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भारतीय मानक

मात्राएँ और इकाइयाँ

भाग 11 भौतिक विज्ञानों और प्रौद्योगिकी में प्रयुक्त गणितीय चिह्न और प्रतीक

(दूसरा पुनरीक्षण)

Indian Standard

QUANTITIES AND UNITS

PART 11 MATHEMATICAL SIGNS AND SYMBOLS FOR USE IN THE PHYSICAL SCIENCES AND TECHNOLOGY

(Second Revision)

UDC 53.081

BUREAU OF INDIAN STANDARDS
MANAK BHAVAN, 9 BAHADUR SHAH ZAFAR MARG
NEW DELHI 110002

August 1995

Price Group 10

NATIONAL FOREWORD

This Indian Standard (Part 11) (Second Revision) which is identical with ISO 31-11 : 1992, issued by the International Organization for Standardization (ISO) was adopted by the Bureau of Indian Standards on the recommendation of the Basic Standards Sectional Committee (MSD 1) and approval of the Management and Systems Division Council.

The text of the ISO Standard has been approved as suitable for publication as Indian Standard without deviations. Certain conventions are, however, not identical to those used in Indian Standards. Attention is particularly drawn to the following:

Comma (,) has been used as a decimal marker while in Indian Standards the current practice is to use a point (.) as the decimal marker.

Wherever the words 'International Standard' appear, referring to this standard, they should be read as 'Indian Standard'.

This standard was first published in 1961 and then revised in 1982. In this second revision, following changes have been made:

a) A new clause on co-ordinate systems has been added, as Introduction to the standard; and

b) Some new items have been added in old clauses such as 11-4.14 to 11-4.17, 11-4.33, 11-5.15, 11-5-16, 11-6.1, 11-6.3, 11-6.4, 11-12.1 to 11-12.3

In the adopted standard, normative references appear to certain International Standards for which Indian Standards also exist. The corresponding Indian Standards which are to be substituted in their place are listed below along with their degree of equivalence for the edition indicated:

International Standard	Corresponding Indian Standard	Degree of Equivalence
IEC 27-1 : 1971	IS 3700 (Part 1) : 1983 Letter symbols and signs used in electric technology : Part 1 General guidance on symbols and subscripts (*first revision*)	Equivalent

ADDITIONAL INFORMATION

IS 1890/ISO 31 consists of the following parts, under the general title 'Quantities and Units':

International Standard	Corresponding Indian Standard	Degree of Equivalence
ISO 31-0 : 1992	IS 1890 (Part 0) : 1995 Quantities and units : Part 0 General principles (*first revision*)	Identical
ISO 31-1 : 1992	IS 1890 (Part 1) : 1995 Quantities and units : Part 1 Space and time (*third revision*)	Identical
ISO 31-2 : 1992	IS 1890 (Part 2) : 1995 Quantities and units : Part 2 Periodic and related phenomena (*second revision*)	Identical

Indian Standard

QUANTITIES AND UNITS

PART 11 MATHEMATICAL SIGNS AND SYMBOLS FOR USE IN THE PHYSICAL SCIENCES AND TECHNOLOGY

(Second Revision)

1 Scope

This part of ISO 31 gives general information about mathematical signs and symbols, their meanings, verbal equivalents and applications.

The recommendations in this part of ISO 31 are intended mainly for use in the physical sciences and technology.

2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this part of ISO 31. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this part of ISO 31 are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 31-0:1992, *Quantities and units — Part 0: General principles*.

3 MATHEMATICAL LOGIC

Item No.	Symbol, sign	Application	Name of symbol	Meaning, verbal equivalent and remarks
11-3.1 (*11-2.1*)	$\wedge$	$p \wedge q$	conjunction sign	p and q
11-3.2 (*11-2.2*)	$\vee$	$p \vee q$	disjunction sign	p or q (or both)
11-3.3 (*11-2.3*)	$\neg$	$\neg p$	negation sign	negation of p; not p; non p
11-3.4 (*11-2.4*)	$\Rightarrow$	$p \Rightarrow q$	implication sign	if p then q; p implies q Can also be written $q \Leftarrow p$. Sometimes $\rightarrow$ is used.
11-3.5 (*11-2.5*)	$\Leftrightarrow$	$p \Leftrightarrow q$	equivalence sign	$p \Rightarrow q$ and $q \Rightarrow p$; p is equivalent to q Sometimes $\leftrightarrow$ is used.
11-3.6 (*11-2-6*)	$\forall$	$\forall x \in A \quad p(x)$ $(\forall x \in A) \quad p(x)$	universal quantifier	for every x belonging to A, the proposition $p(x)$ is true If it is clear from the context which set A is being considered, the notation $\forall x\, p(x)$ can be used. For $x \in A$, see 11-4.1.
11-3.7 (*11-2.7*)	$\exists$	$\exists x \in A \quad p(x)$ $(\exists x \in A) \quad p(x)$	existential quantifier	there exists an x belonging to A for which $p(x)$ is true If it is clear from the context which set A is being considered, the notation $\exists x\, p(x)$ can be used. For $x \in A$, see 11-4.1. $\exists!$ or $\overset{1}{\exists}$ is used to indicate the existence of one and only one element for which $p(x)$ is true.

4 SETS				
Item No.	**Symbol, sign**	**Application**	**Meaning, verbal equivalent**	**Remarks and examples**
11-4.1 (11-1.1)	$\in$	$x \in A$	x belongs to A; x is an element of the set A	
11-4.2 (11-1.2)	$\notin$	$y \notin A$	y does not belong to A; y is not an element of the set A	The symbol $\overline{\in}$ is also used.
11-4.3 (11-1.3)	$\ni$	$A \ni x$	the set A contains x (as element)	$A \ni x$ has the same meaning as $x \in A$.
11-4.4 (11-1.4)	$\not\ni$	$A \not\ni y$	the set A does not contain y (as element)	$A \not\ni y$ has the same meaning as $y \notin A$. The symbol $\overline{\ni}$ is also used.
11-4.5 (11-1.5)	{ }	$\{x_1, x_2, ..., x_n\}$	set with elements $x_1, x_2, ..., x_n$	Also $\{x_i : i \in I\}$, where I denotes a set of indices.
11-4.6 (11-1.6)	{ | }	$\{x \in A \mid p(x)\}$	set of those elements of A for which the proposition $p(x)$ is true	EXAMPLE $\{x \in \mathbb{R} \mid x \leqslant 5\}$ If it is clear from the context which set A is being considered, the notation $\{x \mid p(x)\}$ can be used. EXAMPLE $\{x \mid x \leqslant 5\}$
11-4.7 (—)	card	card (A)	number of elements in A; cardinal of A	
11-4.8 (11-1.7)	$\varnothing$		the empty set	
11-4.9 (11-1.8)	$\mathbb{N}$ **N**		the set of natural numbers; the set of positive integers and zero	$\mathbb{N} = \{0, 1, 2, 3, ...\}$ Exclusion of zero from the sets 11-4.9 to 11-4.13 is denoted by an asterisk, e.g. $\mathbb{N}^*$. $\mathbb{N}_k = \{0, 1, ..., k-1\}$
11-4.10 (11-1.9)	$\mathbb{Z}$ **Z**		the set of integers	$\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$ See remark to 11-4.9.
11-4.11 (11-1.10)	$\mathbb{Q}$ **Q**		the set of rational numbers	See remark to 11-4.9.
11-4.12 (11-1.11)	$\mathbb{R}$ **R**		the set of real numbers	See remark to 11-4.9.
11-4.13 (11-1.12)	$\mathbb{C}$ **C**		the set of complex numbers	See remark to 11-4.9.

4 SETS (continued)

Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-4.14 (—)	[,]	$[a, b]$	closed interval in $\mathbb{R}$ from a (included) to b (included)	$[a, b] = \{x \in \mathbb{R} \mid a \leqslant x \leqslant b\}$
11-4.15 (—)	] ,] (,]	$]a, b]$ $(a, b]$	left half-open interval in $\mathbb{R}$ from a (excluded) to b (included)	$]a, b] = \{x \in \mathbb{R} \mid a < x \leqslant b\}$
11-4.16 (—)	[, [[,)	$[a, b[$ $[a, b)$	right half-open interval in $\mathbb{R}$ from a (included) to b (excluded)	$[a, b[= \{x \in \mathbb{R} \mid a \leqslant x < b\}$
11-4.17 (—)	] , [(,)	$]a, b[$ (a, b)	open interval in $\mathbb{R}$ from a (excluded) to b (excluded)	$]a, b[= \{x \in \mathbb{R} \mid a < x < b\}$
11-4.18 (11-1.13)	$\subseteq$	$B \subseteq A$	B is included in A; B is a subset of A	Every element of B belongs to A. $\subset$ is also used, but see remark to 11-4.19.
11-4.19 (11-1.14)	$\subset$	$B \subset A$	B is properly included in A; B is a proper subset of A	Every element of B belongs to A, but B is not equal to A. If $\subset$ is used for 11-4.18, then $\subsetneqq$ shall be used for 11-4.19.
11-4.20 (11-1.15)	$\nsubseteq$	$C \nsubseteq A$	C is not included in A; C is not a subset of A	$\not\subset$ is also used. The symbols $\nsubseteqq$ and $\not\subset$ are also used.
11-4.21 (11-1.16)	$\supseteq$	$A \supseteq B$	A includes B (as subset)	A contains every element of B. $\supset$ is also used, but see remark to 11-4.22. $A \supseteq B$ has the same meaning as $B \subseteq A$.
11-4.22 (11-1.17)	$\supset$	$A \supset B$	A includes B properly	A contains every element of B, but A is not equal to B. If $\supset$ is used for 11-4.21, then $\supsetneqq$ shall be used for 11-4.22. $A \supset B$ has the same meaning as $B \subset A$.
11-4.23 (11-1.18)	$\nsupseteq$	$A \nsupseteq C$	A does not include C (as subset)	$\not\supset$ is also used. The symbols $\nsupseteqq$ and $\not\supset$ are also used. $A \nsupseteq C$ has the same meaning as $C \nsubseteq A$.
11-4.24 (11-1.19)	$\cup$	$A \cup B$	union of A and B	The set of elements which belong to A or to B or to both A and B. $A \cup B = \{x \mid x \in A \vee x \in B\}$

4 SETS (continued)

Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-4.25 (11-1.20)	$\cup$	$\bigcup\limits_{i=1}^{n} A_i$	union of a collection of sets $A_1, ..., A_n$	$\bigcup\limits_{i=1}^{n} A_i = A_1 \cup A_2 \cup ... \cup A_n$, the set of elements belonging to at least one of the sets $A_1, ..., A_n$. $\cup_{i=1}^{n}$ and $\bigcup\limits_{i \in I}$, $\cup_{i \in I}$ are also used, where I denotes a set of indices.
11-4.26 (11-1.21)	$\cap$	$A \cap B$	intersection of A and B, read as A inter B	The set of elements which belong to both A and B. $A \cap B = \{x \mid x \in A \wedge x \in B\}$
11-4.27 (11-1.22)	$\cap$	$\bigcap\limits_{i=1}^{n} A_i$	intersection of a collection of sets $A_1, ..., A_n$	$\bigcap\limits_{i=1}^{n} A_i = A_1 \cap A_2 \cap ... \cap A_n$, the set of elements belonging to all sets $A_1, A_2, ...$ and A_n. $\cap_{i=1}^{n}$ and $\bigcap\limits_{i \in I}$, $\cap_{i \in I}$ are also used, where I denotes a set of indices.
11-4.28 (11-1.23)		$A \setminus B$	difference between A and B; A minus B	The set of elements which belong to A, but not to B. $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ $A - B$ should not be used.
11-4.29 (11-1.24)	$\complement$	$\complement_A B$	complement of subset B of A	The set of those elements of A which do not belong to the subset B. If it is clear from the context which set A is being considered, the symbol A is often omitted. Also $\complement_A B = A \setminus B$
11-4.30 (11-1.25)	(,)	(a, b)	ordered pair a, b; couple a, b	$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. $\langle a, b \rangle$ is also used.
11-4.31 (11-1.26)	(, ...,)	$(a_1, a_2, ..., a_n)$	ordered n-tuplet	$\langle a_1, a_2, ..., a_n \rangle$ is also used.

4 SETS (*concluded*)				
Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-4.32 (*11-1.27*)	×	$A \times B$	cartesian product of A and B	The set of ordered pairs (a, b) such that $a \in A$ and $b \in B$. $A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$ $A \times A \times \ldots \times A$ is denoted by A^n, where n is the number of factors in the product.
11-4.33 (—)	Δ	Δ_A	set of pairs (x, x) of $A \times A$, where $x \in A$; diagonal of the set $A \times A$	$\Delta_A = \{(x, x) \mid x \in A\}$ id_A is also used.

5 MISCELLANEOUS SIGNS AND SYMBOLS

Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-5.1 (11-3.1)	$=$	$a = b$	a is equal to b	$\equiv$ may be used to emphasize that a particular equality is an identity.
11-5.2 (11-3.2)	$\neq$	$a \neq b$	a is not equal to b	The symbol $\pm$ is also used.
11-5.3 (11-3.3)	$\stackrel{\text{def}}{=}$	$a \stackrel{\text{def}}{=} b$	a is by definition equal to b	EXAMPLE $p \stackrel{\text{def}}{=} mv$, where p is momentum, m is mass and v is velocity. $\stackrel{\text{d}}{=}$ and $:=$ are also used.
11-5.4 (11-3.4)	$\cong$	$a \cong b$	a corresponds to b	EXAMPLES When $E = kT$, 1 eV $\cong$ 11 604,5 K. When 1 cm on a map corresponds to a length of 10 km, one may write 1 cm $\cong$ 10 km.
11-5.5 (11-3.5)	$\approx$	$a \approx b$	a is approximately equal to b	The symbol $\simeq$ is reserved for "is asymptotically equal to". See 11-7.7.
11-5.6 (11-3.6)	$\sim$ $\propto$	$a \sim b$ $a \propto b$	a is proportional to b	
11-5.7 (11-3.7)	$<$	$a < b$	a is less than b	
11-5.8 (11-3.8)	$>$	$b > a$	b is greater than a	
11-5.9 (11-3.9)	$\leqslant$	$a \leqslant b$	a is less than or equal to b	The symbols $\leq$ and $\leqq$ are also used.
11-5.10 (11-3.10)	$\geqslant$	$b \geqslant a$	b is greater than or equal to a	The symbols $\geq$ and $\geqq$ are also used.
11-5.11 (11-3.11)	$\ll$	$a \ll b$	a is much less than b	
11-5.12 (11-3.12)	$\gg$	$b \gg a$	b is much greater than a	
11-5.13 (11-3.13)	∞		infinity	

5	MISCELLANEOUS SIGNS AND SYMBOLS (concluded)			
Item No.	Symbol, sign	Application	Meaning, verbal equivalent	Remarks and examples
11-5.14 (—)	( ) [ ] { } ⟨ ⟩	$(a + b)c$ $[a + b]c$ $\{a + b\}c$ $\langle a + b\rangle c$	$ac + bc$, parentheses $ac + bc$, square brackets $ac + bc$, braces $ac + bc$, angle brackets	In ordinary algebra the sequence of ( ), [ ], { } and ⟨ ⟩ in order of nesting is not standardized. Special uses are made of ( ), [ ], { } and ⟨ ⟩ in particular fields.
11-5.15 (—)	∥	AB ∥ CD	the line AB is parallel to the line CD	
11-5.16 (—)	⊥	AB ⊥ CD	the line AB is perpendicular to the line CD	

6 OPERATIONS

Item No.	Symbol, application	Meaning, verbal equivalent	Remarks and examples
11-6.1 (11-4.1)	$a + b$	a plus b	
11-6.2 (11-4.2)	$a - b$	a minus b	
11-6.3 (—)	$a \pm b$	a plus or minus b	
11-6.4 (—)	$a \mp b$	a minus or plus b	$-(a \pm b) = -a \mp b$
11-6.5 (11-4.3)	$a \cdot b \quad a \times b \quad ab$	a multiplied by b	See also 11-4.32, 11-13.6 and 11-13.7. The sign for multiplication of numbers is a cross ($\times$) or a dot half high ($\cdot$). If a dot is used as the decimal sign, only the cross shall be used for multiplication of numbers. For decimal sign see ISO 31-0:1992, subclause 3.3.2.
11-6.6 (11-4.4)	$\dfrac{a}{b} \quad a/b \quad ab^{-1}$	a divided by b	See also ISO 31-0:1992, subclause 3.1.3.
11-6.7 (11-4.5)	$\displaystyle\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$	Also $\displaystyle\sum_{i=1}^{n} a_i, \sum_i a_i, \sum_i a_i, \sum a_i$
11-6.8 (11-4.6)	$\displaystyle\prod_{i=1}^{n} a_i$	$a_1 \cdot a_2 \cdot \ldots \cdot a_n$	Also $\displaystyle\prod_{i=1}^{n} a_i, \prod_i a_i, \prod_i a_i, \prod a_i$
11-6.9 (11-4.7)	a^p	a to the power p	
11-6.10 (11-4.8)	$a^{1/2} \quad a^{\frac{1}{2}} \quad \sqrt{a} \quad \sqrt{a}$	a to the power 1/2; square root of a	If $a \geqslant 0$, then $\sqrt{a} \geqslant 0$. See remark to 11-6.11.
11-6.11 (11-4.9)	$a^{1/n} \quad a^{\frac{1}{n}} \quad \sqrt[n]{a}$ $\sqrt[n]{a}$	a to the power $1/n$; nth root of a	If $a \geqslant 0$, then $\sqrt[n]{a} \geqslant 0$. If the symbol $\sqrt{\ }$ or $\sqrt[n]{\ }$ acts on a composite expression, parentheses shall be used to avoid ambiguity.
11-6.12 (11-4.10)	$\lvert a \rvert$	absolute value of a; magnitude of a; modulus of a	abs a is also used.

9

6 OPERATIONS (concluded)

Item No.	Symbol, application	Meaning, verbal equivalent	Remarks and examples
11-6.13 (11-4.11)	sgn a	signum a	For real a: $$\text{sgn } a = \begin{cases} 1 \text{ if } a > 0 \\ 0 \text{ if } a = 0 \\ -1 \text{ if } a < 0 \end{cases}$$ For complex a, see 11-10.7.
11-6.14 (11-4.12)	$\bar{a}$ $\langle a \rangle$	mean value of a	The method of forming the mean shall be stated if not clear from the context.
11-6.15 (11-4.13)	$n!$	factorial n	For $n \geqslant 1$: $n! = \prod\limits_{k=1}^{n} k = 1 \times 2 \times 3 \times \ldots \times n$ For $n = 0$: $n! = 1$
11-6.16 (11-4.14)	$\begin{pmatrix} n \\ p \end{pmatrix}$ C_n^p	binomial coefficient n, p	$\begin{pmatrix} n \\ p \end{pmatrix} = \dfrac{n!}{p!(n-p)!}$
11-6.17 (11-4.15)	ent a $E(a)$	the greatest integer less than or equal to a; characteristic of a	ent $2.4 = 2$ ent$(-2,4) = -3$ $[a]$ or int a is sometimes used for ent a, but is now often used with the meaning "integer part of a", e.g. $[2,4] = $ int $2,4 := 2$ $[-2,4] := $ int$(-2,4) = -2$

7 FUNCTIONS

Item No.	Symbol, application	Meaning, verbal equivalent	Remarks and examples
11-7.1 (*11-5.1*)	f	function f	A function may also be denoted by $x \mapsto f(x)$. Letters other than f are also used.
11-7.2 (*11-5.2*)	$f(x)$ $f(x, y, ...)$	value of the function f at x or at $(x, y, ...)$ respectively	
11-7.3 (*11-5.3*)	$f(x)\|_a^b$ $[f(x)]_a^b$	$f(b) - f(a)$	This notation is used mainly when evaluating definite integrals.
11-7.4 (*11-5.4*)	$g \circ f$	the composite function of f and g, read as g circle f	$(g \circ f)(x) = g(f(x))$
11-7.5 (*11-5.5*)	$x \to a$	x tends to a	
11-7.6 (*11-5.6*)	$\lim_{x \to a} f(x)$ $\lim_{x \to a} f(x)$	limit of $f(x)$ as x tends to a	$\lim_{x \to a} f(x) = b$ may be written $f(x) \to b$ as $x \to a$. Limits "from the right" $(x > a)$ and "from the left" $(x < a)$ may be denoted by $\lim_{x \to a+} f(x)$ and $\lim_{x \to a-} f(x)$ respectively.
11-7.7 (*11-5.7*)	$\simeq$	is asymptotically equal to	EXAMPLE $\dfrac{1}{\sin(x-a)} \simeq \dfrac{1}{x-a}$ as $x \to a$.
11-7.8 (*11-5.8*)	$O(g(x))$ $f(x) = O(g(x))$	$\|f(x)/g(x)\|$ is bounded above in the limit implied by the context; f is of the order of g	
11-7.9 (*11-5.9*)	$o(g(x))$ $f(x) = o(g(x))$	$f(x)/g(x) \to 0$ in the limit implied by the context; f is of lower order than g	
11-7.10 (*11-5.10*)	Δx	(finite) increment of x	
11-7.11 (*11-5.11*)	$\dfrac{df}{dx}$ df/dx f'	derivative of the function f of one variable	Df is also used. $\dfrac{df(x)}{dx}$, $df(x)/dx$, $f'(x)$, $Df(x)$ If the independent variable is time t, $\dot{f}$ is also used for $\dfrac{df}{dt}$.

7 FUNCTIONS (continued)

Item No.	Symbol, application	Meaning, verbal equivalent	Remarks and examples
11-7.12 (11-5.12)	$\left(\dfrac{df}{dx}\right)_{x=a}$ $(df/dx)_{x=a}$ $f'(a)$	value at a of the derivative of the function f	$Df(a)$ is also used.
11-7.13 (11-5.13)	$\dfrac{d^n f}{dx^n}$ $d^n f/dx^n$ $f^{(n)}$	nth derivative of the function f of one variable	$D^n f$ is also used. For $n = 2, 3$; f'', f''' are also used for $f^{(n)}$. If the independent variable is time t, $\ddot{f}$ is also used for $\dfrac{d^2 f}{dt^2}$.
11-7.14 (11-5.14)	$\dfrac{\partial f}{\partial x}$ $\partial f/\partial x$ $\partial_x f$	partial derivative of the function f of several variables $x, y, \ldots$ with respect to x	$D_x f$ is also used. $\dfrac{\partial f(x, y, \ldots)}{\partial x}$, $\partial f(x, y, \ldots)/\partial x$, $\partial_x f(x, y, \ldots)$, $D_x f(x, y, \ldots)$ The other independent variables may be shown as subscripts, e.g. $\left(\dfrac{\partial f}{\partial x}\right)_{y\ldots}$ This partial-derivative notation is extended to derivatives of higher order, e.g. $\dfrac{\partial^2 f}{\partial x^2} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial f}{\partial x}\right)$ $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial f}{\partial y}\right)$
11-7.15 (11-5.15)	df	total differential of the function f	$df(x, y, \ldots) = \dfrac{\partial f}{\partial x}\,dx + \dfrac{\partial f}{\partial y}\,dy + \ldots$
11-7.16 (11-5.16)	δf	(infinitesimal) variation of the function f	
11-7.17 (11-5.17)	$\int f(x)\,dx$	an indefinite integral of the function f	

7 FUNCTIONS (concluded)			
Item No.	Symbol, application	Meaning, verbal equivalent	Remarks and examples
11-7.18 (11-5.18)	$\int_a^b f(x)\ \mathrm{d}x$ $\int_a^b f(x)\ \mathrm{d}x$	definite integral of the function f from a to b	Multiple integrals are denoted by, for example: $$\int_{y=c}^{d}\ \int_{x=a}^{b} f(x,y)\ \mathrm{d}x\ \mathrm{d}y$$ Special notations $$\int_C\ \int_S\ \int_V\ \oint$$ are used for integration over a curve C, a surface S and a three-dimensional domain V, and over a closed curve or surface, respectively.
11-7.19 (11-5.19)	δ_{ik}	Kronecker delta symbol	$\delta_{ik} = \begin{cases} 1 \text{ for } i = k \\ 0 \text{ for } i \neq k \end{cases}$ where i and k are integers.
11-7.20 (11-5.20)	ε_{ijk}	Levi-Civita symbol	$\varepsilon_{ijk} = 1$ for $(i, j, k) = (1, 2, 3)$ and is completely antisymmetric in the indices, i.e. $\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = 1$ $\varepsilon_{132} = \varepsilon_{321} = \varepsilon_{213} = -1$ all others being equal to 0.
11-7.21 (11-5.21)	$\delta(x)$	Dirac delta distribution (function)	$\int_{-\infty}^{+\infty} f(x)\ \delta(x)\ \mathrm{d}x = f(0)$
11-7.22 (11-5.22)	$\varepsilon(x)$	unit step function; Heaviside function	$\varepsilon(x) = \begin{cases} 1 \text{ for } x > 0 \\ 0 \text{ for } x < 0 \end{cases}$ $H(x)$ is also used. $\vartheta(t)$ is used for the unit step function of time.
11-7.23 (11-5.23)	$f * g$	convolution of f and g	$(f * g)(x) = \int_{-\infty}^{+\infty} f(y)\ g(x - y)\ \mathrm{d}y$

8 EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-8.1 (—)	a^x	exponential function to the base a of x	Compare 11-6.9.
11-8.2 (11-6.1)	e	base of natural logarithms	$e = \lim\limits_{n \to \infty} \left(1 + \dfrac{1}{n}\right)^n = 2{,}718\ 281\ 8...$
11-8.3 (11-6.2)	e^x exp x	exponential function (to the base e) of x	
11-8.4 (11-6.3)	$\log_a x$	logarithm to the base a of x	log x is used when the base need not be specified.
11-8.5 (11-6.4)	ln x	ln $x = \log_e x$; natural logarithm of x	log x shall not be used in place of ln x, lg x, lb x or $\log_e x$, $\log_{10} x$, $\log_2 x$.
11-8.6 (11-6.5)	lg x	lg $x = \log_{10} x$; common (decimal) logarithm of x	See remark to 11-8.5.
11-8.7 (11-6.6)	lb x	lb $x = \log_2 x$; binary logarithm of x	See remark to 11-8.5.

9 CIRCULAR AND HYPERBOLIC FUNCTIONS

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-9.1 (11-7.1)	π	ratio of the circumference of a circle to its diameter	$\pi = 3{,}141\ 592\ 6...$
11-9.2 (11-7.2)	$\sin x$	sine of x	$(\sin x)^n$, $(\cos x)^n$, etc., are often written $\sin^n x$, $\cos^n x$, etc.
11-9.3 (11-7.3)	$\cos x$	cosine of x	
11-9.4 (11-7.4)	$\tan x$	tangent of x	tg x is still used.
11-9.5 (11-7.5)	$\cot x$	cotangent of x	$\cot x = 1/\tan x$
11-9.6 (11-7.6)	$\sec x$	secant of x	$\sec x = 1/\cos x$
11-9.7 (11-7.7)	$\csc x$	cosecant of x	cosec x is also used. $\csc x = 1/\sin x$
11-9.8 (11-7.8)	$\arcsin x$	arc sine of x	$y = \arcsin x \Leftrightarrow x = \sin y,\ -\pi/2 \leqslant y \leqslant \pi/2$ The function arcsin is the inverse of the function sin with the restriction mentioned above. See remark following 11-9.13.
11-9.9 (11-7.9)	$\arccos x$	arc cosine of x	$y = \arccos x \Leftrightarrow x = \cos y,\ 0 \leqslant y \leqslant \pi$ The function arccos is the inverse of the function cos with the restriction mentioned above. See remark following 11-9.13.
11-9.10 (11-7.10)	$\arctan x$	arc tangent of x	arctg x is still used. $y = \arctan x \Leftrightarrow x = \tan y,\ -\pi/2 < y < \pi/2$ The function arctan is the inverse of the function tan with the restriction mentioned above. See remark following 11-9.13.
11-9.11 (11-7.11)	$\operatorname{arccot} x$	arc cotangent of x	$y = \operatorname{arccot} x \Leftrightarrow x = \cot y,\ 0 < y < \pi$ The function arccot is the inverse of the function cot with the restriction mentioned above. See remark following 11-9.13.

9 CIRCULAR AND HYPERBOLIC FUNCTIONS (*continued*)

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-9.12 (*11-7.12*)	arcsec x	arc secant of x	$y = \text{arcsec } x \Leftrightarrow x = \sec y,\ 0 \leqslant y \leqslant \pi,\ y \neq \pi/2$ The function arcsec is the inverse of the function sec with the restriction mentioned above. See remark following 11-9.13.
11-9.13 (*11-7.13*)	arccsc x	arc cosecant of x	arccosec x is also used. $y = \text{arccsc } x \Leftrightarrow x = \csc y,$ $-\pi/2 \leqslant y \leqslant \pi/2,\ y \neq 0$ The function arccsc is the inverse of the function csc with the restriction mentioned above. **Remark on 11-9.8 to 11-9.13.** The notations $\sin^{-1}x$, $\cos^{-1}x$, etc., for the inverse circular functions shall not be used because they may be mistaken for $(\sin x)^{-1}$ $(\cos x)^{-1}$, etc.
11-9.14 (*11-7.14*)	sinh x	hyperbolic sine of x	sh x is also used.
11-9.15 (*11-7.15*)	cosh x	hyperbolic cosine of x	ch x is also used.
11-9.16 (*11-7.16*)	tanh x	hyperbolic tangent of x	th x is also used.
11-9.17 (*11-7.17*)	coth x	hyperbolic cotangent of x	$\coth x = 1/\tanh x$
11-9.18 (*11-7.18*)	sech x	hyperbolic secant of x	$\text{sech } x = 1/\cosh x$
11-9.19 (*11-7.19*)	csch x	hyperbolic cosecant of x	cosech x is also used. $\text{csch } x = 1/\sinh x$
11-9.20 (*11-7.20*)	arsinh x	inverse hyperbolic sine of x	arsh x and argsh x are also used. $y = \text{arsinh } x \Leftrightarrow x = \sinh y$ The function arsinh is the inverse of the function sinh. See remarks following 11-9.25.
11-9.21 (*11-7.21*)	arcosh x	inverse hyperbolic cosine of x	arch x and argch x are also used. $y = \text{arcosh } x \Leftrightarrow x = \cosh y,\ y \geqslant 0$ The function arcosh is the inverse of the function cosh with the restriction mentioned above. See remarks following 11-9.25.

9 CIRCULAR AND HYPERBOLIC FUNCTIONS (concluded)

| Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-9.22 (11-7.22)	artanh x	inverse hyperbolic tangent of x	arth x and argth x are also used. $y = \text{artanh } x \Leftrightarrow x = \tanh y$ The function artanh is the inverse of the function tanh. See remarks following 11-9.25.
11-9.23 (11-7.23)	arcoth x	inverse hyperbolic cotangent of x	argcoth x is also used. $y = \text{arcoth } x \Leftrightarrow x = \coth y,\ y \neq 0$ The function arcoth is the inverse of the function coth with the restriction mentioned above. See remarks following 11-9.25.
11-9.24 (11-7.24)	arsech x	inverse hyperbolic secant of x	$y = \text{arsech } x \Leftrightarrow x = \text{sech } y,\ y \geqslant 0$ The function arsech is the inverse of the function sech with the restriction mentioned above. See remarks following 11-9.25.
11-9.25 (11-7.25)	arcsch x	inverse hyperbolic cosecant of x	arcosech x is also used. $y = \text{arcsch } x \Leftrightarrow x = \text{csch } y,\ y \neq 0$ The function arcsch is the inverse of the function csch with the restriction mentioned above. **Remarks on 11-9.20 to 11-9.25.** arsinh, arcosh, etc., are also called area hyperbolic sine, area hyperbolic cosine and so on, because the argument is an area. The notations $\sinh^{-1}x$, $\cosh^{-1}x$, etc., for the inverse hyperbolic functions shall not be used because they may be mistaken for $(\sinh x)^{-1}$ $(\cosh x)^{-1}$, etc.

10   COMPLEX NUMBERS

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-10.1 (11-8.1)	i j	imaginary unit, $i^2 = -1$	In electrotechnology, j is generally used.
11-10.2 (11-8.2)	Re z	real part of z	
11-10.3 (11-8.3)	Im z	imaginary part of z	$z = x + iy$, where $x = $ Re z and $y = $ Im z.
11-10.4 (11-8.4)	$\|z\|$	absolute value of z; modulus of z	mod z is also used.
11-10.5 (11-8.5)	arg z	argument of z; phase of z	$z = re^{i\varphi}$, where $r = \|z\|$ and $\varphi = $ arg z, i.e. Re $z = r\cos\varphi$ and Im $z = r\sin\varphi$.
11-10.6 (11-8.6)	z^*	(complex) conjugate of z	Sometimes $\bar{z}$ is used instead of z^*.
11-10.7 (11-8.7)	sgn z	signum z	sgn $z = z/\|z\| = \exp(i\,$arg$\,z)$ for $z \neq 0$, sgn $z = 0$ for $z = 0$.

11 MATRICES

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
			The use of capital letters in this section is not intended to imply that matrices or matrix elements cannot be written with lower-case letters.
11-11.1 (11-9.1)	A $\begin{pmatrix} A_{11} \cdots A_{1n} \\ \vdots \quad\ \vdots \\ A_{m1} \cdots A_{mn} \end{pmatrix}$	matrix A of type m by n	A is the matrix with the elements A_{ij}; m is the number of rows and n is the number of columns. $A = (A_{ij})$ is also used. Square brackets are also used instead of parentheses.
11-11.2 (11-9.2)	AB	product of matrices A and B	$(AB)_{ik} = \sum_j A_{ij} B_{jk}$ where the number of columns of A must be equal to the number of rows of B.
11-11.3 (11-9.3)	$E \quad I$	unit matrix	Any square matrix for which $E_{ik} = \delta_{ik}$. See 11-7.19.
11-11.4 (11-9.4)	A^{-1}	inverse of a square matrix A	$AA^{-1} = A^{-1}A = E$
11-11.5 (11-9.5)	$A^\mathsf{T} \quad \tilde{A}$	transpose matrix of A	$\left(A^\mathsf{T}\right)_{ik} = A_{ki}$
11-11.6 (11-9.6)	A^*	complex conjugate matrix of A	$(A^*)_{ik} = (A_{ik})^* = A_{ik}^*$ In mathematics, $\overline{A}$ is often used.
11-11.7 (11-9.7)	$A^\mathsf{H} \quad A^\dagger$	Hermitian conjugate matrix of A	$\left(A^\mathsf{H}\right)_{ik} = (A_{ki})^* = A_{ki}^*$ In mathematics, A^* is often used.
11-11.8 (11-9.8)	$\det A$ $\begin{vmatrix} A_{11} \cdots A_{1n} \\ \vdots \quad\ \vdots \\ A_{n1} \cdots A_{nn} \end{vmatrix}$	determinant of a square matrix A	
11-11.9 (11-9.9)	$\operatorname{tr} A$	trace of a square matrix A	$\operatorname{tr} A = \sum_i A_{ii}$
11-11.10 (11-9.10)	$\|A\|$	norm of the matrix A	Several matrix norms can be defined, e.g. the Euclidean norm $\|A\| = \left(\operatorname{tr}\left(AA^\mathsf{H}\right)\right)^{1/2}$

12 COORDINATE SYSTEMS

Item No.	Coordinates	Position vector and its differential	Name of coordinate system	Remarks
11-12.1 (—)	x, y, z	$r = xe_x + ye_y + ze_z;$ $dr = dx\ e_x + dy\ e_y + dz\ e_z$	cartesian coordinates	e_x, e_y and e_z form an orthonormal right-handed system. See figure 1.
11-12.2 (—)	ϱ, φ, z	$r = \varrho e_\varrho + ze_z;$ $dr = d\varrho\ e_\varrho +$ $\varrho\ d\varphi\ e_\varphi + dz\ e_z$	cylindrical coordinates	$e_\varrho(\varphi), e_\varphi(\varphi)$ and e_z form an orthonormal right-handed system. See figures 3 and 4. If $z = 0$, then ϱ and φ are the polar coordinates.
11-12.3 (—)	r, ϑ, φ	$r = re_r;$ $dr = dr\ e_r + r\ d\vartheta\ e_\vartheta +$ $r \sin \vartheta\ d\varphi\ e_\varphi$	spherical coordinates	$e_r(\vartheta, \varphi), e_\vartheta(\vartheta, \varphi)$ and $e_\varphi(\varphi)$ form an orthonormal right-handed system. See figures 3 and 5.

NOTE 1   If, exceptionally, a left-handed system (see figure 2) is used for certain purposes, this shall be clearly stated to avoid the risk of sign errors.
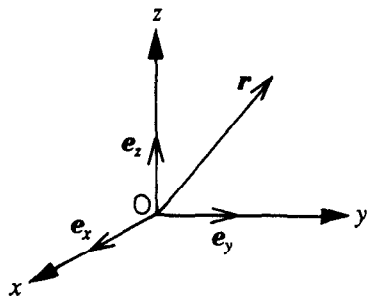
The x-axis is pointing towards the viewer.

Figure 1 — Right-handed cartesian coordinate system
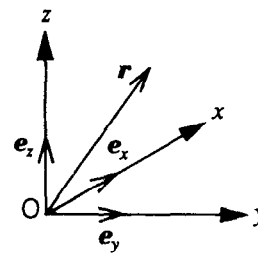
The x-axis is pointing away from the viewer.

Figure 2 — Left-handed cartesian coordinate system
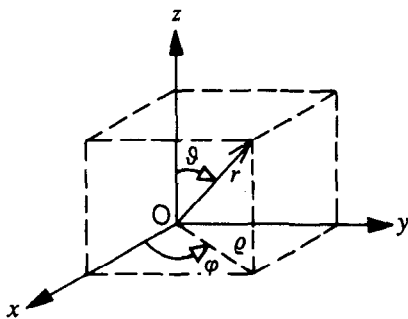

Figure 3 — O*xyz* is a right-handed coordinate system
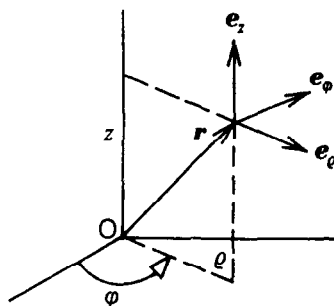

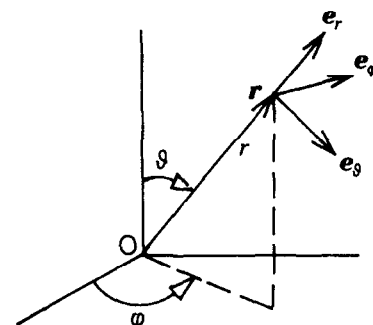**Figure 4 — Right-handed cylindrical coordinates**

Figure 5 — Right-handed spherical coordinates

13 VECTORS AND TENSORS

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
			In this section, only cartesian (orthonormal) coordinates in ordinary space are considered. The more general case requiring covariant and contravariant representations is not treated here. The cartesian coordinates are denoted x, y, z or x_1, x_2, x_3. In the latter case, indices i, j, k, l ranging from 1 to 3 are used, and the following summation convention is adopted: if such an index appears twice in a term, summation over the range of this index is understood.
			Remarks to 11-13.1 to 11-13.20
			A scalar is a tensor of zero order and a vector is a tensor of the first order.
			Vectors and tensors are often represented by general symbols for their components, e.g. a_i for a vector, T_{ij} for a tensor of the second order, $a_i b_j$ for a dyadic product.
11-13.1 (11-10.1)	$\boldsymbol{a}$ $\vec{a}$	vector $\boldsymbol{a}$	An arrow above the letter symbol can be used instead of bold face type to indicate a vector. Any vector $\boldsymbol{a}$ can be multiplied by a scalar or a number k, i.e. $k\boldsymbol{a}$.
11-13.2 (11-10.2)	a $\lvert \boldsymbol{a} \rvert$	magnitude of the vector $\boldsymbol{a}$	$\lVert \boldsymbol{a} \rVert$ is also used.
11-13.3 (11-10.3)	$\boldsymbol{e}_a$	unit vector in the direction of $\boldsymbol{a}$	$\boldsymbol{e}_a = \boldsymbol{a}/\lvert \boldsymbol{a} \rvert$ $\boldsymbol{a} = a\boldsymbol{e}_a$
11-13.4 (11-10.4)	$\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$ $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ $\boldsymbol{e}_i$	unit vectors in the directions of the cartesian coordinate axes	
11-13.5 (11-10.5)	a_x, a_y, a_z a_i	cartesian components of vector $\boldsymbol{a}$	$\boldsymbol{a} = a_x\boldsymbol{e}_x + a_y\boldsymbol{e}_y + a_z\boldsymbol{e}_z$ $a_x\boldsymbol{e}_x$, etc., are the component vectors. If it is clear from the context which are the base vectors, the vector can be written $\boldsymbol{a} = (a_x, a_y, a_z)$. $\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$ is the radius vector (position vector).

13 VECTORS AND TENSORS (*continued*)

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-13.6 (*11-10.6*)	$\boldsymbol{a} \cdot \boldsymbol{b}$	scalar product of $\boldsymbol{a}$ and $\boldsymbol{b}$	$\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z$ $\boldsymbol{a} \cdot \boldsymbol{b} = \sum_i a_i b_i$ often written $a_i b_i$ by convention (see remark preceding 11-13.1). $\boldsymbol{a} \cdot \boldsymbol{a} = \boldsymbol{a}^2 = \lvert \boldsymbol{a} \rvert^2 = a^2$ In special fields, $(\boldsymbol{a}, \boldsymbol{b})$ is also used.
11-13.7 (*11-10.7*)	$\boldsymbol{a} \times \boldsymbol{b}$	vector product of $\boldsymbol{a}$ and $\boldsymbol{b}$	The components are $(\boldsymbol{a} \times \boldsymbol{b})_x = a_y b_z - a_z b_y$, etc., in a right-handed cartesian coordinate system. $(\boldsymbol{a} \times \boldsymbol{b})_i = \sum_j \sum_k \varepsilon_{ijk} a_j b_k$ For ε_{ijk}, see 11-7.20.
11-13.8 (*11-10.8*)	∇ $\vec{\nabla}$	nabla operator	$\nabla = \boldsymbol{e}_x \dfrac{\partial}{\partial x} + \boldsymbol{e}_y \dfrac{\partial}{\partial y} + \boldsymbol{e}_z \dfrac{\partial}{\partial z} = \sum_i \boldsymbol{e}_i \dfrac{\partial}{\partial x_i}$ $\dfrac{\partial}{\partial \boldsymbol{r}}$ is also used.
11-13.9 (*11-10.9*)	$\nabla \varphi$ $\mathbf{grad}\, \varphi$	gradient of φ	grad φ is also used. $\nabla \varphi = \sum_i \boldsymbol{e}_i \dfrac{\partial \varphi}{\partial x_i}$
11-13.10 (*11-10.10*)	$\nabla \cdot \boldsymbol{a}$ div $\boldsymbol{a}$	divergence of $\boldsymbol{a}$	$\nabla \cdot \boldsymbol{a} = \sum_i \dfrac{\partial a_i}{\partial x_i}$
11-13.11 (*11-10.1*)	$\nabla \times \boldsymbol{a}$ $\mathbf{rot}\, \boldsymbol{a}$ $\mathbf{curl}\, \boldsymbol{a}$	rotation of $\boldsymbol{a}$; curl of $\boldsymbol{a}$	The components are $(\nabla \times \boldsymbol{a})_x = \dfrac{\partial a_z}{\partial y} - \dfrac{\partial a_y}{\partial z}$, etc. rot $\boldsymbol{a}$, curl $\boldsymbol{a}$ are also used. $(\nabla \times \boldsymbol{a})_i = \sum_j \sum_k \varepsilon_{ijk} \dfrac{\partial a_k}{\partial x_j}$ For ε_{ijk}, see 11-7.20.
11-13.12 (*11-10.12*)	∇^2 Δ	Laplacian	$\Delta = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$

13 VECTORS AND TENSORS (concluded)

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-13.13 (11-10.13)	$\square$	Dalembertian	$\square = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2} - \dfrac{1}{c^2}\dfrac{\partial^2}{\partial t^2}$
11-13.14 (11-10.14)	$\boldsymbol{T}$ $\overset{\leftrightarrow}{T}$	tensor $\boldsymbol{T}$ of the second order	Two arrows above the letter symbol can be used instead of bold face sans serif type to indicate a tensor of the second order.
11-13.15 (11-10.15)	$T_{xx},\ T_{xy},\ ...,\ T_{zz}$ T_{ij}	cartesian components of tensor $\boldsymbol{T}$	$\boldsymbol{T} = T_{xx}\boldsymbol{e}_x\boldsymbol{e}_x + T_{xy}\boldsymbol{e}_x\boldsymbol{e}_y + ...$ $T_{xx}\boldsymbol{e}_x\boldsymbol{e}_x,$ etc., are the component tensors. If it is clear from the context which are the base vectors, the tensor can be written $\boldsymbol{T} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$
11-13.16 (11-10.16)	$\boldsymbol{ab}\qquad \boldsymbol{a}\otimes\boldsymbol{b}$	dyadic product; tensor product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$	tensor of the second order with components $(\boldsymbol{ab})_{ij} = a_i b_j$
11-13.17 (11-10.17)	$\boldsymbol{T}\otimes\boldsymbol{S}$	tensor product of two tensors $\boldsymbol{T}$ and $\boldsymbol{S}$ of the second order	tensor of the fourth order with components $(\boldsymbol{T}\otimes\boldsymbol{S})_{ijkl} = T_{ij}S_{kl}$
11-13.18 (11-10.18)	$\boldsymbol{T}\cdot\boldsymbol{S}$	inner product of two tensors $\boldsymbol{T}$ and $\boldsymbol{S}$ of the second order	tensor of the second order with components $(\boldsymbol{T}\cdot\boldsymbol{S})_{ik} = \sum_j T_{ij}S_{jk}$
11-13.19 (11-10.19)	$\boldsymbol{T}\cdot\boldsymbol{a}$	inner product of a tensor $\boldsymbol{T}$ of the second order and a vector $\boldsymbol{a}$	vector with components $(\boldsymbol{T}\cdot\boldsymbol{a})_i = \sum_j T_{ij}a_j$
11-13.20 (11-10.20)	$\boldsymbol{T}:\boldsymbol{S}$	scalar product of two tensors $\boldsymbol{T}$ and $\boldsymbol{S}$ of the second order	scalar quantity $\boldsymbol{T}:\boldsymbol{S} = \sum_i \sum_j T_{ij}S_{ji}$

14	SPECIAL FUNCTIONS		
Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-14.1 (11-11.1)	$J_l(x)$	cylindrical Bessel functions (of the first kind)	solutions of $x^2y'' + xy' + \left(x^2 - l^2\right)y = 0$ $J_l(x) = \sum_{k=0}^{\infty} \frac{(-1)^k(x/2)^{l+2k}}{k!\Gamma(l+k+1)} \quad (l \geqslant 0)$ For Γ, see 11-14.19.
11-14.2 (11-11.2)	$N_l(x)$	cylindrical Neumann functions; cylindrical Bessel functions of the second kind	$N_l(x) = \lim_{k \to l} \frac{J_k(x) \cos k\pi - J_{-k}(x)}{\sin k\pi}$ $Y_l(x)$ is also used.
11-14.3 (11-11.3)	$H_l^{(1)}(x)$ $H_l^{(2)}(x)$	cylindrical Hankel functions; cylindrical Bessel functions of the third kind	$H_l^{(1)}(x) = J_l(x) + iN_l(x)$ $H_l^{(2)}(x) = J_l(x) - iN_l(x)$
11-14.4 (11-11.4)	$I_l(x)$ $K_l(x)$	modified cylindrical Bessel functions	solutions of $x^2y'' + xy' - \left(x^2 + l^2\right)y = 0$ $I_l(x) = i^{-l}J_l(ix)$ $K_l(x) = (\pi/2)i^{l+1}(J_l(ix) + iN_l(ix))$
11-14.5 (11-11.5)	$j_l(x)$	spherical Bessel functions (of the first kind)	solutions of $x^2y'' + 2xy' + \left[x^2 - l(l+1)\right]y = 0 \quad (l \geqslant 0)$ $j_l(x) = (\pi/2x)^{1/2}J_{l+1/2}(x)$
11-14.6 (11-11.6)	$n_l(x)$	spherical Neumann functions; spherical Bessel functions of the second kind	$n_l(x) = (\pi/2x)^{1/2}N_{l+1/2}(x)$ $y_l(x)$ is also used.
11-14.7 (11-11.7)	$h_l^{(1)}(x)$ $h_l^{(2)}(x)$	spherical Hankel functions; spherical Bessel functions of the third kind	$h_l^{(1)}(x) = j_l(x) + in_l(x) = (\pi/2x)^{1/2}H_{l+1/2}^{(1)}(x)$ $h_l^{(2)}(x) = j_l(x) - in_l(x) = (\pi/2x)^{1/2}H_{l+1/2}^{(2)}(x)$ Modified spherical Bessel functions are denoted $i_l(x)$ and $k_l(x)$ respectively; compare 11-14.4.
11-14.8 (11-11.8)	$P_l(x)$	Legendre polynomials	solutions of $\left(1 - x^2\right)y'' - 2xy' + l(l+1)y = 0$ $P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l}\left(x^2 - 1\right)^l \quad (l \in \mathbb{N})$

14 SPECIAL FUNCTIONS (*continued*)

Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-14.9 (*11-11.9*)	$P_l^m(x)$	associated Legendre functions	solutions of $$\left(1 - x^2\right)y'' - 2xy' + \left[l(l+1) - \frac{m^2}{1-x^2}\right]y = 0$$ $$P_l^m(x) = \left(1 - x^2\right)^{m/2} \frac{\mathrm{d}^m}{\mathrm{d}x^m} P_l(x)$$ $(l, m \in \mathbb{N}; m \leqslant l)$
11-14.10 (*11-11.10*)	$Y_l^m(\vartheta, \varphi)$	spherical harmonics	solutions of $$\frac{1}{\sin \vartheta} \frac{\partial}{\partial \vartheta}\left(\sin \vartheta \ \frac{\partial y}{\partial \vartheta}\right) +$$ $$\frac{1}{\sin^2 \vartheta} \frac{\partial^2 y}{\partial \varphi^2} + l(l+1)y = 0$$ $$Y_l^m(\vartheta, \varphi) = (-1)^m \left[\frac{(2l+1)}{4\pi} \frac{(l - |m|)!}{(l + |m|)!}\right]^{1/2} \times$$ $$P_l^{|m|}(\cos \vartheta)\mathrm{e}^{\mathrm{i}m\varphi}$$ $(l, |m| \in \mathbb{N}; |m| \leqslant l)$
11-14.11 (*11-11.11*)	$H_n(x)$	Hermite polynomials	solutions of $y'' - 2xy' + 2ny = 0$ $$H_n(x) = (-1)^n \mathrm{e}^{x^2} \frac{\mathrm{d}^n}{\mathrm{d}x^n} \mathrm{e}^{-x^2} \qquad (n \in \mathbb{N})$$
11-14.12 (*11-11.12*)	$L_n(x)$	Laguerre polynomials	solutions of $xy'' + (1-x)y' + ny = 0$ $$L_n(x) = \mathrm{e}^x \frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(x^n \mathrm{e}^{-x}\right) \qquad (n \in \mathbb{N})$$
11-14.13 (*11-11.13*)	$L_n^m(x)$	associated Laguerre polynomials	solutions of $xy'' + (m + 1 - x)y' + (n - m)y = 0$ $$L_n^m(x) = \frac{\mathrm{d}^m}{\mathrm{d}x^m} L_n(x) \qquad (m, n \in \mathbb{N}; m \leqslant n)$$
11-14.14 (*11-11.14*)	$F(a, b; c; x)$	hypergeometric functions	solutions of $x(1-x)y'' + [c - (a + b + 1)x]y' - aby = 0$ $F(a, b; c; x) =$ $$1 + \frac{ab}{c}x + \frac{a(a+1)b(b+1)}{2!c(c+1)}x^2 + \ldots$$

14 SPECIAL FUNCTIONS (*continued*)			
Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-14.15 (*11-11.15*)	$F(a; c; x)$	confluent hypergeometric functions	solutions of $$xy'' + (c - x)y' - ay = 0$$ $$F(a; c; x) = 1 + \frac{a}{c} x + \frac{a(a + 1)}{2!c(c + 1)} x^2 + \dots$$
11-14.16 (*11-11.16*)	$F(k, \varphi)$	(incomplete) elliptic integral of the first kind	$$F(k, \varphi) = \int_0^\varphi \frac{d\vartheta}{\sqrt{1 - k^2 \sin^2\vartheta}}$$ $K(k) = F(k, \pi/2) \quad (0 < k < 1)$ is the complete elliptic integral of the first kind.
11-14.17 (*11-11.17*)	$E(k, \varphi)$	(incomplete) elliptic integral of the second kind	$$E(k, \varphi) = \int_0^\varphi \sqrt{1 - k^2 \sin^2\vartheta}\, d\vartheta$$ $E(k) = E(k, \pi/2) \quad (0 < k < 1)$ is the complete elliptic integral of the second kind.
11-14.18 (*11-11.18*)	$\Pi(k, n, \varphi)$	(incomplete) elliptic integral of the third kind	$$\Pi(k, n, \varphi) = \int_0^\varphi \frac{d\vartheta}{\left(1 + n \sin^2\vartheta\right)\sqrt{1 - k^2 \sin^2\vartheta}}$$ $\Pi(k, n, \pi/2) \quad (0 < k < 1)$ is the complete elliptic integral of the third kind.
11-14.19 (*11-11.19*)	$\Gamma(x)$	gamma function	$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\, dt \quad (x > 0)$$ $\Gamma(n + 1) = n! \quad (n \in \mathbb{N})$
11-14.20 (—)	$B(x, y)$	beta function	$$B(x, y) = \int_0^1 t^{x-1}(1 - t)^{y-1}\, dt$$ $(x, y \in \mathbb{R}; x > 0, y > 0)$ $B(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x + y)$
11-14.21 (*11-11.20*)	$\mathrm{Ei}\, x$	exponential integral	$$\mathrm{Ei}\, x = \int_x^\infty \frac{e^{-t}}{t}\, dt$$

14 SPECIAL FUNCTIONS (concluded)			
Item No.	Sign, symbol, expression	Meaning	Remarks and examples
11-14.22 (11-11.21)	erf x	error function	$\mathrm{erf}\ x = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^x e^{-t^2}\, dt$ $\mathrm{erf}(\infty) = 1$ The function erfc $x = 1 - \mathrm{erf}\ x$ is called the complementary error function. In statistics the distribution function $\Phi(x) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^x e^{-t^2/2}\, dt$ is used.
11-14.23 (11-11.22)	$\zeta(x)$	Riemann zeta function	$\zeta(x) = \dfrac{1}{1^x} + \dfrac{1}{2^x} + \dfrac{1}{3^x} + \dots \quad (x > 1)$

International Standard	Corresponding Indian Standard	Degree of Equivalence
ISO 31-3 : 1992	IS 1890 (Part 3) : 1995 Quantities and units : Part 3 Mechanics (*second revision*)	Identical
ISO 31-4 : 1992	IS 1890 (Part 4) : 1982* Quantities, units and symbols : Part 4 Heat (*first revision*)	Technically Equivalent
ISO 31-5 : 1992	IS 1890 (Part 5) : 1995 Quantities and units : Part 5 Electricity and magnetism (*first revision*)	Identical
ISO 31-6 : 1992	IS 1890 (Part 6) : 1983* Quantities, units and symbols : Part 6 Light and related electromagnetic radiations	Technically Equivalent
ISO 31-7 : 1992	IS 1890 (Part 7) : 1995 Quantities and units : Part 7 Acoustics (*first revision*)	Identical
ISO 31-8 : 1992	IS 1890 (Part 8) : 1995 Quantities and units : Part 8 Physical chemistry and molecular physics (*first revision*)	Identical
ISO 31-9 : 1992	IS 1890 (Part 9) : 1995 Quantities and units : Part 9 Atomic and nuclear physics (*first revision*)	Identical
ISO 31-10 : 1992	IS 1890 (Part 10) : 1995 Quantities and units : Part 10 Nuclear reactions and ionizing radiation (first revision)	Identical
ISO 31-11 : 1992	IS 1890 (Part 11) : 1995 Quantities and units : Part 11 Mathematical signs and symbols for use in the physical science and technology (*second revision*)	Identical
ISO 31-12 : 1992	IS 1890 (Part 12) : 1995 Quantities and units: Part 12 Characteristic numbers (*first revision*)	Identical
ISO 31-13 : 1992	IS 1890 (Part 13), 1983* Quantities, units and symbols : Part 13 Solid state physics	Technically Equivalent

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This Indian Standard has been developed from Doc : No. MSD 01 (94)

Amendments Issued Since Publication

Amend No.	Date of Issue	Text Affected

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