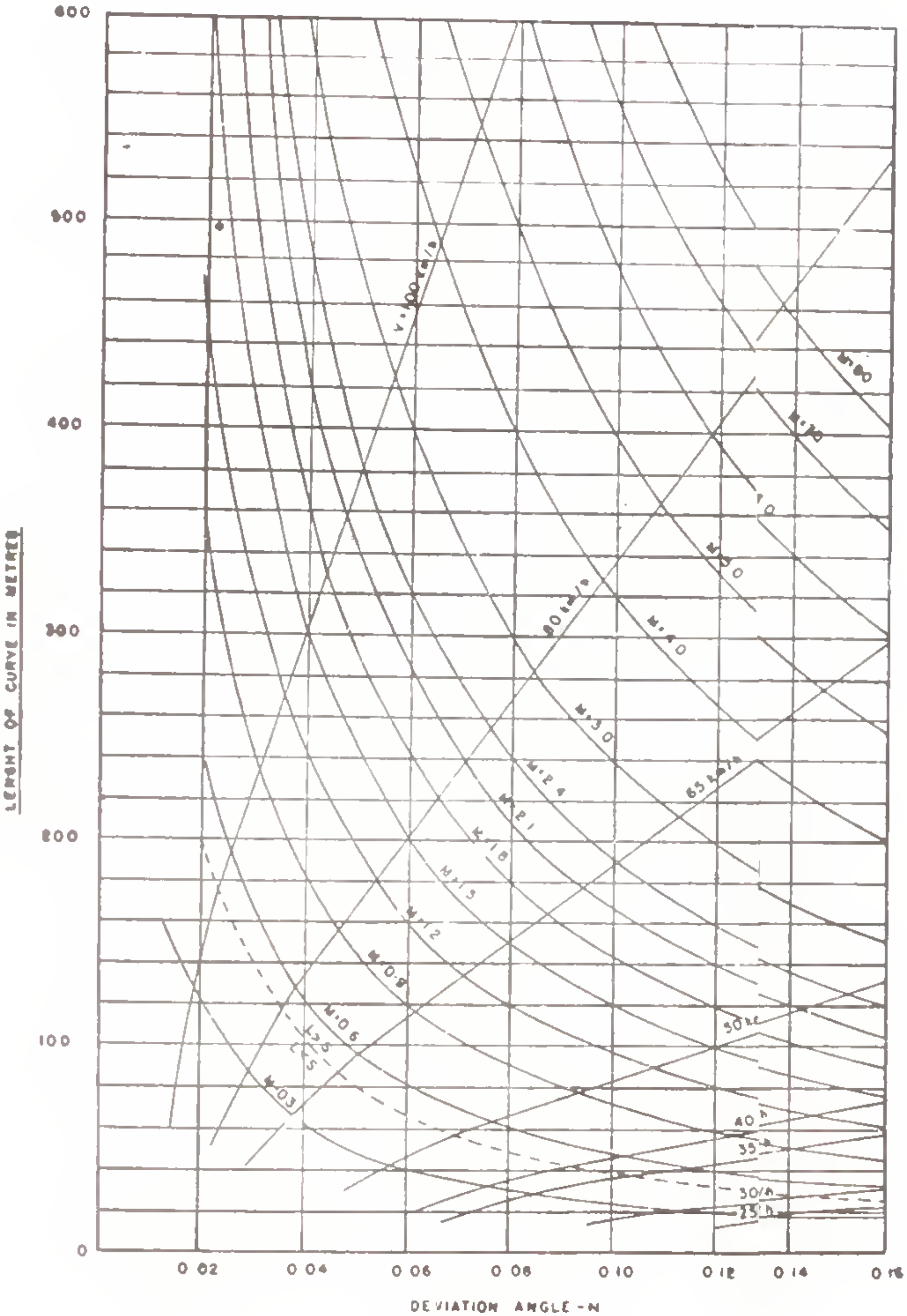
Fig. 20.

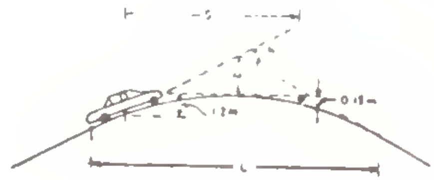
$$\text{R.L of } B = 100 + \frac{0.3}{4} = 100.075$$

$$\text{R.L of } D = 100.6 - \frac{0.3}{4} = 100.525$$

Levels of in-between points as also of the approach on the other side can be calculated in a similar way.

PLATE I





$$L = \frac{NS^2}{4.4} \quad (L > S)$$

$$L = 2S - \frac{4.4}{N} \quad (L < S)$$

$$M = \frac{NL}{8}$$

WHERE L = LENGTH OF SUMMIT CURVE

S = STOPPING SIGHT DISTANCE

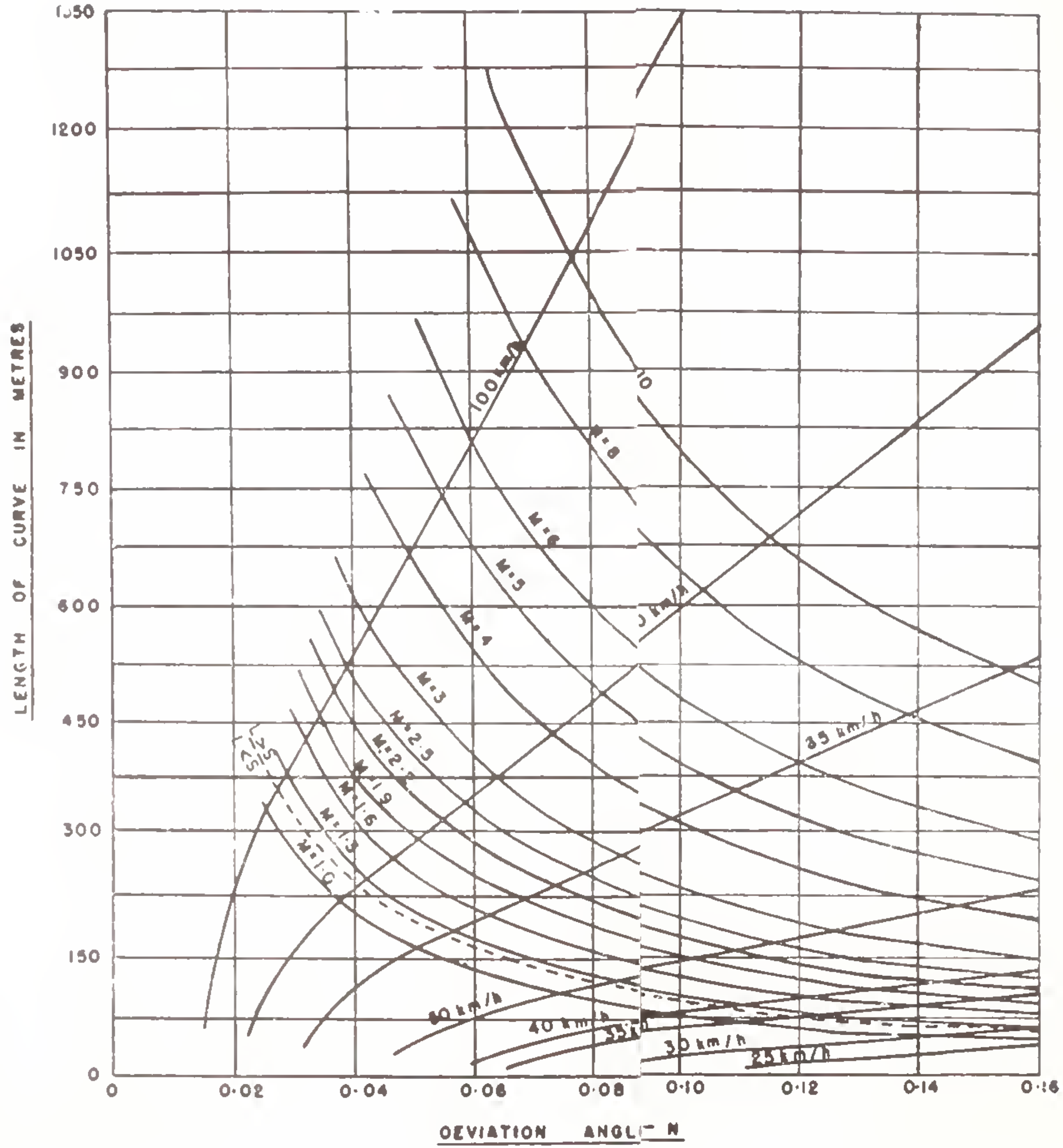
N = DEVIATION ANGLE

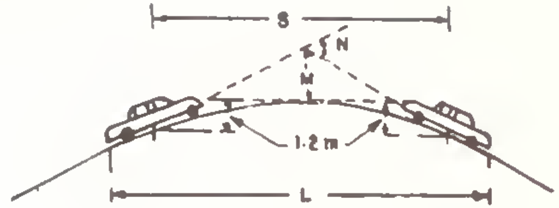
M = ORDINATE TO SUMMIT CURVE FROM THE INTERSECTION POINT OF GRADE LINES

NOTE — FOR MINIMUM LENGTH OF CURVE, SEE TABLE 7

LENGTH OF SUMMIT CURVE FOR STOPPING SIGHT DISTANCE

PLATE 2





$L = \frac{NS^2}{9 \cdot 6}$ (L > S)

$L = 2S - \frac{9 \cdot 6}{N}$ (L < S)

$M = \frac{NL}{8}$

WHERE L = LENGTH OF SUMMIT CURVE

S = INTERMEDIATE SIGHT DISTANCE

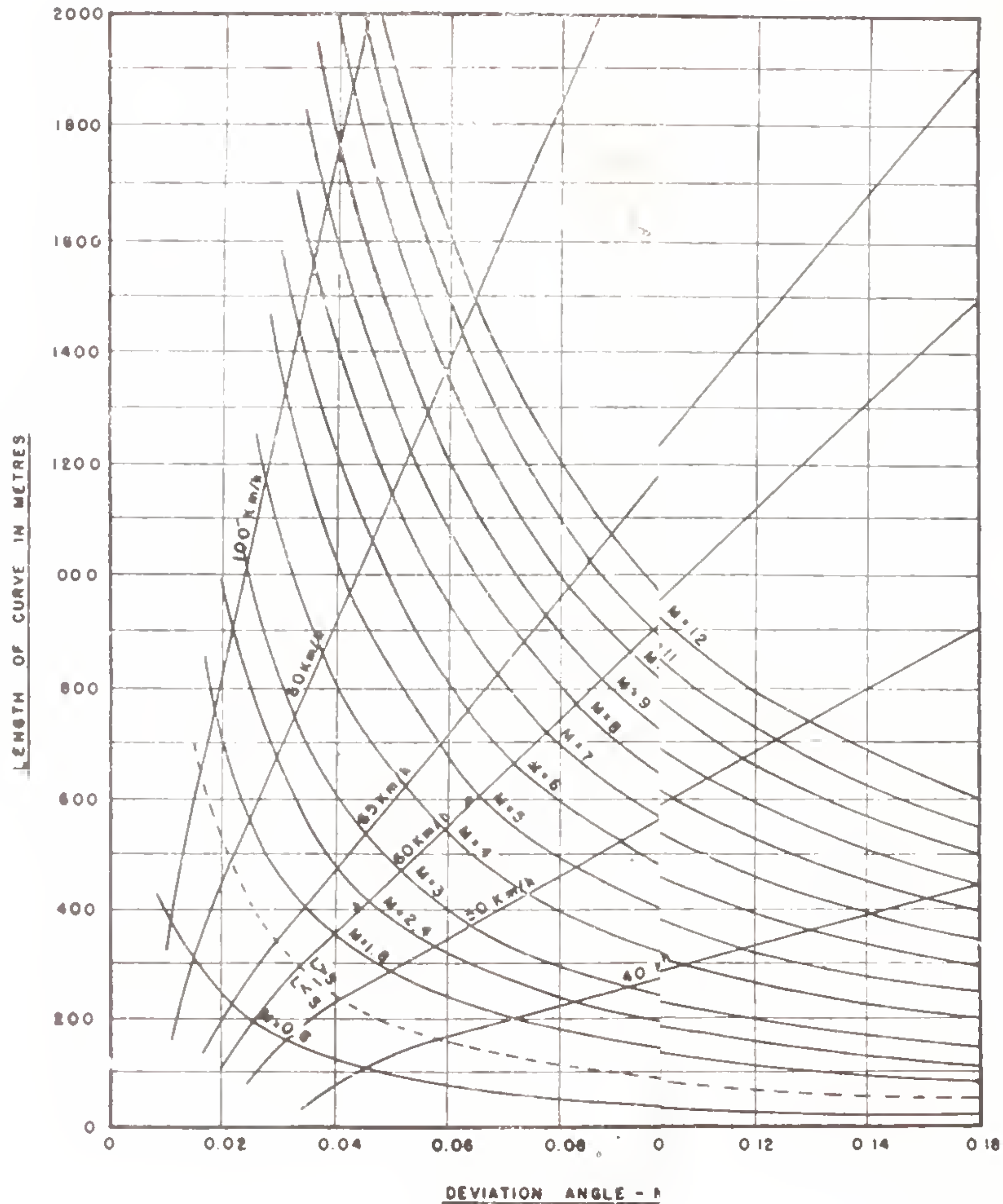
N = DEVIATION ANGLE

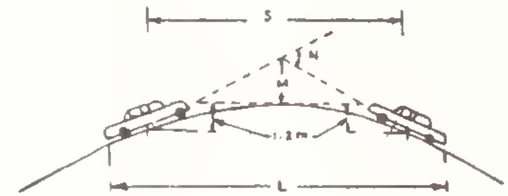
M = ORDINATE TO SUMMIT CURVE FROM THE INTERSECTION POINT OF GRADE LINES

NOTE:- FOR MINIMUM LENGTH OF CURVE SEE TABLE 7

LENGTH OF SUMMIT CURVE OF INTERMEDIATE SIGHT DISTANCE

PLATE 3





$L = \frac{NS^2}{9.6}$ (L>S)

$L = 2S - \frac{9.6}{N}$ (L<S)

$M = \frac{NL}{8}$

WHERE L = LENGTH OF SUMMIT CURVE

S = OVERTAKING SIGHT DISTANCE

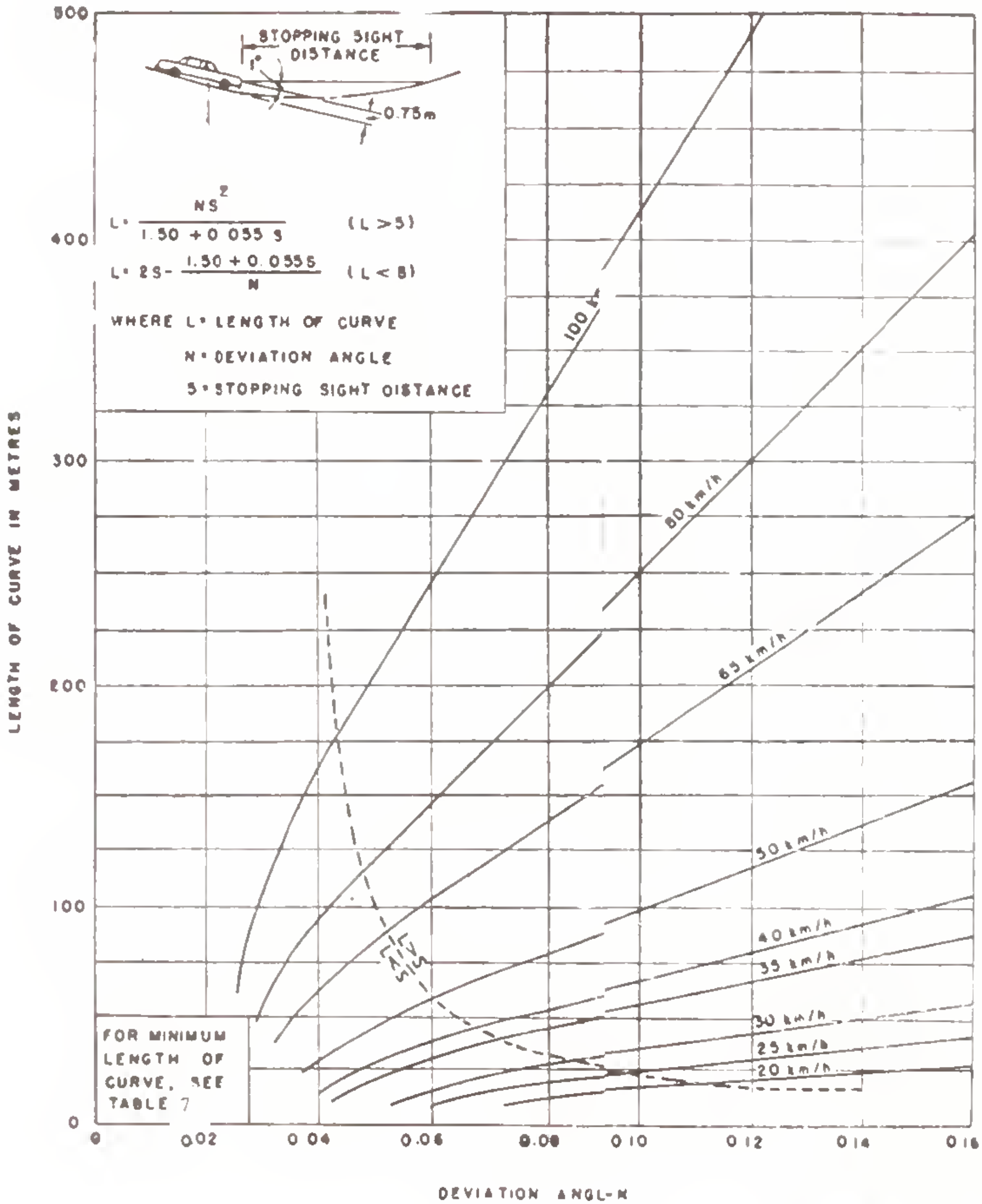
N = DEVIATION ANGLE

M = ORDINATE TO SUMMIT CURVE FROM THE INTERSECTION POINT OF GRADE LINES

NOTE — FOR MINIMUM LENGTH OF CURVE, SEE TABLE 7

LENGTH OF SUMMIT CURVE FOR OVERTAKING SIGHT DISTANCE

PLATE 4



LENGTH OF VALLEY CURVE