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ISI HANDBOOK
FOR
STRUCTURAL ENGINEERS

No. 6

ISI HANDBOOK
FOR
STRUCTURAL ENGINEERS

6. APPLICATION OF PLASTIC THEORY IN DESIGN OF STEEL STRUCTURES

INDIAN STANDARDS INSTITUTION
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FOREWORD

This Handbook, which has been processed by the Structural Engineering Sectional Committee, SMBDC 7, the composition of which is given in Appendix D, had been approved for publication by the Structural and Metals Division Council and the Civil Engineering Division Council of ISI.

Steel, which is a very important basic raw materials for industrialization, had been receiving attention from the Planning Commission even from the very early stages of the country's First Five Year Plan period. The Planning Commission not only envisaged an increase in production capacity in the country, but also considered the question of even greater importance, namely, taking of urgent measures for the conservation of available resources. Its expert committees came to the conclusion that a good proportion of the steel consumed by the structural steel industry in India could be saved if higher efficiency procedures were adopted in the production and use of steel. The Planning Commission, therefore, recommended to the Government of India that the Indian Standards Institution should take up a Steel Economy Project and prepare a series of Indian Standard specifications and codes of practice in the field of steel production and utilization.

Over fifteen years of continuous study in India and abroad, and the deliberations at numerous sittings of committees, panels and study groups resulted in the formulation of a number of Indian Standards in the field of steel production, design and use, a list of which is given in Appendix E.

This Handbook which relates to the application of plastic theory in design of steel structures is intended to present the important principles and assumptions involved in the plastic method of structural analysis, and to provide illustrative examples for the guidance of the designer in the analysis of practical design problems.

The subject is introduced by considering the various limits of usefulness of a steel structure, the limits that are function (in part) of the mechanical properties of steel. Knowledge of these properties is used in Section A to show how the maximum strength of some simple structures may be computed. The historical development of the plastic theory of structures is also dealt with in brief.

Section B answers the question 'Why plastic design'. It is shown that stress is an inadequate design criterion for a large number of practical engineering structures. The experimental verification of the plastic theory (which bases the design of structures on the maximum strength) has also been indicated. The basic theoretical work is dealt with in Sections C and D. The concepts of plastic bending and redistribution of moments are described and the methods of analysis has been indicated. Section E contains general comments on design procedures. Although this section covers a few examples relating to multistorey frames, it is proposed to deal with the subject in detail in a supplement in due course. The limitations, modifications and design details have been described under the heading 'Secondary Design Consideration'. Proper attention should be given to the effect of shear force, axial force, local and lateral buckling, etc. Further, the beams, columns and connections should be designed to meet the requirements of plastic hinge formation.

The section on design examples treats a number of building frames of different profiles. The secondary design considerations are checked throughout. Section 7 describes simplified procedures of solving design problems with the use of formulas, charts and graphs.

In Appendix A is given a list of selected references for further detailed information on plastic theory of structures.

What will plastic design mean? To the 'sidewalk superintendent', it will mean nothing. The structure will look just the same as a conventionally designed structure. To the engineer, it will mean a more rapid method of analysis. To the owner, it will mean economy, because plastic design requires less steel than conventional design. For the building authority, it would mean more efficient operations because designs may be checked faster. To steel industry, it would mean more efficient use of its products. Finally, to a nation, it will mean better use of her natural resources.

This Handbook is based on and requires reference to the following publications issued by ISI:

- IS: 226-1969. Specification for structural steel (standard quality) (*fourth revision*)
- IS: 800-1962 Code of practice for use of structural steel in general building construction (*revised*)
- IS: 875-1964 Code of practice for structural safety of buildings: Loading standards (*revised*)
- IS: 2062-1969 Specification for structural steel (fusion welding quality) (*first revision*)
- IS: 4000-1967 Code of practice for assembly of structural joints using high tensile friction grip fasteners

In the preparation of this handbook, the technical committee has derived valuable assistance from Dr Lynn S. Beedle, Professor of Structural Engineering, Lehigh University, Bethlehem, USA. Dr Beedle prepared the preliminary draft of this handbook. This assistance was made available to ISI through Messrs Ramseyer & Miller, Inc, Iron, and Steel Industry Consultants, New York, by the Technical Co-operation Mission to India of the Government of India under their Technical Assistance Programme.

No handbook of this kind may be made complete for all times to come at the very first attempt. As designers and engineers begin to use it, they will be able to suggest modifications and additions for improving its utility. They are requested to send such valuable suggestions to ISI which will be received with appreciation and gratitude.

SYMBOLS

Symbols used in this handbook shall have the meaning assigned to them as indicated below:

A = Area of cross-section
A_F = Area of both flanges of WF shape
A_{st} = Area of split-tee
A_W = Area of web between flanges
b = Flange width
c = Distance from neutral axis to the extreme fibre
d = Depth of section
E = Young's modulus of elasticity
E_{st} = Strain-hardening modulus $= \dfrac{d\sigma_{st}}{d\epsilon_{st}}$
E_t = Tangent modulus
e = Eccentricity
F = Load factor of safety
f = Shape factor $= \dfrac{M_p}{M_y} = \dfrac{S}{Z}$
f = Fixity factor for use in evaluating and restraint coefficient
G = Modulus of elasticity in shear
G_{st} = Modulus of elasticity in shear at onset of strain-hardening
H = Hinge rotation required at a plastic hinge
H_B = Portion of hinge rotation that occurs in critical (buckling) segment of beam
I = Moment of inertia
I_e = Moment of inertia of elastic part of cross-section
I_p = Moment of inertia of plastic part of cross-section
J = Number of remaining redundancies in a structure that is redundant at ultimate load
K = Euler length factor
k = Distance from flange face to end of fillet
KL = Effective (pin end) length of column
L = Span length; actual column length
L_{cr} = Critical length for lateral buckling
M = Moment

m = Number of plastic hinges developed in a structure that is redundant at ultimate load

M_h = Moment at the haunch point

M_o = End moment; a useful maximum moment; hinge moment

M_p = Plastic moment

M_P = Plastic moment capacity of a beam section

M_{pc} = Plastic hinge moment modified to include the effect of axial compression

M_{ps} = Plastic hinge moment modified to include effect of shear force

M_s = Maximum moment of a simply-supported beam

M_y = Moment at which yield point is reached in flexure

M_{yc} = Moment at which initial outer fibre yield occurs when axial thrust is present

M_w = Moment at the working load

N = Number of possible plastic hinges

n = Number of possible independent mechanisms

P = Concentrated load

P_{cr} = Useful column load. A load used as the 'maximum column load'

P_e = Euler buckling load

P_r = Reduced modulus load

P_s = Stabilizing load

P_t = Tangent modulus load

P_u = Theoretical ultimate load

P_w = Working load

P_y = Axial load corresponding to yield stress level; $P = A\sigma_y$

R = Rotation capacity

r = Radius of gyration

S = Section modulus, I/c

S_e = Section modulus of elastic part of cross-section

T = Force

t_f = Flange thickness

t_s = Stiffner thickness

t_w = Web thickness

V = Shear force

V_C = Shear carrying capacity of a section

u, v, w = Displacements in x, y, and z directions

W = Total distributed load

W_{EXT} = External work due to virtual displacement
W_{INT} = Internal work due to virtual displacement
w = Distributed load per unit of length
wd = Thickness of the wet doublers
w_u = Total uniformity distributed load
X = Number of redundancies
x = Longitudinal coordinate
x = Distance to position of plastic hinge under distributed load
y = Transverse coordinate
y = Distance from neutral axis to centroid of half-area
Z = Plastic modulus $= \dfrac{M_p}{\sigma_y}$
Z_e = Plastic modulus of elastic portion
Z_p = Plastic modulus of plastic portion
z = Lateral co-ordinate
ΔL = Equivalent length of connection
δ = Deflection
ϵ = Strain
ϵ_{st} = Strain at strain-hardening
ϵ_y = Strain corresponding to first attainment of yield stress level
θ = Measured angle change; rotation. Rotation
μ = Poisson's ratio
ρ = Radius of curvature
σ = Normal stress
σ_{ly} = Lower yield point
σ_p = Proportion limit
σ_r = Residual stress
σ_{ult} = Ultimate tensilo strength of material
σ_{uy} = Upper yield point
σ_w = Working stress
σ_y = Yield stress level
τ = Shear stress
ϕ = Rotation per unit length, or average unit rotation; curvature
ϕ_y = Curvature corresponding to first yield in flexure

SECTION A

INTRODUCTION

1. SCOPE

1.1 It is the purpose of this handbook to present the fundamental concepts involved in plastic design and to justify its application to structural steel frames. The methods of plastic analysis will be described together with the design procedures that have so far been developed. Secondary design considerations are also included.

1.2 Specific application may be made to statically loaded frames of structural steel to continuous beams, to single-storeyed industrial frames and to such other structures whose condition of loading and geometry are consistent with the assumptions involved in the theory. Numerous applications will undoubtedly be made to other types of structures such as rings and arches, but for the time being the scope of application is limited to the indicated structural types.

2. GENERAL

2.1 Steel possesses ductility, a unique property that no other structural material exhibits in quite the same way. Through ductility structural steel is able to absorb large deformations beyond the elastic limit without the danger of fracture.

2.2 Although there are a few instances where conscious use has been made of this property, by and large the engineer has not been able to fully exploit this feature of ductility in structural steel. As a result of these limitations it turns out that considerable sacrifice of economy is involved in the so-called 'conventional' design procedures.

2.3 Engineers have known of this ductility for years, and since the 1920's have been attempting to see if some conscious use could be made of this property in design. Plastic design is the realization of that goal. This goal has been achieved because two important conditions have been satisfied. First, the theory concerning the plastic behaviour of continuous steel frames has been systematized and reduced to simple design procedures. Secondly, every conceivable factor that might tend to limit the load-carrying capacity to something less than that predicted by the simple plastic theory has been investigated and rules have been formulated to safeguard against such factors.

3. STRUCTURAL STRENGTH

3.1 The design of any engineering structure, be it a bridge or building, is satisfactory if it is possible to built it with the needed economy and if throughout its useful life it carries its intended loads and otherwise performs its intended function. As already mentioned, in the process of selecting suitable members for such a structure, it is necessary to make a general analysis of structural strength and secondly to examine certain details to assure that local failure does not occur.

3.2 The ability to carry the load may be termed 'structural strength'. Broadly speaking, the structural strength or design load of a steel frame may be determined or controlled by a number of factors, factors that have been called 'limits of structural usefulness'. These are: first attainment of yield point stress (conventional design), brittle fracture, fatigue, instability, deflections, and finally the attainment of maximum plastic strength.

3.3 Strictly speaking, a design based on any one of the above-mentioned six factors could be referred to as a 'limit design', although the term usually has been applied to the determination of ultimate load as limited by buckling or maximum strength[1*]. 'Plastic design' as an aspect of limit design and as applied to continuous beams and frames embraces, then, the last of the limits — the attainment of maximum plastic strength.

3.4 Thus, plastic design is first a design on the basis of the maximum load the structure will carry as determined from an analysis of strength in the plastic range (that is, a plastic analysis). Secondly it consists of a consideration by rules or formulas of certain factors that might otherwise tend to prevent the structure from attaining the computed maximum load. Some of these factors may be present in conventional (elastic) design. Others are associated only with the plastic behaviour of the structure. But the unique feature of plastic design is that the *ultimate load* rather than the *yield stress* is regarded as the design criterion.

3.5 It has long been known that whenever members are rigidly connected, the structure has a much greater load-carrying capacity than indicated by the elastic stress concept. Continuous or 'rigid' frames are able to carry increased loads above 'first yield' because structural steel has the capacity to yield in a ductile manner with no loss in strength;

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*This number refers to the serial number of the selected references given in Appendix A.

indeed, with frequent increase in resistance. Although the phenomenon will be described in complete detail later, in general terms what happens is this:

> As load is applied to the structure, the cross-section with the greatest bending moment will eventually reach the yield moment. Elsewhere the structure is elastic and the 'peak' moment values are less than yield. As load is added, a zone of yielding develops at the first critical section; but due to the ductility of steel, the moment at that section remains about constant. The structure, therefore, calls upon its less-heavily stressed portions to carry the increase in load. Eventually, zones of yielding are formed at other sections until the moment capacity has been exhausted at all necessary critical sections. After reaching the maximum load value, the structure would simply deform at constant load.

3.6 At the outset it is essential to make a clear distinction between elastic design and plastic design. In conventional elastic design practice, a member is selected such that the maximum allowable bending stress is equal to 1 650 kgf/cm² at the working load. As shown in Fig. 1 such a beam has a reserve of strength of 1·65 if the yield point stress is 2 400 kgf/cm². Due to the ductility of steel there is an

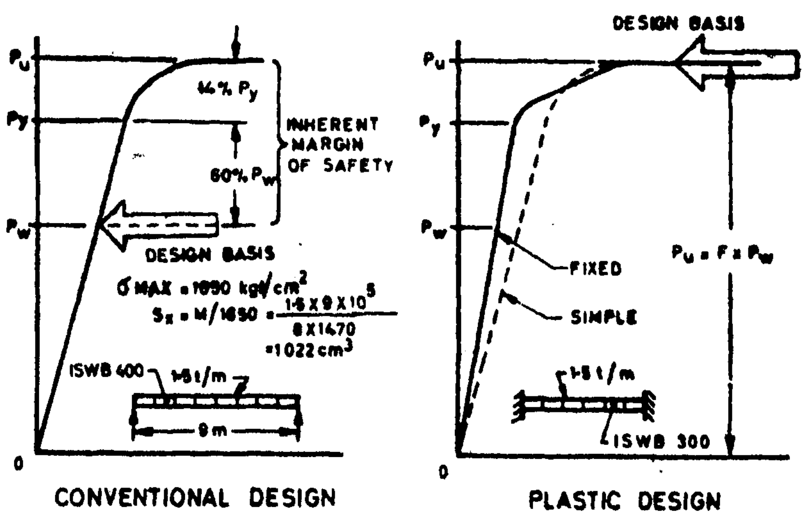

Fig. 1 Plastic Design Compared with Elastic Design

additional reserve which amounts to 12 to 14 percent for a wide flange shape. Thus the total inherent overload factor of safety is equal to $1{\cdot}65 \times 1{\cdot}12 = 1{\cdot}85$ as an average value.

3.7 In plastic design, on the other hand, the design commences with the *ultimate* load. (As will be evident later, it is much easier to analyze an indeterminate structure for its ultimate load than to compute the yield load.) Thus the working load, P_w, is multiplied by the same load factor (1·85) and a member is selected that will reach this factored load.

3.8 The load *v* deflection curve for the restrained beam is shown in Fig. 1. It has the same ultimate load as the conventional design of the simple beam and the member is elastic at working load. The important thing to note is that the factor of safety is the same in the plastic design of the indeterminate structure as it is in the conventional design of the simple-beam.

3.9 While there are other features here, the important point to get in mind at this stage is that in conventional procedures one computes the maximum moment under the *working load* and selects a member such that the maximum stress is not greater than 1 650 kgf/cm² on the other hand in plastic design one multiples the working load by $F = 1{\cdot}85$ and selects a member which will just support the *ultimate load*.

3.10 Terminology — Plastic design naturally involves the use of some new terms. Actually these are few in number, but for convenience are listed below:

Limit Design — A design based on any chosen limit of structural usefulness.

Plastic Design — A design based upon the ultimate load-carrying capacity (maximum strength) of the structure. The term ' plastic ' is derived from the fact that the ultimate load is computed from a knowledge of the strength of steel in the plastic range.

Ultimate Load (P_u) *or Maximum Strength* — The highest load a structure will carry. (It is *not* to be confused with the term as applied to the ultimate load carried by an ordinary tensile test specimens.) In the design P_u is determined by multiplying the expected working load (P_w) by the load factor (*see* below).

Plastification — The development of full plastic yield of the cross-section.

Plastic Moment (M_p)— Maximum moment of resistance of a fully yielded cross-section.

Plastic Modulus *(Z)*— Combined static moments about the neutral axis of the cross-sectional areas above and below the neutral axis.

Plastic Hinge — A yielded section of a beam which acts as if it were hinged, except with a constant restraining moment.

Shape Factor *(f)*— The ratio of the maximum resisting moment of a cross-section (M_p) to the yield moment (M_y).

Mechanism — A 'hinge system', a system of members that can move without an increase in load.

Redistribution of Moment — A process which results in the successive formation of plastic hinges until the ultimate load is reached. By it, the less-highly stressed portions of a structure also may reach the (M_p)-value.

Load Factor *(F)*— A safety factor. The term is selected to emphasize the dependence upon load-carrying capacity. It is the number by which the working load is multiplied to obtain P_u.

4. MECHANICAL PROPERTIES OF STEEL

4.1 An outstanding property of steel, which (as already mentioned) sets it apart from other structural materials, is the amazing ductility which it possesses. This is characterized by Fig. 2 which shows in somewhat idealized form the stress-strain properties of steel in the initial portion of the curve. In Fig. 3 are shown partial tensile stress-strain curves for a number of different steels. Note that when the elastic limit is reached, elongations from 8 to 15 times the elastic limit take place without any decrease in load. Afterwards some increase in strength is exhibited as the material strain hardens.

4.2 Although the first application of plastic design is to structures fabricated of structural grade steel, it is not less applicable to steels of

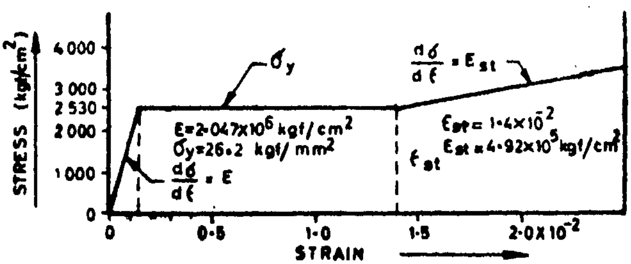

FIG. 2 STRESS-STRAIN CURVE OF ST-42 STEEL IDEALIZED

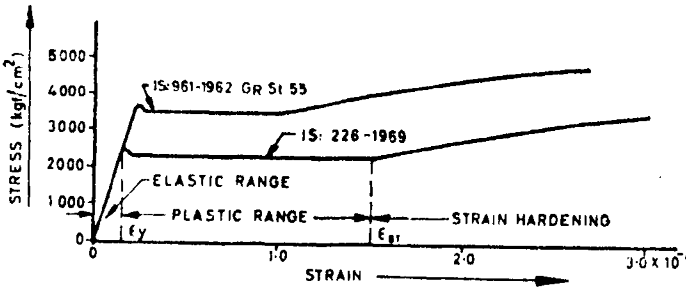

FIG. 3 STRESS-STRAIN CURVE OF ST-42 AND St-55 STRUCTURAL STEELS

higher strength as long as they possess the necessary ductility. Figure 3 attests to the ability of a wide range of steels to deform plastically with characteristics similar to steel conforming to IS: 226-1969*.

4.3 It is important to bear in mind that the strains shown in Fig. 3 are really very small. As shown in Fig. 4, for ordinary structural steel, final failure by rupture occurs only after a specimen has stretched some

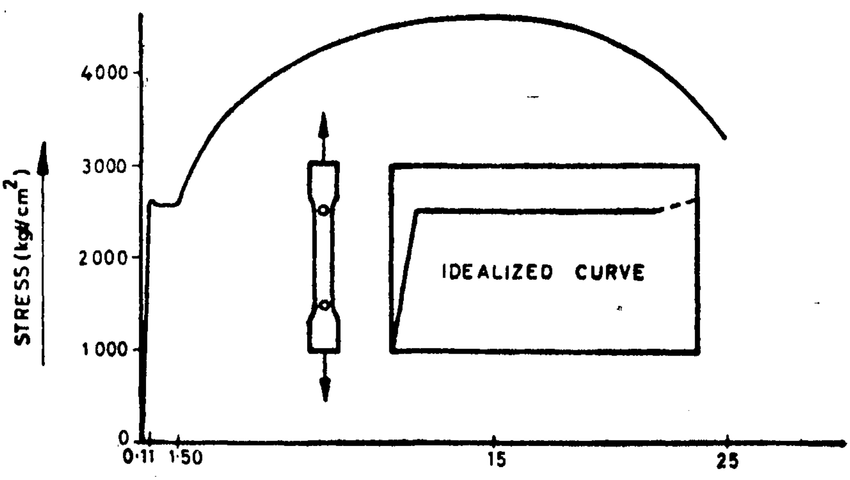

FIG. 4 COMPLETE STRESS-STRAIN CURVE OF STRUCTURAL STEEL

*Structural steel (standard quality) (*fourth revision*).

15 to 25 times the maximum strain that is encountered in plastic design. Even in plastic design, at ultimate load the critical strains will not have exceeded percentage elongation of about 1·5. Thus, the use of ultimate load as the design criterion still leaves available a major portion of the reserve ductility of steel which may be used as an added margin of safety. This maximum strain of 1·5 percent is a strain at ultimate load in the structure not at working load. In most cases under working load the strains will still be below the elastic limit.

5. MAXIMUM STRENGTH OF SOME ELEMENTS

5.1 On the basis of the ductility of steel (characterized by Fig. 2) it is now possible to calculate quickly the maximum carrying capacity of certain elementary structures.

As a first example take a tension member such as an eye bar (Fig. 5). The stress is $\sigma = P/A$.

The load *v* deflection relationship will be elastic until the yield point is reached. As shown in Fig. 5 deflection at the elastic limit is given by $\delta_y = P_u L/AE$.

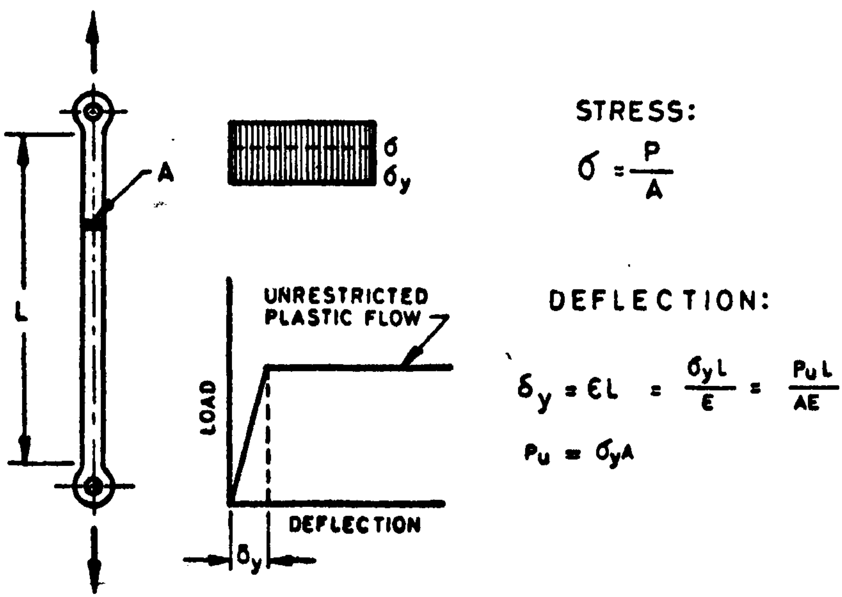

FIG. 5 MAXIMUM STRENGTH OF AN EYE BAR (DETERMINATE STRUCTURE)

Since the stress distribution is uniform across the section, unrestricted plastic flow will set in when the load reached the value given by

$$P_u = \sigma_y A$$

This is, therefore, the ultimate load. It is the maximum load the structure will carry without the onset of unrestricted plastic flow.

As a second example consider the three-bar structure shown in Fig. 6. It is not possible to consider the state of stress by statics alone and thus it is indeterminate. Consider the elastic state. From the equilibrium condition there is obtained:

$$2T_1 + T_2 = P \qquad \dots \qquad \dots \qquad \dots \qquad \dots(1)$$

where T_1 is the force in bars 1 and 3 and T_2 the force in the bar 2.

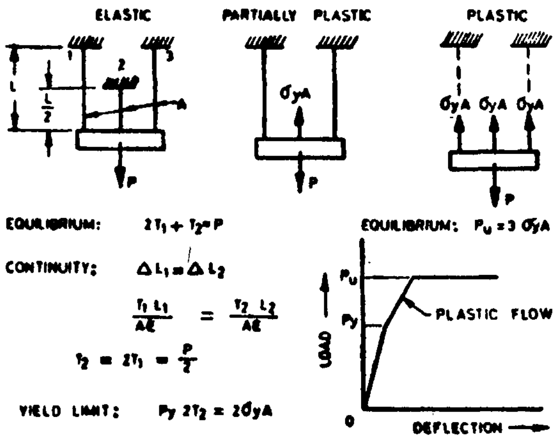

FIG. 6 PLASTIC AND ELASTIC ANALYSIS OF AN INDETERMINATE SYSTEM

The next condition to consider is continuity. For a rigid cross bar, the total displacement of Bar 1 will be equal to that of Bar 2. Therefore:

$$\frac{T_1 L_1}{AE} = \frac{T_2 L_2}{AE} \qquad \dots \qquad \dots \qquad \dots \qquad \dots(2)$$

$$T_1 = \frac{T_2}{2} \text{ (as } L_1 = 2L_2\text{)} \qquad \dots \qquad \dots \qquad \dots(3)$$

With this relationship between T_1 and T_2 obtained by the continuity condition, using Eq (1) it is found that:

$$T_2 = \frac{P}{2} \qquad \dots \qquad \dots \qquad \dots \qquad \dots \qquad \dots(4)$$

The load at which the structure will first yield may then be determined by substituting in Eq (4) the maximum load which T_2 can reach, namely, $\sigma_y A$.
Thus,

$$P_y = 2T_2 = 2\sigma_y A \qquad \ldots \qquad \ldots \qquad \ldots(5)$$

The displacement at the yield load would be determined from:

$$\delta_y = \sigma_y{}^{-1} L_2 = \frac{\sigma_y L}{2E} \qquad \ldots \qquad \ldots \qquad \ldots(6)$$

Now, when the structure is partially plastic it deforms as if it were a two-bar structure except that a constant force equal to $\sigma_y A$ is supplied by Bar 2 (the member is in the plastic range). This situation continues until the load reaches the yield value in the two outer bars. Notice how easily it is possible to compute the ultimate load:

$$P_u = 3\sigma_y A \qquad \ldots \qquad \ldots \qquad \ldots(7)$$

The basic reason for this simplicity is that the continuity condition need not be considered when the ultimate load in the plastic range is being computed.

The load-deflection relationship for the structure shown in Fig. 6 is indicated at the bottom. Not until the load reaches that value computed by a plastic analysis (Eq 7) did the deflections commence to increase rapidly. The deflection when the ultimate load is first reached can be computed from:

$$\delta_u = \sigma_y{}^{-} L_1 = \frac{\sigma_y L}{E} \qquad \ldots \qquad \ldots \qquad \ldots(8)$$

The three essential features of this simple plastic analysis are as follows:

a) Each portion of the structure (each bar) reached a plastic yield condition,

b) The equilibrium condition was satisfied at ultimate load, and

c) There was unrestricted plastic flow at the ultimate load.

These same features are all that are required to complete the plastic analysis of an indeterminate beam or frame, and in fact, this simple example illustrates all of the essential features of a plastic analysis.

6. HISTORICAL DEVELOPMENT

6.1 The concept of design based on ultimate load as the criterion is more than 40 years old ! The application of plastic analysis to structural design appears to have been initiated by Dr Gabor Kazinszy, a Hungarian, who published results of his Tests[2] of Clamped Girders as early as

1914. He also suggested analytical procedures similar to those now current, and designs of apartment-type buildings were actually carried out.

6.2 In his Strength of Materials[3], Timoshenko refers to early suggestions to utilize ultimate load capacity in the plastic range and states as follows:

> Such a procedure appears logical in the case of steel structures submitted to the action of stationary loads, since in such cases a failure owing to the fatigue of metal is excluded and only failure due to the yielding of metals has to be considered.

Early tests in Germany were made by Maier-Leibnitz[4] who showed that the ultimate capacity was not affected by settlement of supports of continuous beams. In so doing he corroborated the procedures previously developed by others for the calculation of maximum load capacity. The efforts of Van den Broek[1] in USA and J. F. Baker[6,10] and his associates in Great Britain to utilize actually the plastic reserve strength as a design criterion are well known. Progress in the theory of plastic structural analysis (particularly that at Brown University) has been summarized by Symonds and Neal[7].

6.3 For more than ten years the American Institute of Steel Construction, the Welding Research Council, the Navy Department and the American Iron and Steel Institute have sponsored studies at Lehigh University[5,8,9]. These studies have featured not only the verification of this method of analysis through appropriate tests on large structures, but have given particular attention to the conditions that should be met to satisfy important secondary design requirements.

6.4 Plastic design has now 'come of age'. It is already a part of the British Standard specifications and numerous structures both in Europe and North America have been constructed to designs based upon the plastic method. IS: 800-1962* permits the use of Plastic Theory in the design of steel structures (*see* **13.5.1** of IS: 800-1962*).

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*Code of practice for use of structural steel in general building construction (*revised*).

SECTION B

JUSTIFICATION FOR PLASTIC DESIGN

7. WHY PLASTIC DESIGN

7.1 What is the justification for plastic design? One could reverse the question by asking, 'why use elastic design?' If the structure will support the load and otherwise meet its intended function, are the magnitudes of the stresses really important?

7.2 It is true that in simple structures the concept of the hypothetical yield point as a limit of usefulness is rational. This is because the ultimate load capacity of a simple beam is but 10 to 15 percent greater than the hypothetical yield point, and deflections start increasing very rapidly at such a load. While it would seem logical to extend elastic stress analysis to indeterminate structures, such procedures have tended to overemphasize the importance of stress rather than strength as a guide in engineering design and have introduced a complexity that now seems unnecessary for a large number of structures.

7.3 Actually the idea of design on the basis of ultimate load rather than allowable stress is a return to the realistic point of view that had to be adopted by our forefathers in a very crude way because they did not possess knowledge of mathematics and statics that would allow them to compute stresses.

7.4 The introduction of welding, of course, has been a very real stimulus to studies of the ultimate strength of frames. By welding it is possible to achieve complete continuity at joints and connections — and to do it economically. The full strength of one member may thus be transmitted to another.

7.5 It has often been demonstrated that elastic stress analysis cannot predict the real stress-distribution in a building frame with anything like the degree of accuracy that is assumed in the design. The work done in England by Prof. Baker and his associates as a forerunner to their ultimate strength studies clearly indicated this.

7.6 Examples of 'imperfections' that cause severe irregularity in measured stresses are: differences in beam-column connection fit-up and flexibility, spreading of supports, sinking of supports, residual stresses, flexibility assumed where actually there is rigidity (and *vice-versa*), and points of stress concentration. Such factors, however, usually do not influence the maximum plastic strength.

7.7 Assuming that stress is not the most rational design criteria, in order to justify our further consideration of maximum strength as the design criteria there must be other advantages. There are two such advantages: economy and simplicity.

7.8 Since there is considerable reserve of strength beyond the elastic limit and since the corresponding ultimate load may be computed quite accurately, then structural members of smaller size will adequately support the working loads when design is based on maximum strength. Numerous demonstrations of this will be made later in this bandbook.

7.9 The second feature was 'simplicity'. An analysis based upon ultimate load possesses an inherent simplicity because the elastic condition of continuity need no longer be considered. This was evident from a consideration of the three-bar truss in Section A (Fig. 6) and the examples of Section D will demonstrate this further. Also the 'imperfections' mentioned above usually may be disregarded.

7.10 As already mentioned the concept is more rational. By plastic analysis the engineer can determine with an accuracy that far exceeds his presently available techniques the real maximum strength of a structure. Thereby the factor of safety has more real meaning than at present. It is not unusual for the factor of safety to vary from 1·65 up to 3 or more for structures designed according to conventional elastic methods.

7.11 Thus the application of plastic analysis should be considered seriously because it provides a less-expensive structure, it is a similar design office technique, and it constitutes a rational design basis. Further, these concepts are verified by tests and (as we shall now see) they have been used consciously or unconsciously in conventional design practice.

8. INADEQUACY OF STRESS AS THE DESIGN CRITERION

8.1 The question immediately arises, will it not be possible simply to change the allowable stress and retain the present stress concept? While theoretically possible, the practical answer is 'no'. It would mean a different working stress for every type of structure and would vary for different loading conditions.

8.2 To a greater extent than we may realize, the maximum strength of a structure has always been the dominant design criteria. When the permissible working stress of 1 400 kg/cm² has led to designs that were consistently too conservative, then that stress has been changed. Thus the benefits of plasticity have been used consciously or unconsciously in design. It is also evident to most engineers that present design procedures completely disregard local over-stressing at points of stress-concentration like bolt holes, notches, etc. Long experience with similar structures so designed shows that this is a safe procedure. Thus, the stresses that are calculated for design purposes are not true maximum stresses at all, they simply provide an index for structural design.

8.3 A number of examples will now be given in which the ductility of steel has been counted upon (knowingly or not) in elastic design. It should be borne in mind that plastic analysis has not generally been used as a basis for determining these particular design rules and as a result the so-called elastic stress formulas have been devised in a rather haphazard fashion. A rational basis for the design of a complete steel frame (as well as its details) can only be attained when the maximum strength in the plastic range is adopted as the design criterion.

8.4 Such examples are the following and are listed in two categories: (a) factors that are neglected because of the compensating effect of ductility; and (b) instances in which the working stresses have been revised because the 'normal' value was too conservative. Several examples of each are given:

a) *Factors that are neglected*:

1) Residual stresses (in the case of flexure due to cooling after rolling);
2) Residual stresses resulting from the cambering of beams;
3) Erection stresses;
4) Foundation settlements;
5) Over-stress at points of stress-concentration (holes, etc);
6) Bending stresses in angles connected in tension by one leg only;
7) Over-stress at points of bearing;
8) Non-uniform stress-distribution in splices, leading to design of connections on the assumption of a uniform distribution of stresses among the rivets, bolts, or welds;
9) Difference in stress-distribution arising from the 'cantilever' as compared with the 'portal' method of wind stress analysis;

10) IS: 800-1962 specifies the following values for bending stresses:
 1 650 kgf/cm² for rolled sections,
 1 575 kgf/cm² for plate girders, and
 1 890 kgf/cm² in flat bases.

b) *Revisions in working stress due to reserve plastic strength:*

1) Bending stress of 2 109 kgf/cm² (or 30 ksi) in round pins (in AISC specification);
2) Bearing stress of 2 812 kgf/cm² (or 40 ksi) on pins in double shear;
3) Bending stress of 1 687 kgf/cm² (or 24 ksi) in framed structures at points of interior supports;
4) Bending stress of 1 650 kgf/cm² and 1 575 kgf/cm² for rolled sections and plate girders respectively (in IS: 800-1962*); and
5) Bending stress of 1 890 kgf/cm² in slab bases (in IS: 800-1962*).

Consider Item (a) (1) for example. All rolled members contain residual stresses that are formed due to cooling after rolling or due to cold-straightening. A typical wide flange shape with a typical residual stress pattern shown in Fig. 7. When load carrying bending stresses are applied, the resulting strains are additive to the residual strains already present. As a result, the 'final stress' could easily involve yielding at working load. In the example of Fig. 7, such yielding has

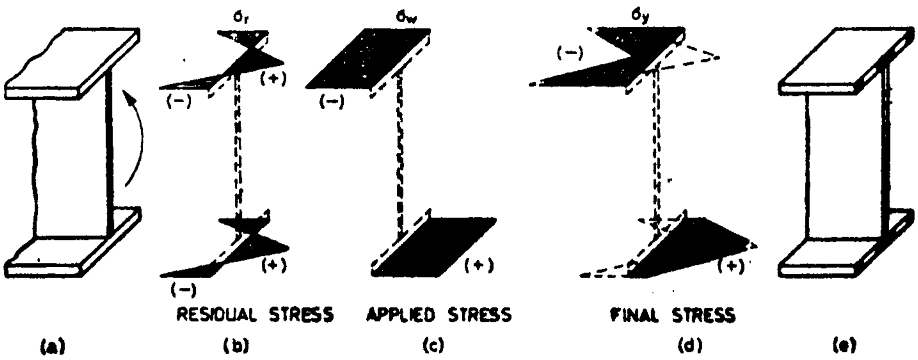


FIG. 7 RESIDUAL STRESSES IN A ROLLED BEAM SECTION

*Code of practice for use of structural steel in general building construction (*revised*).

occurred both at the compression flange tips and at the centre of the tension flange. Thus, it is seen that cooling residual stresses (whose influence is neglected and yet which are present in all rolled beams) cause yielding in the flange tips even below the working load.

8.5 Structural members experience yield while being straightened in the mill, fabricated in a shop or forced into position during erection. Actually, it is during these three operations that ductility of steel beyond the yield point is called upon to the greatest degree. Having permitted such yielding in the mill, shop and field, there is no valid basis to prohibit it thereafter, provided such yield has no adverse effect upon the structure. As an illustration of item (a) (3) in the list in **8.4**, Fig. 8 shows how erection forces will introduce bending moment into a structure prior to the application of external load (*see* first line for $P = 0$). Although the yield-point load is reduced as a result of these 'erection moments' (in the second line of the figure, the yield-point load has been reached for case 2), *there is no effect whatever on the maximum strength.* The reason for this is that redistribution of moment followed the onset of yielding at the corners (case 2) until the plastic moment was reached at the beam centre; therefore, the ultimate load moment diagrams for cases 1 and 2 are identical.

8.6 Consider, next, the design of a riveted or bolted joint [Item (a) (8) in **8.4**]. The common assumption is made that each fastener carries the

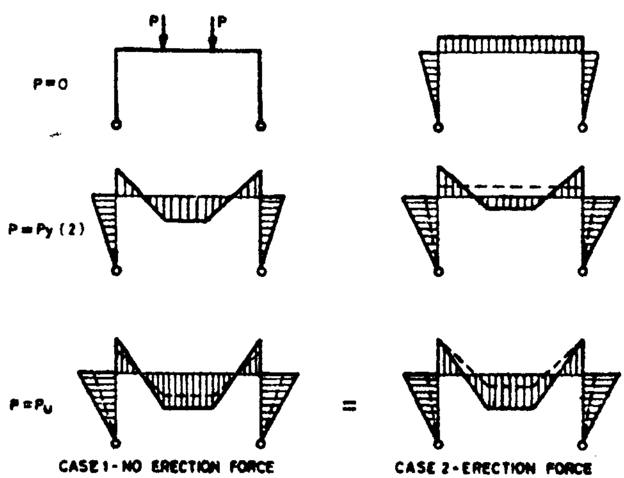


FIG. 8 DEMONSTRATION THAT ERECTION STRESSES DO NOT INFLUENCE ULTIMATE LOAD

same shear force. This is true only in the case of two fasteners. When more are added (Fig. 9), then as long as the joint remains elastic, the outer fastners should carry the greater portion of the load. For example, with four rivets, if each rivet transmitted the same load, then, between rivets C and D one plate would carry perhaps three times the force in the other. Therefore, it would stretch three times as much and would necessarily force the outer rivet D to carry more load. The actual forces would look something like these shown under the heading ' Elastic '. What eventually happens is that the outer rivets yield, redistributing forces to the inner rivets until all forces are about equal. Therefore, the basis for design of a rivet joint is really its ultimate load and not the attainment of first yield.

8.7 A ' revised working stress ' example [*see* Item (b) (1) in **8.4**] is shown in Fig. 10 and is concerned with the design of a round pin. In a simple beam with wide flanges, when the maximum stress due to bending reaches the yield point, most of the usable strength has been exhausted. However, for some cross-sectional shapes, much additional load may be carried without excessive deflections. The relation between bending moment and curvature for wide flange and round beams is shown in Fig. 10. The upper curve is for the pin, the lower for a typical wide flange beam, the non-dimensional plot being such that the two curves coincide in the elastic range. The maximum bending strength of the wide flange beam is 1·14 M_y, whereas that of the pin is 1·70 M_y. The permissible design stresses (for steel with yield stress 36 ksi) according to specifications of the American Institute of Steel Construction are 1 550 kgf/cm² (or 22 ksi) for the wide flange beam and 2 320 kgf/cm² (or 30 ksi) for the round pin.

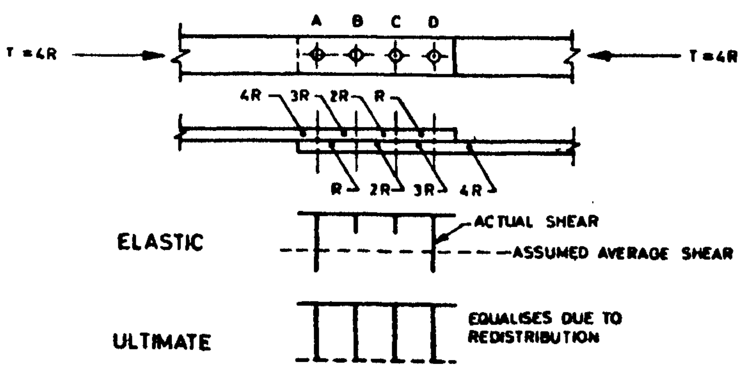


Fig. 9 Redistribution of Shear in the Fasteners of a Lap Joint

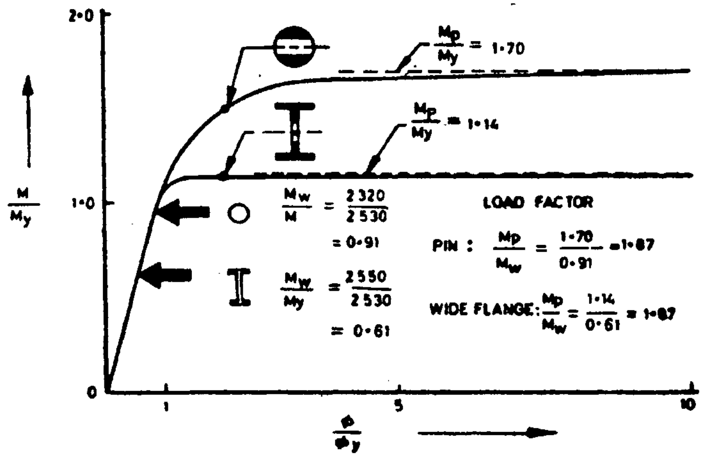


Fig. 10 Maximum Strength of a Round Pin Compared with that of a Wide Flange Beam

Expressing these stresses as ratios of yield point stress:

$$\text{Wide flange:} \quad \frac{\sigma_w}{\sigma_y} = \frac{1\,550}{2\,530} = 0{\cdot}61$$

$$\text{Pin:} \quad \frac{\sigma_w}{\sigma_y} = \frac{2\,320}{2\,530} = 0{\cdot}91$$

For a simply-supported beam the stresses, moments, and load all bear a linear relationship to one another in the elastic range and thus:

$$\frac{P}{P_y} = \frac{\sigma}{\sigma_y} = \frac{M}{M_y}$$

Therefore, the moment at allowable working stress (M_w) in the wide flange beam is 0·61 M_y; for the pin, on the other hand, $M_w = 0{\cdot}91\ M_y$. What is the true load factor of safety for each case?

$$\text{Wide flange:}\ F = \frac{P_{Max}}{P_w} = \frac{M_{Max}}{M_w} = \frac{1{\cdot}14\ M_y}{0{\cdot}61\ M_y} = 1{\cdot}87$$

$$\text{Pin:} \quad F = \frac{P_{Max}}{P_w} = \frac{1{\cdot}70\ M_y}{0{\cdot}91\ M_y} = 1{\cdot}87$$

The exact agreement between the true factors of safety with respect to ultimate load in the two cases, while somewhat of a coincidence, is indicative of the influence of long years of experience on the part of engineers which has resulted in different permissible working stresses for various conditions resulting in practice. Probably no such analysis as the foregoing influenced the choice of different unit stresses that give identical factors of safety with various sections; nevertheless, the choice of such stresses is fully justified on this basis. When years of experience and common sense have led to certain empirical practices these practices are usually justified on a scientific basis.

8.8 Permitting a 20 percent increase in the allowable working stress at points of interior support in continuous beams represents another case in which both experience ***and*** a 'plastic analysis' justify a revision in working stresses.

9. EXPERIMENTAL VERIFICATION

9.1 In the previous clauses some of the important concepts of the plastic theory are described. How well does structural behaviour bear out the theory? Do structures really contain the ductility assumed? If we test a 'full size' structure with rolled members will it actually carry the load predicted by plastic analysis?

9.2 The important assumptions made with regard to the plastic behaviour of structures are recapitulated in Fig. 11). In Lecture 4 of Ref 12 (*see* Appendix A), the experimental confirmation of these assumptions is given, demonstrating the ductility of steel, the development of plastic hinges in beams and connections, and redistribution of moment. In the last analysis, the most important verification of plastic theory is that given by the results of full-scale tests and some of these will now be presented.

9.3 Typical structures were tested both in USA and other countries. The structure carried the predicted ultimate load, the load-deflection curve being shown in Fig. 12.

9.4 Further tests conducted on frames fabricated from rolled sections have shown that the actual strength of even the weakest structure was within 5 percent of its predicted ultimate load an agreement much better than obtained at the so-called 'elastic limit'[4,15,20,39,40,41]. In tests on beams with three supports, applying the vertical load, the central support was raised until the yield point was first reached, with the result that application of the first increment of external load caused the structure to yield. In spite of this, the computed ultimate load was attained. In the tests conducted on pinned and fixed basis and with flat, saw tooth and gabled roofs, the ultimate load computed by the plastic theory was reached and in numerous cases it exceeded[42,43,44].

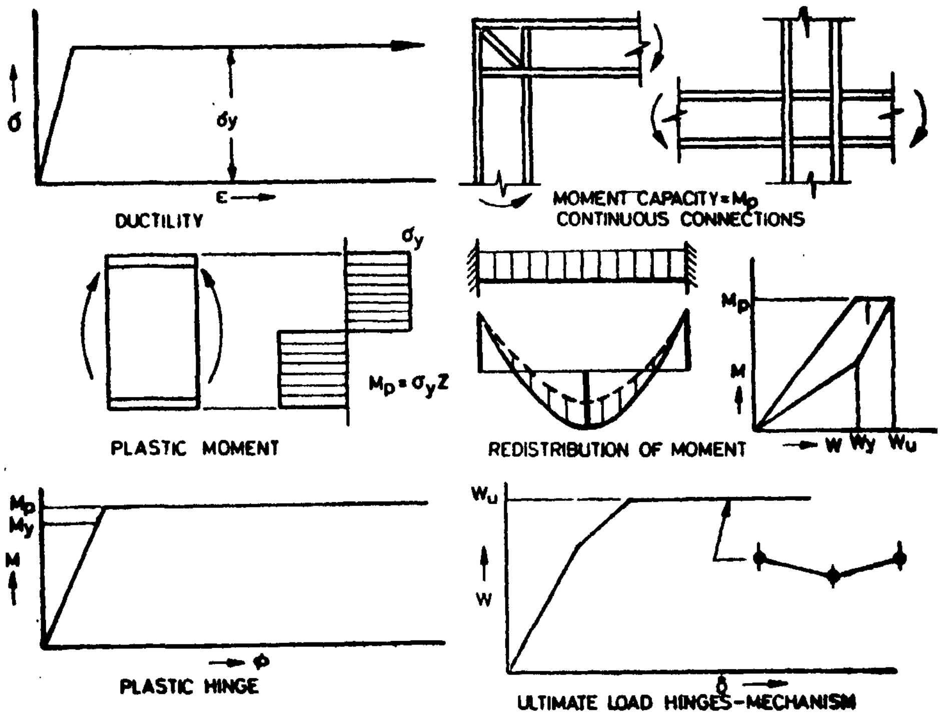


FIG. 11 ASSUMPTIONS MADE IN REGARD TO PLASTIC BEHAVIOUR OF STRUCTURES

10. THE CASE FOR PLASTIC DESIGN

10.1 As summarized in the preceding paragraphs the results of tests have verified the theory of plastic analysis. Is the engineer now justified in giving further attention to the method of plastic analysis, in studying it, and in applying it to the appropriate design problems? The answer is 'yes'.

The case for plastic design is illustrated by the following observations:

a) The reserve in strength above conventional working loads is considerable in indeterminate steel structures. Indeed, in some instances as much load-carrying capacity is disregarded as is used in conventional design.

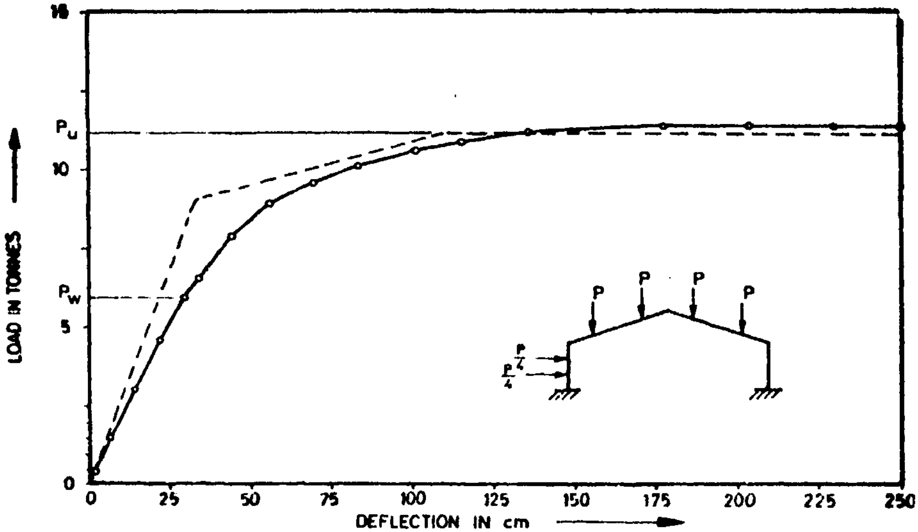


Fig. 12 Load-Deflection Curve of a Test Frame

b) Use of ultimate load as the design criterion provides at least the same margin of safety as is presently afforded in the elastic design of simple beams (Fig. 1).

c) At working load the structure is still in the so-called elastic range (Fig. 1).

d) In most cases, a structure designed by the plastic method will deflect no more at working load than will a simply-supported beam designed by conventional methods to support the same load (Fig. 1).

e) Plastic design gives promise of economy in the use of steel, of savings in the design office by virtue of its simplicity, and of building frames more logically designed for greater over-all strength.

It is important to bear in mind that dependence may be placed upon the maximum plastic strength only when proper attention is given to 'details'. These are the secondary design considerations mentioned earlier and treated in Section E.

SECTION C

FLEXURE OF BEAMS

11. ASSUMPTIONS AND CONDITIONS

11.1 Certain of the fundamental concepts of plastic analysis were presented in Section A (*see* **3** and **4**). The examples there, however, were limited to cases of simple tension and compression. The next objective is to determine how a beam deforms beyond the elastic limit under the action of bending moments, that is, to determine the moment-curvature ($M-\phi$) relationship.

The assumptions and conditions used in the following development are:

a) strains are proportional to the distance from the neutral axis (plane sections under bending remain plane after deformation).

b) the stress-strain relationship is idealized to consist of two straight lines:

$$\left.\begin{array}{ll}\sigma = E & (0<\epsilon<\epsilon_y)\\ \sigma = \sigma_y & (\epsilon_y<\epsilon<\infty)\end{array}\right\} \quad \ldots(9)$$

The complete stress-strain diagram is shown in Fig. 4 and is shown in an idealized form in Fig. 2. The properties in compression are assumed to be the same as those in tension. Also, the behaviour of fibres in bending is the same as in tension.

c) The equilibrium conditions are as given by Eq (10):

$$\left.\begin{array}{ll}\text{Normal force:} & P = \int_{Area} \sigma dA\\ \text{Moment:} & M = \int_{Area} \sigma dA.y\end{array}\right\} \quad \ldots(10)$$

where σ is the stress at distance y from the neutral axis.

d) Deformations are sufficiently small so that $\phi = \tan\phi$ (ϕ = curvature).

12. BENDING OF RECTANGULAR BEAM

12.1 The moment-curvature relationship in the plastic range and the magnitude of the maximum plastic moment are developed by following the same processes as in elastic analysis, that is, consider the deformed structure and obtain the corresponding curvature and moment. The development of strain and stress distribution as a rectangular beam is bent in successive stages beyond the elastic limit (Stage 1) and up to

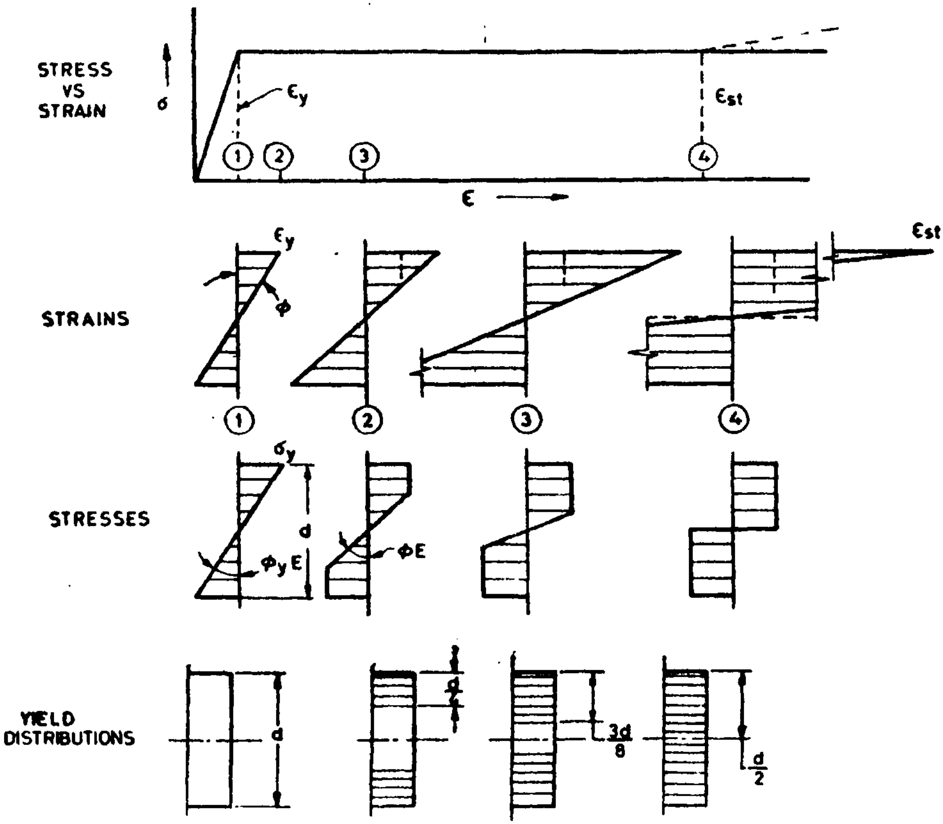


FIG. 13 PLASTIC BENDING OF RECTANGULAR BEAMS

the plastic limit (Stage 4) is shown in Fig. 13. The strain distribution is first selected or assumed and this fixes the stress-distribution.

12.2 Let us now trace the stages of yield stress penetration in a rectangular beam subjected to a progressive increase in bending moment. At the top of Fig. 13 is replotted for reference purposes the stress-strain relationship. At Stage 1, as shown in the next line of Fig. 13, the strains have reached the yield strain. When more moment is applied (say to Stage 2), the extreme fibre strains are twice the elastic limit value. The situation is similar for Stage 3 ($\epsilon_{Max} = 4\epsilon_y$). Finally, at Stage 4 the extreme fibre has strained to ϵ_{st}.

12.3 What are the stress distributions that correspond to these strain diagrams ? These are shown in the next line of Fig. 13. As long as the strain is greater than the yield value ϵ_y, as could be noticed from the stress-strain curve that the stress remains constant at σ_y. The stress distributions, therefore, follow directly from the assumed strain distributions.

As a limit we obtain the 'stress block'— a rectangular pattern which is very close to the stress distribution at Stage 4.

A new term introduced in Fig. 13 is the curvature. This is the relative rotation of two sections a unit distance apart. According to the first assumption (as in elastic bending):

$$\phi = \frac{1}{\rho} = \frac{\epsilon}{y} = \frac{\sigma}{E.y} \quad \ldots \quad \ldots \quad \ldots \quad \ldots(11)$$

where ρ = radius of curvature and ϵ, the strain at a distance y from the neutral axis. Just as it is basic to the fundamentals of elastic analysis, the relationship of bending moment to this curvature, ϕ, is a basic concept in plastic analysis.

The expressions for curvature and moment (and, thus, the resulting $M-\phi$ curve) follow directly from Fig. 13. Curvature at a given stage is obtained from a particular stress-distribution*. The corresponding moment-value is obtained by integration of stress-areas. The derivation of expressions for curvature and moment now follow.

Stage 2 of the example shown in Fig. 13 is shown in Fig. 14. From Eq 11 the curvature thus becomes:

$$\phi = \frac{\sigma_y}{Ey_o} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots(12)$$

where y_o is the ordinate to the neutral axis to the farthest still elastic fibre.

To compute the bending moment for this same Stage 2, the stress distribution of Fig. 14 is divided into parts in Fig. 15. The moment of

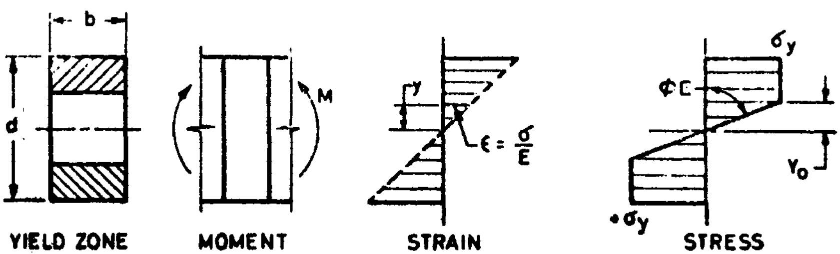


FIG. 14 STRESS DISTRIBUTION IN A PARTIALLY PLASTIC RECTANGULAR CROSS-SECTION

*Even though curvature is a measure of strain distribution, the stress-distribution diagram is used, since in the elastic range, the stress varies linearly with strain.

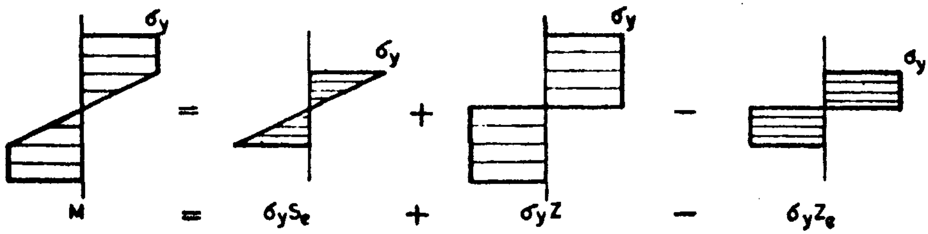


FIG. 15 STRESS ELEMENTS OF A PARTIALLY PLASTIC DESIGN

resistance may thus be considered as being made up of an elastic ($\sigma_y S_e$) and a plastic part ($\sigma_y Z_p$), or:

$$M = \sigma_y S_e + \sigma_y Z_p = \sigma_y S_e + \sigma_y Z - \sigma_y Z_e \qquad \dots \qquad \dots(13)$$

where the subscripts 'e' and 'p' refer to the elastic and plastic portions of the cross-section, respectively.

Equation 13 may also be derived directly from Eq 10. Referring to Fig. 14:

$$M = \int_A \sigma dA.y$$

$$= 2\int_o^{y_o} \sigma.bdy.y + 2\int_{y_o}^{d/2} \sigma_y.bdy.y = 2\int_o^{y_o} \sigma_y.\frac{y}{y_o}.bdy.y + 2\int_{y_o}^{d/2} \sigma_y.bdy.y$$

$$= \sigma_y \frac{2\int_o^{y_o} y^2.bdy}{y_o} + \sigma_y 2\int_{y_o}^{d/2} y.bdy \qquad \dots \qquad \dots \qquad \dots(13a)$$

$$= \sigma_y S_e + \sigma_y Z_p$$

The quantity Z is a property of a cross-section that corresponds in importance to the section modulus, S. It is called the plastic modulus, and (for symmetric sections) represents twice the statical moment (taken about the neutral axis) of the plastic section area above or below that axis. General methods for computing Z will be discussed later.

For the rectangular section, necessary values for section modulus, S, and plastic modulus, Z for use in Eq 13 are:

$$Z_e = 2by_o \frac{y_o}{2} = by_o^2$$

$$S_e = \frac{b(2y_o)^2}{6} = \frac{2}{3} by_o^2 = \frac{2}{3} Z_e$$

$$Z_p = Z - Z_e$$

$$Z = \frac{bd^2}{4} \qquad \dots \qquad \dots \qquad \dots \qquad \dots(14)$$

Thus, the bending moment in terms of Z is given by:

$$M = \sigma_y \left(Z - \frac{Z_e}{3} \right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots(15)$$

The maximum moment is obtained when the elastic part is reduced to zero or:

$$M_p = \sigma_y Z \quad \ldots \quad \ldots \quad \ldots \quad \ldots(16)$$

M_p is called the 'plastic moment'.

From the equations just derived for curvature and moment, we are now in a position to write the desired moment-curvature relationship. In terms of y_o, then:

$$M = \sigma_y \left(Z - \frac{b y_o^2}{3} \right) \quad \ldots \quad \ldots \quad \ldots(17)$$

In terms of ϕ, using Eq 12:

$$M = \sigma_y \left(Z - \frac{b \sigma_y^2}{3E^2 \phi^2} \right) \quad (\phi_y < \phi < \infty) \quad \ldots \quad \ldots(18)$$

The following non-dimensional relationship is obtained by dividing both sides of Eq 18 by $M_y = \sigma_y S$:

$$\frac{M}{M_y} = \frac{3}{2} \left[1 - \frac{1}{3} \frac{(\phi_y)^2}{(\phi)} \right] \quad \ldots \quad \ldots \quad \ldots(19)$$

The resulting non-dimensional $M - \phi$ curve for a rectangle is shown in Fig. 16. The numbers in circles in Fig. 16 correspond to 'stages' of

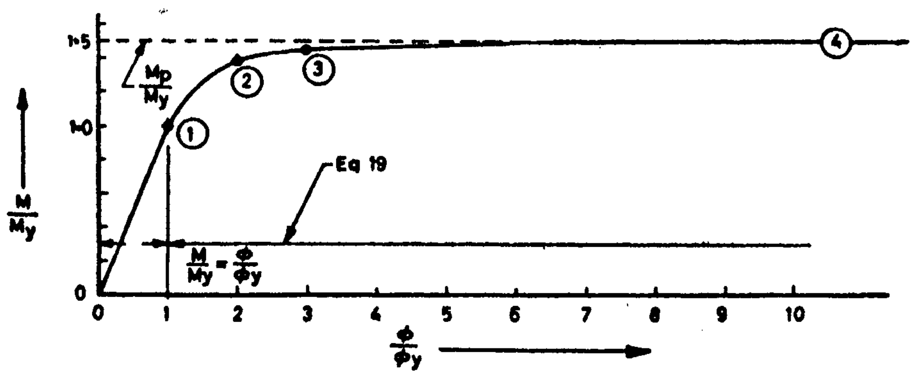


FIG. 16 NON-DIMENSIONAL MOMENT-CURVATURE RELATIONSHIP FOR RECTANGULAR BEAM

Fig. 13. Stage 4, approached as a limit, represents complete plastic yield of the cross-section, where $M_p = \sigma_y Z$. Note that there is a 50 percent increase in strength above the computed elastic limit (Stage 1) due to this 'plastification' of the cross-section. This represents one of the sources of reserve strength beyond the elastic limit of a rigid frame.

The ratio of the plastic moment (M_p) to the yield moment (M_y), representing the increase of strength due to plastic action, will be a function of the cross-section form or shape. Thus the 'shape factor' is given by:

$$f = \frac{M_p}{M_y} = \frac{\sigma_y Z}{\sigma_y S} = \frac{Z}{S} \qquad \ldots \qquad \ldots \qquad \ldots (20)$$

For the rectangular beam being considered, $f = \frac{bd^2}{4} + \frac{bd^2}{6} = 1{\cdot}50$ as indicated in Fig. 16.

13. BENDING OF WIDE FLANGE BEAM

13.1 The action of a wide flange beam under bending moment is diagrammatically shown in Fig. 17. If it is assumed that all of the material in a wide flange shape is concentrated in the flanges then (when the elastic limit is reached) the compression flange shortens at constant load and the tension flange lengthens at constant load. The resulting moment is, therefore, constant; the member acts just like a hinge except that deformation occurs under constant moment (the plastic-hinge moment).

13.2 A more realistic picture of the moment-curvature relationship of a wide flange shape is shown in Fig. 18. Point 1 is the elastic limit; at Point 2 the member is partially plastic and at Point 3 the cross-section approaches a condition of full plastic yield.

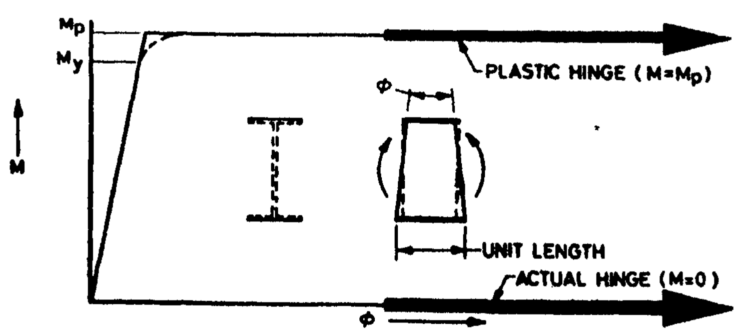


FIG. 17 IDEALIZED MOMENT-CURVATURE RELATIONSHIP FOR WIDE FLANGE BEAM

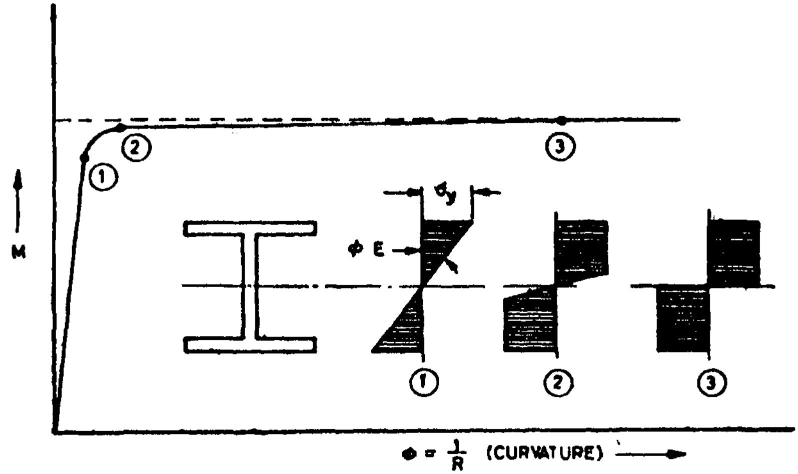


FIG. 18 TYPICAL THEORETICAL MOMENT-CURVATURE RELATIONSHIP OF WIDE FLANGE BEAM

The magnitude of the moment, M_p, may be computed directly from the stress distribution shown for Point 3. As shown in Fig. 19 it is equal to the couple created by the tensile and compressive forces. The moment due to each of these forces is equal to the product of the yield stress, σ_y, and the area above the neutral axis ($A/2$) multiplied by the distance $\bar{y}$ measured to the centre of gravity of that area.

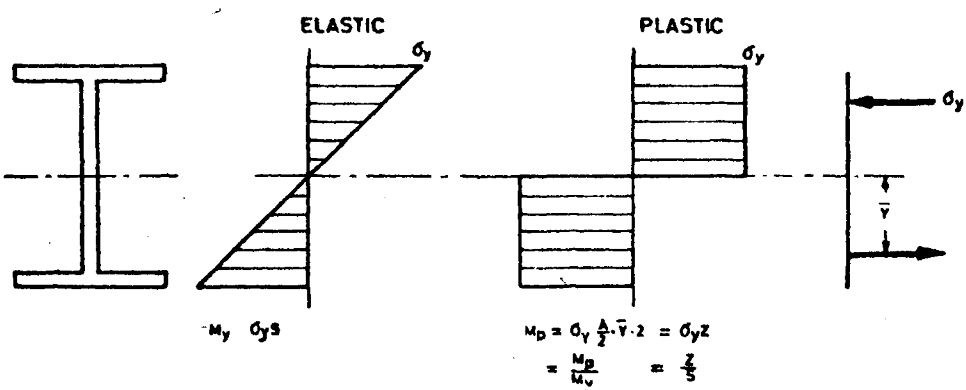


FIG. 19 ELASTIC AND PLASTIC LIMIT MOMENTS

Thus:

$$M_p = 2\ \sigma_y \frac{A}{2}\ \bar{y} = \sigma_y A \bar{y} \quad \ldots \quad \ldots \quad \ldots(21)$$

The quantity $A\bar{y}$ is called the 'plastic modulus' and is denoted by Z; therefore, as before,

$$M_p = \sigma_y Z \quad \ldots \quad \ldots \quad \ldots(16)$$

The plastic modulus, Z, is thus equal to the combined statical moments of the cross-sectional areas above and below the neutral axis, since the stress at every point on these areas is the same.

The moment-curvature relationship may be developed for wide-flange shapes by the same procedure as outlined for a rectangular cross-section. Due to variation of width of section with depth, separate expressions are necessary when yielding is limited to the flanges and when yielding has penetrated to the web.

For Case 1, in which the yield zone has penetrated part way through the flanges (Fig. 20), the non-dimensional $M-\phi$ curve becomes:

$$\frac{M}{M_y} = \frac{\phi}{\phi_y}\left(1 - \frac{bd^2}{6S}\right) + \frac{bd^2}{4S}\left[1 - \frac{1}{3}\left(\frac{\phi_y}{\phi}\right)^2\right] \quad \ldots \quad \ldots(22)$$

$$\left(1 < \frac{\phi}{\phi_y} < \frac{d/2}{(d/2 - t)}\right)$$

For Case 2, in which yielding has penetrated through the flanges and into the web (Fig. 21), the non-dimensional $M-\phi$ curve becomes:

$$\frac{M}{M_y} = f - \frac{wd^2}{12S}\left(\frac{\phi_y}{\phi}\right)^2$$

$$\left(\frac{d/2}{d/2 - K} < \frac{\phi}{\phi_y} < \infty\right) \quad \ldots \quad \ldots(23)$$

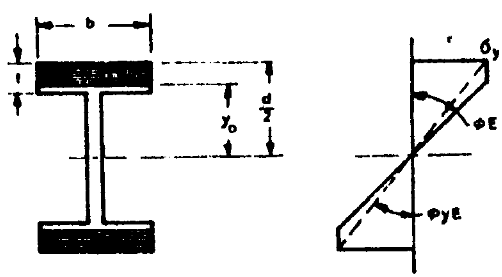


Fig. 20 Plastic Stress Distribution in Wide Flange Beam — Case 1: Partial Yielding

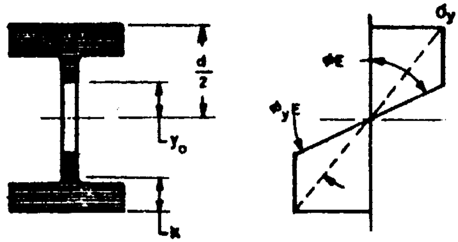


FIG. 21 PLASTIC STRESS DISTRIBUTION IN WIDE FLANGE BEAM — CASE 2: PARTIAL YIELDING

The curve resulting from Eq 22 and 23 is shown in Fig. 18 for a typical wide flange shape. (The stress-distributions correspond to the numbered points on the $M-\phi$ curve.) It will be noted that the shape factor is smaller than for the rectangle (compare Fig. 16), the average value of 'f' for all wide flange beams being 1·14. Correspondingly there is a more rapid approach to M_p when compared with rectangle. As a matter of fact when the curvature is twice the elastic limit value (Stage 2 of Fig. 18) the moment has reached to within 2 percent of the full M_p-value.

14. PLASTIC HINGE

14.1 The reason a structure will support the computed ultimate load is that plastic hinges are formed at certain critical sections. What is the plastic hinge? What factors influence its formation? What is its importance?

The $M-\phi$ curve is characteristic of the plastic hinge (Fig. 18). Two features are particularly important:

a) the rapid approach to $M = M_p = \sigma_y Z$; and

b) the indefinite increase in ϕ at constant M.

An idealized $M-\phi$ curve is obtained by assuming (for a wide flange shape) that all of the material is concentrated in the flanges as shown in Fig. 17. The behaviour shown there is of basic importance to plastic analysis. According to it, the member remains elastic until the moment reaches M_p. Thereafter, rotation occurs at constant moment; that is, the member acts as if it were hinged except with a constant restraining moment, M_p. This, then, is the plastic hinge.

According to the idealization of Fig. 17, plastic hinges form at discrete points of zero length. Actually the hinge extends over that part of the beam whose bending moment is greater than M_p. That length is dependent on the loading and geometry. It is justified to neglect this 'distribution', however, and the length of hinge is assumed to be zero. Closely related to the plastic hinge is the plastic modulus, Z. It has already been defined for the symmetrical sections as twice the statical moment about the neutral axis, of the half sectional area. As noted earlier, $Z = Ay$. For wide flange beam shapes, the quantity y may be determined directly from the properties of split tees and thus:

$$Z_{\text{wide flange}} = A\left(\frac{d}{2} - y_1\right) \qquad \ldots \qquad \ldots \qquad \ldots (24)$$

where y_1 is the distance from the flange to the centre of gravity.

The shape factors, already defined as $f = Z/S$, varies for wide flange shapes from 1·09 to 1·22. The mode is 1·12 and the average is 1·14 for I shapes. Examples of the ratio of $Z/S = f$ for symmetrical shapes other than the wide flange are shown in Fig. 22 and 23.

For sections with symmetry only about an axis in the plane of bending, the neutral axis at the plastic moment condition follows directly from Eq 10. The general definition for Z is 'The combined statical moments of the cross-sectional areas above and below the neutral axis'. Since $P = O$, and $\sigma = \sigma_y$, for equilibrium the area above the neutral axis should equal that below. Thus, for a triangular-section in Fig. 22 the elastic neutral axis is at a distance of $2/3d$ from the toe, while the plastic' neutral axis is at a distance of $d\sqrt{2}$.

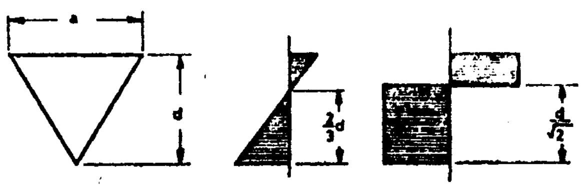


FIG. 22 NEUTRAL AXIS OF A TRIANGULAR SECTION

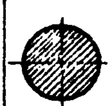
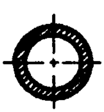
					WIDE FLANGE STRONG AXIS	WIDE FLANGE WEAK AXIS
SECTION						
$f = \frac{Z}{S}$	2·00	1·70	1·55	1·27	~1·14	~1·50

FIG. 23 SHAPE FACTORS OF COMMON SYMMETRICAL SECTIONS

In addition to the shape factor whose influence on strength has already been described, several other factors influence the ability of members to form plastic hinges. Some of these factors are important from the design point of view (such as shear, axial force, instability) and are treated in Section E. Others are primarily of academic interest in so far as routine design applications are concerned. Factors affecting the bending strength and stiffness of beams have been listed in Chapter 2 of Ref 9 (*see* Appendix A) with further reference being made to other sources of information on each factor.

The following definitions or principles briefly summarize this clause and are important to a later understanding of plastic analysis:

a) ***A plastic hinge is a zone of yielding due to flexure in a structural member*** — Although its length depends on the geometry and loading, in most of the analytical work it is assumed that all plastic rotation occurs at a point. At those sections where plastic hinges are located, *the member acts as if it were hinged, except with a constant restraining moment* M_p, (Fig. 17).

b) ***Plastic hinges form at points of maximum moment*** — Thus in a framed structure with prismatic members, it would be possible for plastic hinges to form at points of concentrated load, at the end of each member meeting at a connection involving a change in geometry, and at the point of zero shear in a span under distributed load.

c) The plastic moment, M_p, equals $\sigma_y Z$.

d) The shape factor $\left(f = Z/S = \dfrac{A\bar{y}}{I/c}\right)$ is one source of reserve strength beyond the elastic limit.

Application of the plastic hinge concept to analysis is illustrated in **15**.

15. REDISTRIBUTION OF MOMENT

15.1 The second factor contributing to the reserve of strength is called 'redistribution of moment' and is due to the action of the plastic hinges. As load is added to a structure eventually the plastic moment is reached at a critical section — the section that is most highly stressed in the elastic range. As further load is added, this plastic moment value is maintained while the section rotates. Other less highly-stressed sections maintain equilibrium with the increased load by a proportionate increase in moment. This process of redistribution of moment due to the successive formation of plastic hinges continues until the ultimate load is reached.

15.2 This is exactly the process that took place in the case of the three-bar truss of Fig. 6 except that tensile forces instead of moments were involved. When the force in Bar 2 reached the yield condition it remained constant there while the forces continued to increase in Bars 1 and 3. The ultimate load was reached when all critical bars became plastic.

15.3 The phenomenon of redistribution of moment will now be illustrated with the case shown in Fig. 24, a fixed ended beam with a concentrated load off-centre. As the load P is increased the beam reaches its elastic limit at the left end (Stage 1). The moments at sections B and C are less than the maximum moment. Note that in this example we will consider the idealized $M-\phi$ relationship as shown in the lower left-hand portion. (The dotted curve shows the more 'precise' behaviour).

As the load is further increased, a plastic hinge eventually forms at Section B. The formation of the plastic hinge at A will permit the beam to rotate there without absorbing any more moment. Referring to the load-deflection curve immediately below the moment diagrams the deflection is increasing at a greater rate.

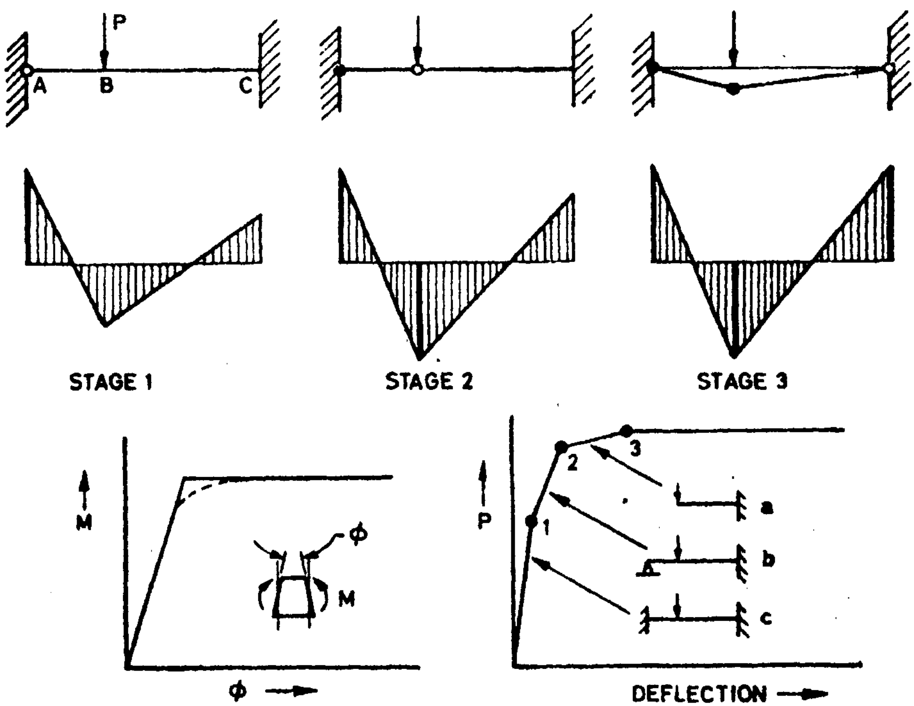


Fig. 24 Re-Distribution of Moments

Eventually, at Stage 3, when the load is increased sufficiently to form a plastic hinge at Point C, all of the available moment capacity of the beam will have been exhausted and the ultimate load reached.

It is evident from the load-deflection curve shown in the lower part of the figure that the formation of each plastic hinge acts to remove one of the indeterminates in the problem, and the subsequent load-deflection relationship will be that of a new (and simpler) structure. In the elastic range, the deflection under load can be determined for the completely elastic beam. Starting from Point 1 the Segment 1-2 represents the load-deflection curve of the beam in sketch *b* loaded within the elastic range. Likewise the load-deflection curve of the beam in sketch *c* looks similar to the portion 2-3.

Beyond Stage 3 the beam will continue to deform without an increase in load just like a link mechanism if the plastic hinges were replaced by real hinges. This situation is called a 'mechanism' in the somewhat special condition that motion is possible without an *increase* in load.

Two further fundamental concepts in addition to the four listed in **14** are in summary of this clause and are demonstrated by Fig. 24:

a) The formation of plastic hinges allows a subsequent redistribution of moment until M_p is reached at each critical (maximum) section.

b) The maximum load will be reached when a sufficient number of plastic hinges have formed to create a mechanism.

On the basis of the principles just discussed one may readily visualize how to compute the ultimate load: Simply sketch a moment diagram such that plastic hinges are formed at a sufficient number of sections to allow 'mechanism motion'. Thus in Fig. 25, the bending moment diagram for the uniformly-loaded, fixed-ended beam would be drawn such

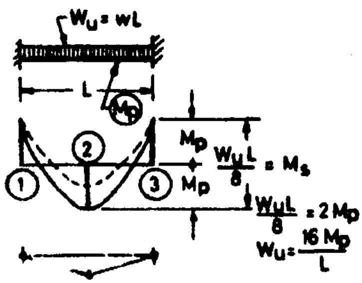


FIG. 25 MOMENT DIAGRAM AT VARIOUS STAGES FOR FIXED-ENDED BEAM WITH UNIFORMLY DISTRIBUTED LOAD

that M_p is reached at the two ends and the centre. In this way a mechanism is formed. By equilibrium:

$$\frac{W_u L}{8} = 2M_p$$

$$W_u = \frac{16M_p}{L}$$

How does this compare with the load at first yield ? At the elastic limit (*see* dotted moment-diagram in Fig. 25) we know from a consideration of continuity that the centre moment is one-half the end moment. Thus:

$$\frac{W_y L}{8} = M_y + \frac{M_y}{2} = \frac{3M_y}{2}$$

$$W_y = \frac{12M_y}{L}$$

Therefore, the reserve strength due to redistribution of moment is:

$$\frac{W_u}{W_y} = \frac{16M_p/L}{12M_y/L} = \frac{4}{3}\,\frac{M_p}{M_y}$$

Considering the average shape factor of wide flange beams, the total reserve strength due to redistribution and shape factor (plastification) is:

$$\frac{W_u}{W_y} = \frac{4}{3} \times 1{\cdot}14 = 1{\cdot}52$$

For this particular problem, then, the ultimate load was 52 percent greater than the load at first yield, representing a considerable margin that is disregarded in conventional design.

There are other methods for analyzing a structure for its ultimate load, in particular the 'Mechanism Method' (to be described later) which starts out with an assumed mechanism instead of an assumed moment diagram. But in every method, there are always these two important features:

a) the formation of plastic hinges, and

b) the development of a mechanism.

With these fundamental concepts regarding the mechanical properties of steel and the flexure of beams we are now in a position to examine the methods of plastic analysis.

SECTION D

PLASTIC ANALYSIS

16. FUNDAMENTAL PRINCIPLES

16.1 With the evidence presented in Section B that full-size structures behave as predicted by plastic theory and having considered in Section C the plastic behaviour of beams, we may now proceed to a consideration of the methods of plastic analysis. The objective of this Section is to describe briefly the fundamental principles upon which plastic analysis rests and then to describe how these principles are used in analyzing continuous beams and frames.

The basis for computing the 'ultimate load' (or maximum plastic strength) is the strength of steel in the plastic range. As shown in **3**, structural steel has the ability to deform plastically after the yield-point is reached. The resulting flat stress-strain characteristic assures dependable plastic strength, on the one hand, and provides an effective 'limit' to the strength of a given cross-section making it independent of further deformation. Thus, when certain parts of a structure reach the yield stress, they maintain that stress while other less-highly stressed parts deform until they, too, reach the yield condition. Since all critical sections eventually reach the yield condition, the analysis is considerably simplified because only this fact need be considered. It is not of interest *how* the stresses are redistributed; we should only ascertain that they *did*. We are thus freed from the often laborious calculations that result from the necessity of considering the 'continuity' conditions that are essential to elastic analysis.

While elastic and plastic analysis were compared at the outset in **2** from the design point of view, it is of interest now, to compare them as regards to the fundamental conditions satisfied by each.

Whatever method of plastic analysis is used, it should satisfy the following three conditions that may be deduced from what has been said in **15**:

a) Mechanism condition (ultimate load is reached when a mechanism forms),

b) Equilibrium condition (the structure must be in equilibrium), and

c) Plastic moment condition (the moment may nowhere be greater than M_p).

Actually these conditions are similar to those in elastic analysis which requires a consideration of the *continuity*, the *equilibrium* and the *limiting stress* conditions. The similarity is demonstrated in Fig. 26. With regard to continuity, in plastic analysis, the situation is just the reverse. Theoretically, plastic hinges interrupt continuity, so the requirement is that sufficient plastic hinges form to allow the structure (or part of it) to deform as a mechanism. This could be termed a *mechanism* condition. The *equilibrium* condition is the same, namely, the load should be supported. Instead of initial yield, the limit of usefulness is the attainment of plastic hinge moments, not only at one cross-section but at each of the critical sections; this will be termed a *plastic moment* condition.

As will be discussed further, two useful methods of analysis take their name from the particular conditions being satisfied:

a) Mechanical Method ——— satisfies: Mechanical condition; Equilibrium condition

b) Statical (Equilibrium) Method —— satisfies: Equilibrium condition; Plastic moment condition

In the first method, a mechanism is assumed and the resulting equilibrium equations are solved for the ultimate load. This value is only correct if the plastic moment condition is also satisfied. On the other hand, in the statical or 'equilibrium' method, an equilibrium moment diagram is drawn such that $M \leqslant M_p$. The resulting ultimate load is only the correct value if sufficient plastic hinges were assumed to create a mechanism.

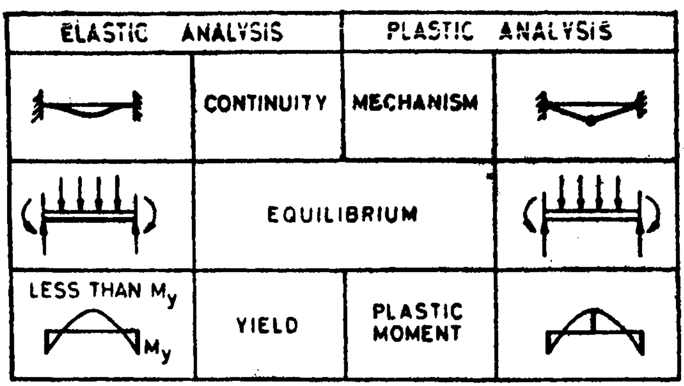


FIG. 26 CONDITIONS FOR ELASTIC AND PLASTIC ANALYSIS

Having considered these three necessary and sufficient conditions, it will next be of interest to examine certain additional principles and assumptions upon which the plastic methods rest. Although the plastic design procedures do not require a direct use of these principles (or assumptions) they will be stated for background purposes.

a) Virtual Displacements

The principle of virtual displacements is as follows*:

> If a system of forces in equilibrium is subjected to a virtual displacement, the work done by the external forces equals the work done by the internal forces.

This is simply a means of expressing an equilibrium condition. If the internal work is called W_I and the external work is called W_E, we may write:

$$W_E = W_I \qquad \dots \qquad \dots \qquad \dots(25)$$

Application of this equation will be demonstrated in **18**.

b) Upper and Lower Bound Theorems

It is not generally possible to solve all three of the necessary conditions (mechanism, equilibrium and plastic moment) in one operation. Although the Equilibrium condition will always be satisfied, a solution arrived at on the basis of an assumed mechanism will give a load-carrying capacity that is either correct or *too high*. On the other hand, one that is arrived at by drawing a statical moment diagram that does not violate the plastic moment condition will either be correct or *too low*. Thus, depending on how the problem is solved, we will obtain an upper 'limit' or 'bound' below which the correct answer should certainly lie, or we will determine a lower 'limit' or 'bound' which is certainly less than the true load capacity.

The important upper and lower bound theorems or principles were proved by Greenberg and Prager. When both theorems have been satisfied in any given problem, then the solution is in fact the correct one. The two principles will now be stated and illustrated.

> **Upper Bound Theorem** — A load computed on the basis of an assumed mechanism will always be greater than or at best equal to the true ultimate load.

Consider the fixed-ended beam in Fig. 27(A). If we assume a mechanism on the basis of a guess that the plastic hinge in the beam forms at

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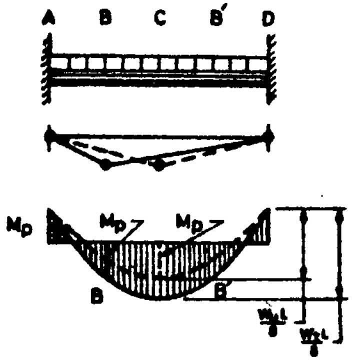
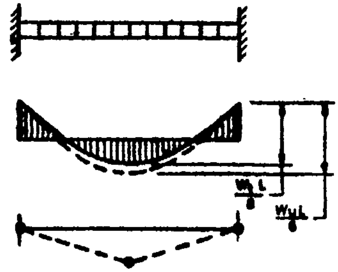
*Reference 21 contains an excellent discussion of the principle of virtual displacements.



27A Upper Bound Theorem



27B Lower Bound Theorem

FIG. 27 UPPER AND LOWER BOUND THEOREMS

B, then the equilibrium moment diagram would be as shown by the solid line in Fig. 27(A). The beam would have to be reinforced over the length *BB'* to carry the 'trial' load, W_t; the load is too great. Only when the mechanism is selected such that the plastic moment value is nowhere exceeded (*see* the dotted lines) is the correct (lowest) value obtained.

Lower Bound Theorem — A load computed on the basis of an assumed equilibrium moment diagram in which the moments are not greater than M_p is less than or at best equal to the true ultimate load.

Illustrating with the fixed-ended beam of Fig. 27(B), if we select the redundants such that the moment is never greater than M_p, then the corresponding trial load, W_t, may be less than W_u, [Fig. 27(B)]. We have not used the full load capacity of the beam because the centre line moment is less than M_p. Only when the load is increased to the stage where a mechanism is formed (dotted) will the correct value be obtained.

Thus, if the problem is approached from the point of view of assuming a mechanism, an upper bound to the correct load will be obtained. But this could violate the plastic moment condition. On the other hand, if we approach it from the aspect of making arbitrary assumptions as to the moment diagram, then the load might not be sufficiently great to create a mechanism.

Incidentally, Fig. 27(B) demonstrates that conventional (elastic) design is a 'lower bound' solution. This is the explanation as to why the 'local yielding' involved in many of our current design assumptions has not resulted in unsafe structures.

It is seen, then, that the 'statical (equilibrium)' method of analysis is based on the lower bound principle. The mechanism method, on the other hand, represents an upper limit to the true ultimate load.

c) **Further Assumptions** — In addition to the assumptions of **11** the following further assumptions are necessary:

a) The theory considers only first order deformations. The deformations are assumed to be sufficiently small such that equilibrium conditions can be formulated for the undeformed structure (just as in the case of elastic analysis).

b) Instability of the structure will not occur prior to the attainment of the ultimate load (this is assured through attention to secondary design considerations).

c) The connections provide full continuity such that the plastic moment, M_p, can be transmitted (*see* Section E).

d) The influence of normal and shearing forces on the plastic moment, M_p, are neglected (*see* Section E for necessary modifications).

e) The loading is proportional, that is, all loads are such that they increase in fixed proportions to one another. However, independent increase can be allowed, provided no local failure occurs (*see* Section E for repeated loading).

With the Principles of Virtual Displacements, the Upper and Lower Bound Theorems, and the additional assumptions noted above, it is now possible to consider the various methods of analysis.

17. STATICAL METHOD OF ANALYSIS

17.1 As noted in **16**, the 'statical' method of analysis is based on the Lower Bound Principle. The procedure is first described and then several examples are solved.

17.2 Method of Analysis by Statical Method — By the following procedure find an equilibrium moment diagram in which $M \leqslant M_p$ such that a mechanism is formed:

a) Select redundant(s),

b) Draw moment diagram for determinate structure,

c) Draw moment diagram for structure loaded by redundant(s),

d) Sketch composite moment diagram in such a way that a mechanism is formed (sketch mechanism),

e) Compute value of ultimate load by solving equilibrium equation, and

f) Check to see that $M \leqslant M_p$.

Example 1:

Fixed-ended, uniformly loaded beam, Fig. 25
(*indeterminate to second degree*)

The problem (already treated in **15**) is to find the ultimate load, W_u, that a beam of moment capacity M_p will support. For redundants, one could select the end moments. The resulting moment diagram for the determinate structure would be the solid parabola in Fig. 25, with:

$$M_s = \frac{W_u L}{8} \quad \ldots \quad \ldots \quad \ldots(26)$$

The moment diagram for the structure loaded by the redundants would be a uniform moment along the beam.

The composite moment diagram is actually what has been sketched in Fig. 25 since, we can immediately see that a hinge must also form at point 2. Notice that if the 'fixing line' had been drawn in any other position than that which divides $M_s = \frac{W_u L}{8}$ in half, then no mechanism would have been formed. The correct mechanism is sketched in the lower portion and $M = M_p$ at the locations of maximum moment.

The equilibrium equation, from Fig. 25 (at location 2), is:

$$\frac{W_u L}{8} = M_p + M_p$$

and the ultimate load is given by:

$$W_u = \frac{16 M_p}{L} \quad \ldots \quad \ldots \quad \ldots(27)$$

Example 2:

Two-span continuous beam, Fig. 28
(*indeterminated to first degree*)

The redundant is selected as the moment at $C(M_c)$. The resultant loadings are shown in Fig. 28(a) and 28(b).

Moment diagrams due to loads and redundants are shown in Fig. 28(c) and 28(d).

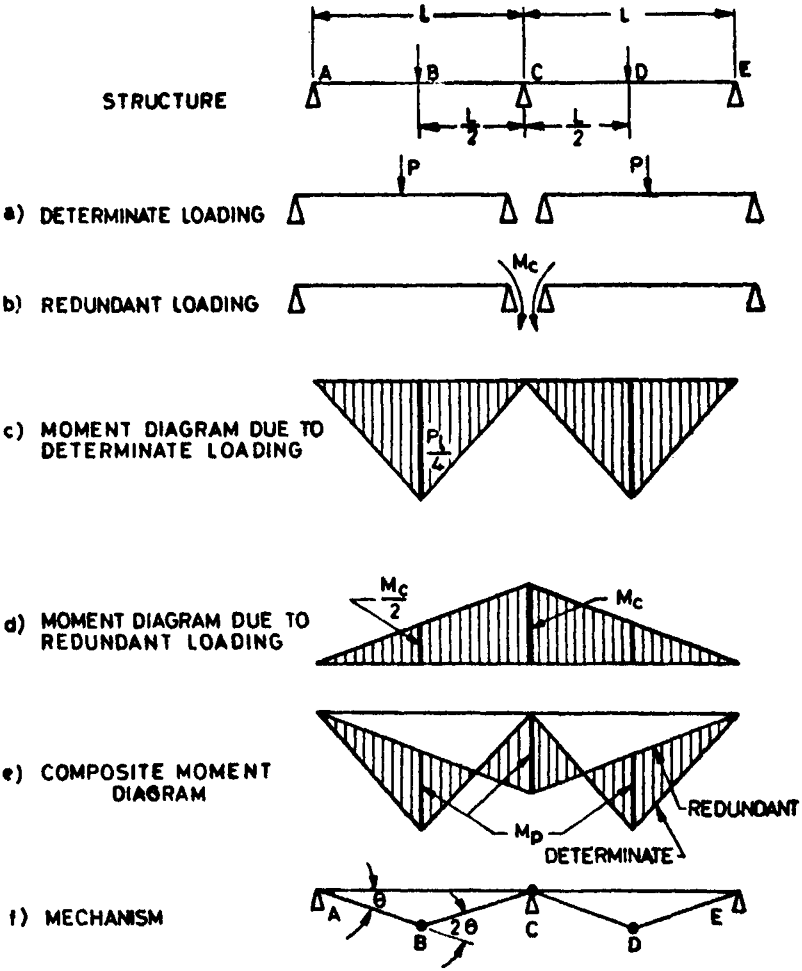


FIG. 28 PLASTIC ANALYSIS OF TWO-SPAN CONTINUOUS BEAM (STATICAL METHOD)

The composite moment diagram is sketched in Fig. 28(e) in such a way that the necessary mechanism is formed, Fig. 28(f), with maximum moments, M_p, at locations B, C and D.

The equilibrium equation is obtained by summing the moments at location B:

$$\frac{P_u L}{4} = M_p + M_{p/2}$$

$$P_u = \frac{6M_p}{L} \quad \ldots \quad \ldots \quad \ldots (28)$$

Since all three of the necessary conditions are satisfied (Mechanism, Equilibrium, and Plastic Moment), this is the correct answer. Further examples of the use of this method are given in Section F, Design Examples 1, 2, 4 and 6.

18. MECHANISM METHOD OF ANALYSIS

18.1 General Procedure — As the number of redundants increases, the number of possible failure mechanisms also increases. Thus it may become more difficult to construct the correct equilibrium moment diagram. For such cases the mechanism method of plastic analysis may be used to find various 'upper bounds'. The correct mechanism will be the one which results in the lowest possible load (upper bound theorem) and for which the moment does not exceed the plastic moment at any section of the structure (lower bound theorem). Thus the objective is to find a mechanism such that the plastic moment condition is not violated.

The following, then, is the general procedure.

18.2 Method of Analysis by Mechanism Method — Find a mechanism (independent or composite) such that $M = M_p$:

a) determine location of possible plastic hinges (load points, connections, point of zero shear in a beam span under distributed load);

b) select possible independent and composite mechanisms;

c) solve equilibrium equation (virtual displacement method) for the lowest load; and

d) check to see that $M = M_p$ at all sections.

Example 3:

Rectangular Portal Frame, Fig. 29

Given a rectangular frame of uniform section whose plastic moment capacity is M_p, what is the ultimate load it will carry?

In the frame shown in Fig. 29(a) locations of possible plastic hinges are at locations 2, 3, 4. Now, in the previous examples there was

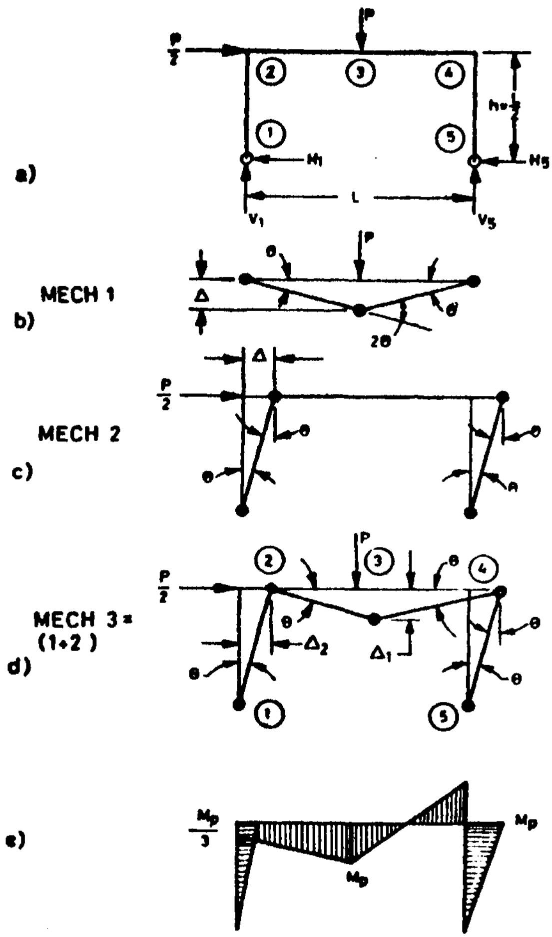


FIG. 29 MECHANISM METHOD OF ANALYSIS APPLIED TO A RECTANGULAR PORTAL FRAME WITH PINNED BARS

only one possible failure mechanism. However, in this problem there are several possibilities. 'Elementary' or 'independent' Mechanisms 1 and 2 correspond to the action of the different loads, whereas Mechanism 3, Fig. 29(d), is a 'composite' mechanism formed by combination of Mechanisms 1 and 2 to eliminate a plastic hinge at location 2. Which is the correct one? It is the one which results in the lowest critical load P_u.

The method of virtual displacements may be used to compute the critical load. After the ultimate load is reached, the frame is allowed to move through a small additional displacement such as shown by Δ in Fig. 29(b). For equilibrium, the external work done by the loads as they move through small displacements shall equal the internal work absorbed at each hinge as it rotates through a corresponding small angle, or

$$W_E = W_I$$

The following equations are obtained for the various mechanisms:

Mechanism 1: (Beam)

$$P\Delta = M_p\theta + M_p(2\theta) + M_p\theta \quad \ldots \quad \ldots(29)$$

$$\frac{PL\theta}{2} = M_p(4\theta)$$

$$P_1 = \frac{8M_p}{L} \quad \ldots \quad \ldots \quad \ldots(30)$$

Mechanism 2: (Panel)

$$\frac{P\Delta}{2} = M_p(\theta + \theta) \quad \ldots \quad \ldots \quad \ldots(31)$$

$$\frac{P}{2}\,\frac{L\theta}{2} = 2M_p\,\theta \text{ (as } h = L/2)$$

$$P_2 = \frac{8M_p}{L} \quad \ldots \quad \ldots \quad \ldots(32)$$

Mechanism 3: (Composite)

$$P\Delta_1 + \frac{P}{2}\Delta_2 = M_p(2\theta) + M_p(2\theta) \quad \ldots \quad \ldots(33)$$

$$P\frac{L\theta}{2} + \frac{P}{2}\,\frac{L\theta}{2} = 4M_p\theta$$

$$P_3 = \frac{16}{3}\,\frac{M_p}{L} = P_u \quad \ldots \quad \ldots \quad \ldots(34)$$

The lowest value is P_3 which is, therefore the true ultimate load, P_u.

To make sure that some other possible mechanism was not overlooked it is necessary to check the plastic moment condition to see that $M < M_p$ at all sections. To do this the complete moment diagram is drawn as shown in Fig. 29. The moment at location 2 is determined as follows:

$$H_5 = \frac{M_p}{L/2} = \frac{2M_p}{L}$$

$$H_1 = \frac{P}{2} = H_5 = \frac{16M_p}{3L}\,\frac{1}{2} = \frac{2M_p}{L} = \frac{2}{3}\,\frac{M_p}{L}$$

$$M_2 = H_1 h = \frac{2}{3}\,\frac{M_p}{L}\,\frac{L}{2} = \frac{M_p}{3}$$

Since the moment is nowhere greater than M_p, we have obtained the correct answer and the problem is solved.

In Example 3, the virtual work equation was solved anew for the 'composite' mechanism. An alternate procedure for computing the ultimate load for a composite mechanism is to add together the virtual work equations for each mechanism in the combination, being careful to subtract the internal work done in an elementary mechanism at any hinges being eliminated by the combination. Using this procedure for Example 3, there is obtained from the previous set of equations:

Mechanism	*Virtual Work Equation*	*Hinges Cancelled*
Mechanism 1: (Beam)	$\frac{PL\theta}{2} = 4M_p\theta$	$-M_p\theta$
Mechanism 2: (Panel)	$\frac{PL}{4}\,\theta = 2M_p\theta$	$-M_p\theta$
Mechanism 3: (Composite)	$\frac{3PL}{4}\,\theta = [6M_p\theta]$	$-2M_p\theta$

$$P_3 = 16M_p/3L$$

This is the same answer as obtained in Eq 34.

In the previous examples there were a sufficiently small number of possible mechanism so that the combinations were almost obvious. Further, the geometry in the deformed position could be developed with no difficulty. A number of guides and techniques will now be discussed that are useful in solving more involved problems.

18.3 Types of Mechanism — First of all, for convenience in referring to different mechanisms of structures given in Fig. 30(a) there are the following types which are illustrated in Fig. 30:

a) *Beam Mechanism* Fig. 30(b)
(Four examples are given here of the displacement of single spans under load)

b) *Panel Mechanism* Fig. 30(c)
(This motion is due to side-sway)

c) *Gable Mechanism* Fig. 30(d)
(This is a characteristic mechanism of gabled frames, involving spreading of the column tops with respect to the bases)

d) *Joint Mechanism* Fig. 30(e)
(This independent mechanism forms at the junction of three or more members and represents motion under the action of a moment)

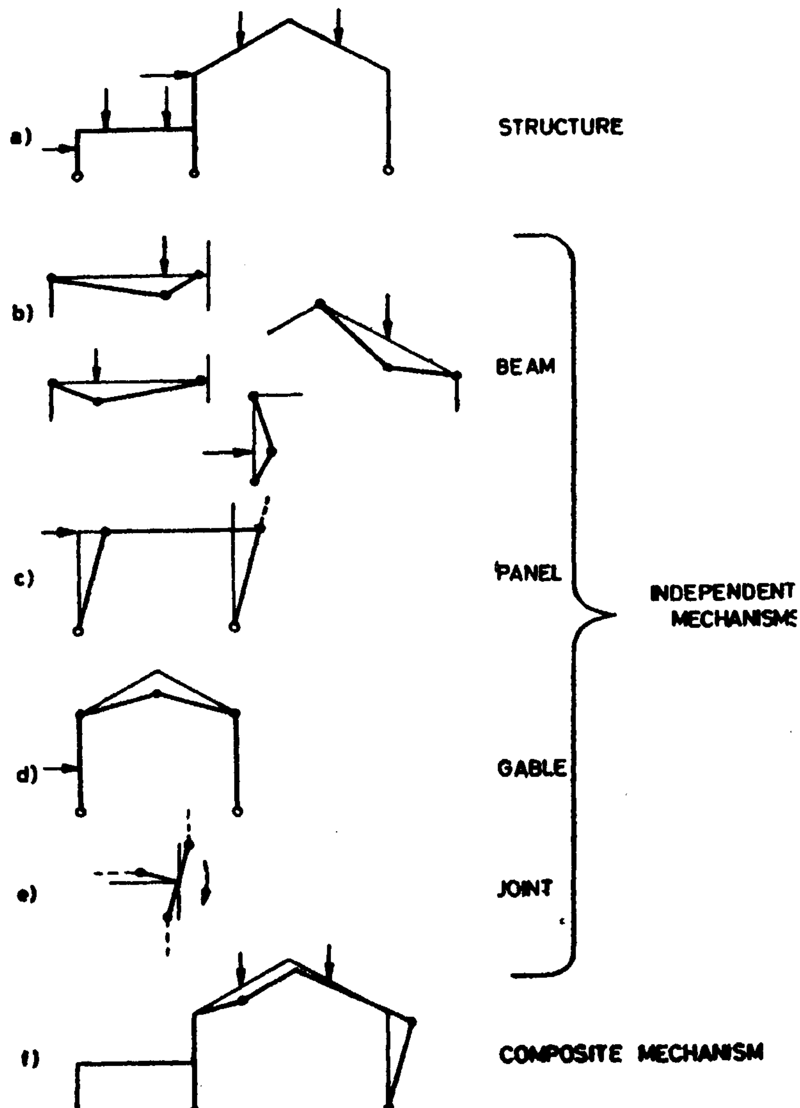


FIG. 30 TYPES OF MECHANISMS

e) *Composite Mechanism* Fig. 30(f)
(Various combinations of the independent mechanism may be made. The one shown is a combination of a beam and a gable mechanism)

18.3 Number of Independent Mechanisms — If it were known in advance how many independent mechanisms existed, then combinations could be made in a systematic manner and there would be less likelihood of overlooking a possible combination. Fortunately, the following simple procedure is available for determining this.

If the number of possible plastic hinges is N and if the number of redundancies is X, then the number of possible independent mechanism, n, may be found from

$$n = N - X \qquad \ldots \qquad \ldots \qquad \ldots (35)$$

Thus, in Example 3 there are 3 possible plastic hinges (locations 2, 3 and 4), the frame is indeterminate to the first degree, and, therefore, there are two elementary mechanisms (Mechanisms 1 and 2).

This correlation is no coincidence because each independent mechanism corresponds to the action of a different loading system. Said in another way, each mechanism corresponds to an independent equation of equilibrium. In Example 3 Mechanism 1 corresponds to equilibrium between applied vertical load and vertical shear. Mechanism 2 corresponds to equilibrium between applied horizontal load ($P/2$) and horizontal shear in the two columns. These force systems are 'elementary' or 'independent' and hence the term.

Equation 35 may be seen in this way. For a determinate system, if a plastic hinge develops, the structure becomes a mechanism. Thus, for each possible plastic hinge there corresponds a mechanism; if there are 'n' possible plastic hinges, there will be 'n' mechanism [*see* Fig. 31(a)]. As we add redundants to the structure, we add a plastic hinge for each redundant but do not change the number of mechanisms. Where the member was free to deform beforehand (at a real hinge), it is now restrained; however, the number of basic mechanisms remains unchanged [*see* Fig. 31(b) and 31 (c)]. Thus the number of possible plastic hinges, N, equals the number of mechanisms n, plus the number of redundants, X, or $n = (N - X)$.

18.4 Composite Mechanisms — Equation 35 is useful because it enables us to set out all the possible 'elements' from which combinations may later be made. These combinations are to be made in such a way as to make the external work a maximum or the internal work a minimum, since by this means the lowest possible load, P, is obtained. Therefore, the

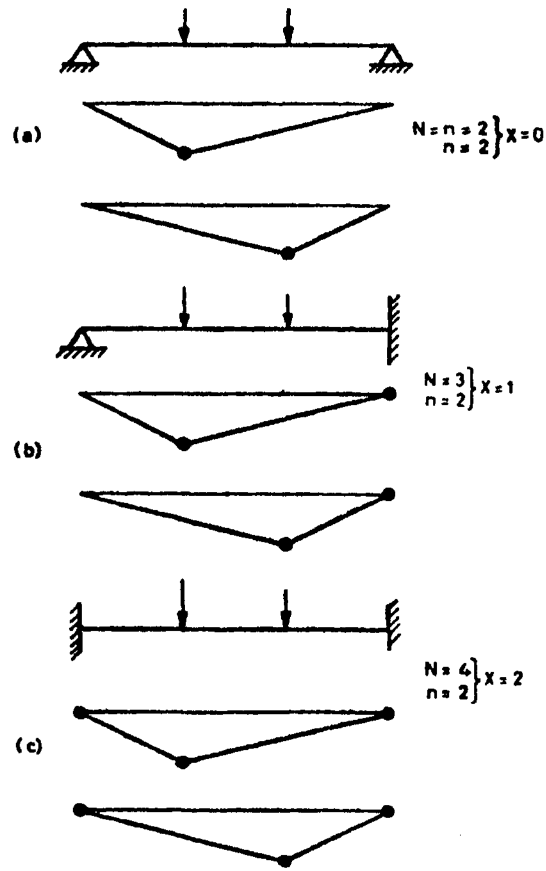


FIG. 31 EXAMPLES OF PROCEDURE FOR DETERMINING THE NUMBER OF MECHANISMS

procedure generally is to make combinations that involve mechanism motion by as many *loads* as possible and the elimination or cancellation of plastic *hinges* — as was done in composite Mechanism 3 of Example 3.

18.5 Indeterminacy — In order to determine the number of redundants, X, for use in Eq 35 it is merely necessary to cut sufficient supports and structural members such that all loads are carried by simple beam or cantilever action. The number of redundants is then equal to the number of forces and moments required to restore continuity. (In

Example 3, cutting the horizontal reaction at Section 5 -- supplying a roller — creates simple beam action; thus $X = 1$).

18.6 Geometry of Mechanisms (Instnataneous Centres)— As will later be evident, in cases involving sloping roofs [Fig. 30(f)], computation of the geometrical relationship of the displacement in the direction of the load as the structure moves through the mechanism may become somewhat tedious. In such cases, the method of instantaneous centres may be used, a term borrowed from mechanical engineering and the consideration of linkages.

Although the use of instantaneous centres was not needed in the solution of Example 3, consider its application to Mechanism 3 of this problem [Fig. 29(d) and Fig. 32]. When the structure moves, Segment 1-2-3 pivots around the base at 1. Member 5-4 pivots about Point 5. About what centre does Segment 3-4 move? The answer is obtained by considering how the ends of the Segment move.

Point 4 is constrained to move perpendicular to line 4-5 and thus its centre of rotation (as part of Segment 3-4) must be somewhere along line 5-4 extended. Point 3, on the other hand moves about point 1 since it is a part of Segment 1-2-3. Therefore it moves normal to line 1-3 and its centre of rotation as part of Segment 3-4 should be along line 1-3 extended. Point 1 satisfies both conditions and therefore Segment 3-4 rotates about Point 1, that point being its ' instantaneous centre ' of rotation.

What are the ' kink angles ' at the plastic hinges? The rotation at both column bases is θ. The horizontal motion of Point 4 is thus $(\theta)(L/2)$. Since the Length 1-4 is also equal to $L/2$, then the rotation of 3-4 about I is $\theta \dfrac{L/2}{L/2} = \theta$. The total rotation at location 4, therefore, equals 2θ and that at 3 is also 2θ since the Lengths 1-3 and 3-1 are equal.

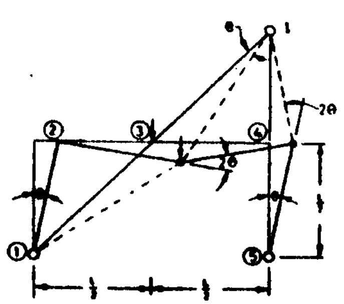


Fig. 32 Location of Instantaneous Centre for the Rectangular Frame Mechanism of Fig. 29

What is the vertical motion of the load at Point 3 ? Since no hinge forms in joint 2 it remains as a right angle and the rotation of 2-3 with respect to the horizontal is also equal to 0. The vertical motion is, therefore, $\theta L/2$. This answer for vertical displacement and that in the previous paragraph for kink angles are identical, of course, to those obtained in Example 3, Eq 33.

Example 4:

The suitable application of 'instantaneous centres' is to the case of gabled frames. Consider, for example, the structure shown in Fig. 33. Assign the value θ to the arbitrarily small rotation of

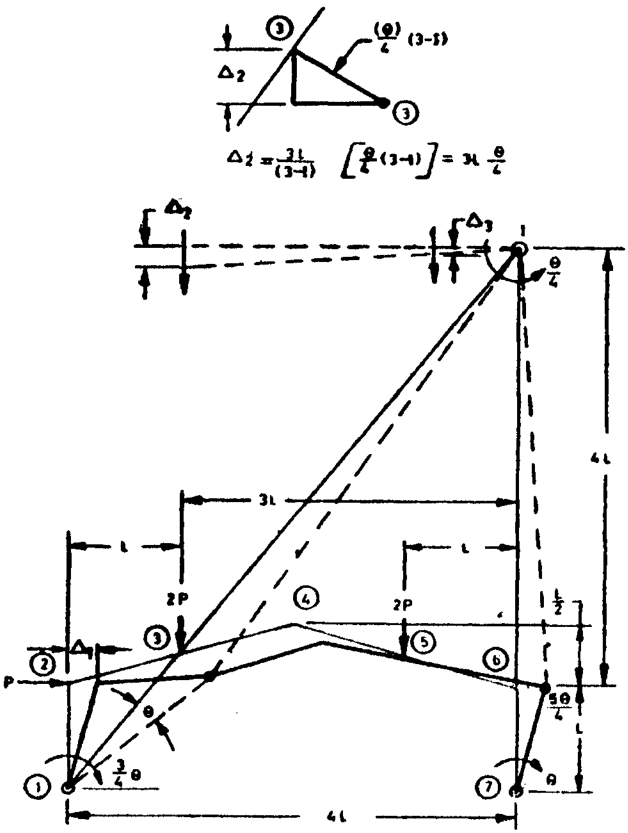


FIG. 33 LOCATION OF INSTANTANEOUS CENTRE OF A GABLED FRAME MECHANISM

member 6-7 about Point 7. Segment 1-2-3 will rotate about Point 1 an amount yet to be determined. To find the instantaneous centre of Segment 3-4-6, find the common point about which both *ends* rotate. Point 6, being constrained to move normal to line 7-6 will have its centre along that line. Similarly, the centre of 3 will be along 1-3 extended. Thus Point 1 is located.

By geometry the Length I-7 is equal to $(5L/4)(4)=5L$, therefore, 6-$I = 4L$. Since the horizontal displacement of Point 6 is θL, the rotation at $I = \theta/4$*. By similar triangles, the ratio of 3-I to 1-3 is 3: 1. Thus the rotation at 1 is given by

$$\frac{\theta}{4}\left(\frac{3}{1}\right)=\frac{3\theta}{4}$$

Kink angles and displacements in the direction of load may now be computed. The rotation at $6 = \theta+\theta/4 = (5/4)\theta$. The rotation at $3 = \theta/4+3/4\theta = \theta$. The displacements of the loads in the direction of application are as follows:

$$\begin{aligned} &\text{Horizontal load:} && \Delta_1=(3/4)(\theta)(L) \\ &\text{Left vertical load:} && \Delta_2=(\theta/4)(3L) \qquad \ldots \qquad \ldots(35a) \\ &\text{Right vertical load:} && \Delta_3=(\theta/4)(L) \end{aligned}$$

The accuracy of the last two equations may be seen in two ways. If the loads are imagined as hung from the dotted positions shown, then it is evident that the vertical displacements are as shown above and in Fig. 33(a). Alternatively, working out the geometry on the basis of similar triangles as shown in Fig. 33(b), the vertical component of the mechanism motion of Point 3 (for example) is equal to the rotation about the appropriate instantaneous centre multiplied by the distance to that centre measured normal to the line of action.

To complete the example the ultimate load for this *mechanism* is given by:

$$P\left(\frac{3\theta L}{4}\right)+2P\left(\frac{3\theta L}{4}\right)+2P\left(\frac{\theta L}{4}\right)=M_p\left(\theta+\frac{5\theta}{4}\right) \qquad \ldots(35b)$$

$$\begin{pmatrix}\text{Horizontal}\\ \text{Load}\end{pmatrix}\begin{pmatrix}\text{Vertical}\\ \text{Load } L\end{pmatrix}\begin{pmatrix}\text{Vertical}\\ \text{Load } R\end{pmatrix}\begin{pmatrix}\text{location}\\ 3\end{pmatrix}\begin{pmatrix}\text{location}\\ 6\end{pmatrix}$$

$$P_u=\frac{9}{11}\frac{M_p}{L} \qquad \ldots \qquad \ldots \qquad \ldots(35c)$$

Construction of the moment diagram shows that the moment is nowhere greater than M_p, so this is the correct answer.

---

*Note that the rotation at I is in general equal to the rotation at the column base multiplied by the ratio of the distances 7-6 to 6-I.

Precisely the same answer would have been obtained, of course, had we worked out the deformation at the various joints through a consideration of the frame geometry in the deformed position. The convenience of the use of 'instantaneous centres' should be evident, however.

19. FURTHER CONSIDERATIONS

19.1 Further Methods of Analysis — In addition to the statical and mechanism methods of analysis, there are additional techniques for determining the ultimate load which a structure will support. Two methods in particular are the 'method of inequalities'[23] and a psuedo 'Moment Distribution Technique'[24,25]. In a great majority of cases, however, recourse to those methods will not be necessary and, therefore, no further discussion is presented here. The interested reader may see to the indicated references.

19.2 Distributed Load — A slight modification of procedure is necessary in case the load is distributed. In the event that a mechanism involves formation of a hinge within the beam that is (between supports) the precise location of the hinge in the beam is not known in advance.

Take the case shown in Fig. 34 — a portion of a continuous beam — in which the M_p values are as shown in the circles. If the load is actually distributed along the member, then the correct value of the ultimate load is obtained by determining the distance to the point of maximum moment. The distance x can be computed by writing the virtual work equation in terms of x and either minimizing the loads by differenciation or by solving for x by making a few trials. Alternatively x may be found by plotting the uniform load parabola, A-$\underline{B}$-C-D, from the base line A-D.

To illustrate the computations, from the mechanism of Fig. 34(b) the virtual work Eq 25 gives:

$$W\frac{X}{2}\theta^{*} = 3M_p\theta\left(1+\frac{X}{L-X}\right)+2M_p\theta\frac{X}{L-X}$$

$$W = \frac{2M_p}{X}\left(3+\frac{3X}{L-X}+\frac{2X}{L-X}\right)$$

$$W = \frac{2M_p}{X}\left(3+\frac{5X}{L-X}\right) \quad \ldots \quad \ldots \quad \ldots\ldots(36)$$

---

*The external work for a mechanism under distributed load may conveniently be written as the load/unit length times area swept during mechanism motion. In this example 'area' $= (L)\ (\theta x)\ (1/2)$; work $= wL(\theta x)(1/2) = W\frac{X}{2}\theta$.

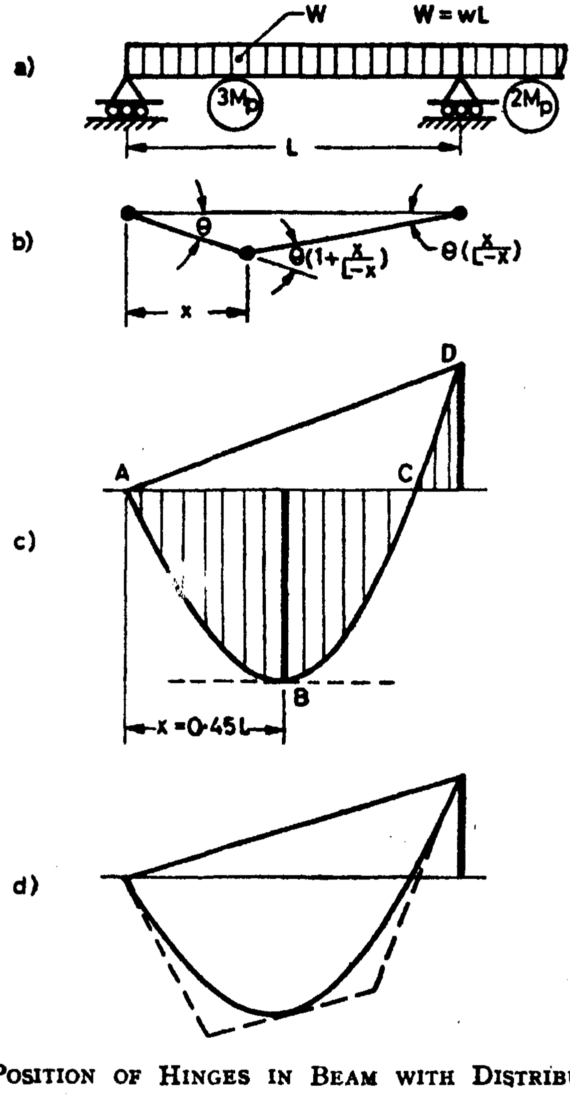


FIG. 34 POSITION OF HINGES IN BEAM WITH DISTRIBUTED LOAD

Selecting values of x and solving for W, the value of x to give the minimum value is:

$$x = 0{\cdot}44L$$

and

$$W_u = \frac{31{\cdot}3M_p}{L} \qquad \ldots \qquad \ldots \qquad \ldots(37)$$

The graphical method was used in Fig. 34 and a value $x = 0{\cdot}45L$ was obtained.

With errors that are usually slight, the analysis could be made on the basis that the distributed load is replaced by a set of equivalent concentrated loads. Thus in Fig. 35, if the distributed load, $wL = P$, is concentrated in the various ways shown, the uniform load parabola is always circumscribed (giving the same maximum shear). The result is always conservative because the *actual* moment in the beam is always less than or at most equal to the assumed moment. Of course, the more concentrated loads assumed, the closer is the approximation to actuality.

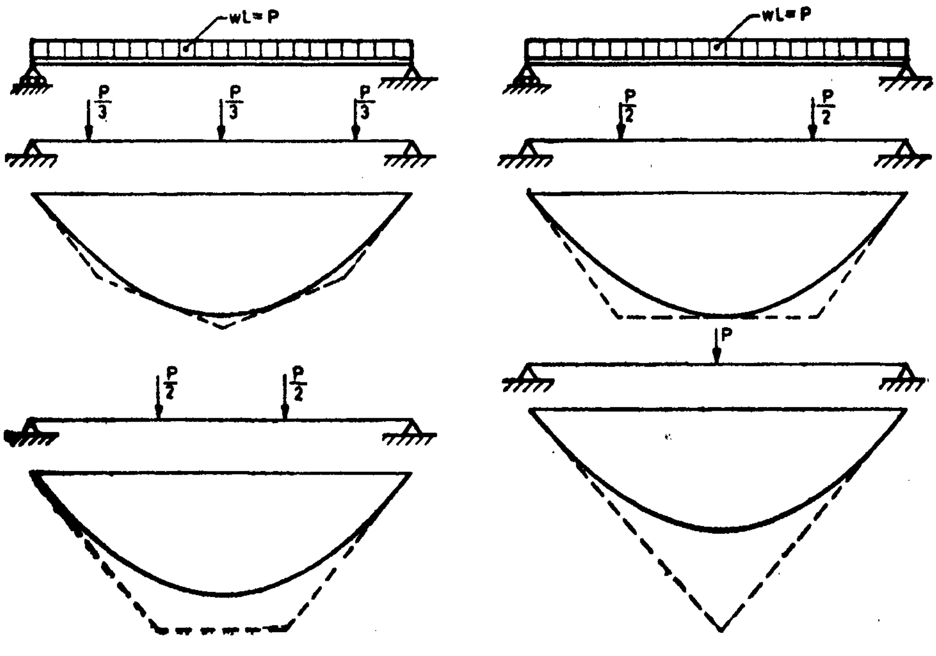


Fig. 35 Effect of Replacing a Distributed Load by an Equivalent Set of Concentrated Loads

Of course, if the distributed load is actually brought to the main frame through purlins and girts, the uniform load may be converted, at the outset, to actual purlin reactions (on the basis of assumed purlin spacing). The analysis is then made on the basis of the actual concentrated loads. The only difficulty with this procedure is that numerous additional possible plastic hinges are created — one at each purlin. And for every possible hinge position there is another possible mechanism. Of course, with experience the designer will be able to tell as to how many of these mechanisms he should investigate.

19.3 Moment Check — One of the conditions that a 'plastic' solution must satisfy is that the moment is nowhere greater than the plastic moment (*see* **16**). In the case of the Statical method (*see* **17**), there is no particular problem, because the moment used in the equations equilibrium presumed $M \leqslant M_p$. However, in the mechanism method the solution loads to an upper bound and it is consequently necessary to see if the solution also satisfies equilibrium with $M \leqslant M_p$ throughout the frame. Otherwise it is possible to overlook a more favourable combination of mechanisms which would have resulted in a lower load.

When the structure is *determinate* at ultimate load, the equations of simple statics are all that are necessary to determine the moments in all parts of the frame. However, when the structure is indeterminate at ultimate load, an elastic analysis would be required to determine precisely the moments in those segments that do not contain plastic hinges at their ends. However, in solutions by plastic analysis, the *precise* magnitude of moment at a section that remains elastic is not of interest. If a mechanism has already been created, it is only necessary to show that moments elsewhere are not greater than M_p. As a result, approximations may be used to find a possible equilibrium moment diagram. If the plastic moment condition is met, then the solution satisfies the lower bound principle, and the computed load should be the correct value.

Prior to considering the partially indeterminate cases further, it should be pointed out that a design which leads to such a condition (that is, part of the structure indeterminate) is probably not the best design. The design objective is to make *all* of the structure perform as efficiently as possible. If the frame is still indeterminate at ultimate load, it should be obvious that it is possible to save material somewhere in the structure, bringing moments up to their plastic values. What this means is that simple statics will usually be adequate for making the 'moment check'. As a routine procedure it will not be required to carry out what would otherwise be a more complicated checking operation, because a structure that turns out to be partially redundant would be redesigned for lighter structure.

Further examples of the moment check do not appear necessary here for the determinate cases. Example 3 given in **18.1**, and Design Examples 5 and 7 given at the end of this handbook are illustrative.

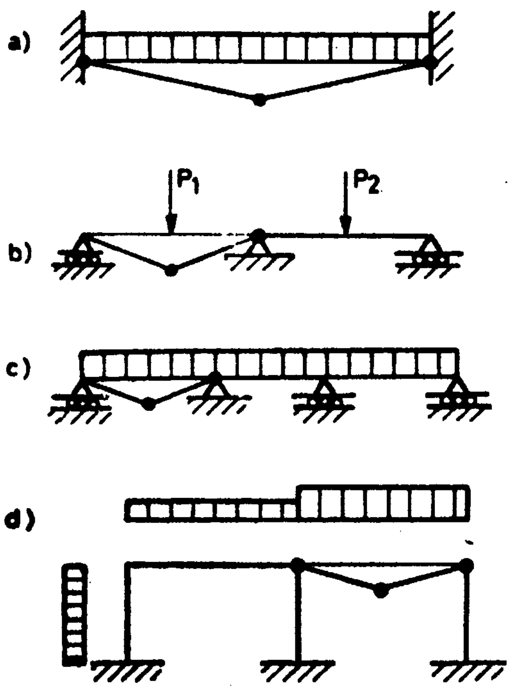
The first step in the case of indeterminate structures is to check on the redundancy. The following rule may be stated to indicate whether or not the structure at failure is determinate.

If X = number of redundancies in the original structure, and
M = number of plastic hinges developed

Then I, the number of remaining redundancies, is given by

$$I = X - (M - 1) \qquad \ldots \qquad \ldots \qquad \ldots (38)$$

In Fig. 36 are shown three continuous beams and a two span fixed base frame. Equation 38 correctly indicates the number of remaining redundants in Fig. 36(c) and 36(d). The structures are redundant at failure.



X = 2
M = 3
I = X − (M−1) = 2−2 = 0 ---(O K)

X = 1
M = 2
I = X−(M−1) = 1−1 = 0 ---- (OK)

X = 2
M = 2
I = X−(M−1) = 2−1 = 1 -----(OK)

X = 6
M = 3
I = X−(M−1) = 4 ------(OK)

FIG. 36 EXAMPLE OF PROCEDURE FOR DETERMINING THE NUMBER OF REMAINING REDUNDANCIES IN A STRUCTURE

If, now the frame is redundant, two methods are convenient for determining a *possible* equilibrium configuration. One is a 'trial and error' method and the other a 'moment-balancing' method. Where there are only one or two remaining redundancies (try Eq 38), the 'trial and error' method is most suitable. Since this covers most ordinary cases and since partial redundancy means inefficient design, the second method will not be treated[9]. By the 'trial and error' method, then, values for the remaining 'I' moments are guessed and the equilibrium equations solved for the remaining unknown.

Example 5:

Given a three-span continuous beam of uniform section, M_p, and with concentrated loads in each span (Fig. 37). Assume that the answer has been obtained on the basis of the assumed mechanism shown in Fig. 37(b). For this case:

$$P_u = \frac{3M_p}{L}^{*} \quad \ldots \quad \ldots \quad \ldots(39)$$

The remaining redundancies from Eq 38 are $I = X-(M-1) = 2-(2-1) = 1$ (namely the moment at E).

The next step is to assume a value for this moment (say $M_E = M_p$). Solving the equilibrium equation for spans CE and EG,

$$M_D = \frac{M_C}{2} + \frac{M_E}{2} + \frac{P_L}{4} = -\frac{M_p}{2} - \frac{M_p}{2} + \frac{3M_p}{4}^{\dagger}$$

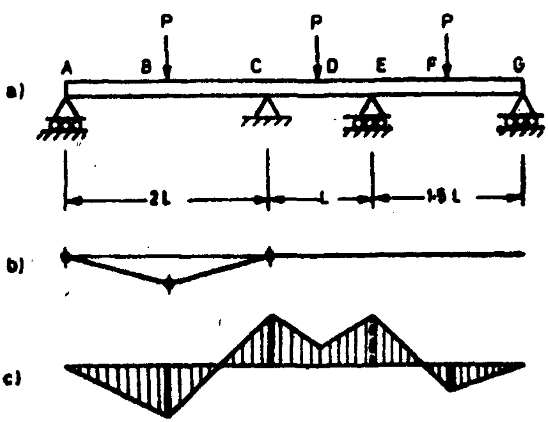


FIG. 37 MOMENT CHECK USING THE 'TRIAL AND ERROR' METHOD

*See Eq 28.
†See Eq 29.

$$M_D = -\frac{M_p}{4}$$

$$M_F = \frac{M_E}{2} + \frac{M_G}{2} + \frac{1{\cdot}5PL}{4} = -\frac{M_p}{2} + \frac{9}{8}M_p{}^*$$

$$M_E = +\frac{5M_p}{8}$$

The resulting moment diagram is shown by the dotted lines. Since $M \leqslant M_p$ throughout, the trial solution is correct and $P_u = \frac{3M_p}{L}$. Quite evidently, more efficient use of material would result if the design were revised to supply only the ***required*** plastic moment for each span.

In summary, this section has presented the basis for and the techniques of two methods of plastic analysis: the 'statical' and the 'mechanism' methods. Application to design will be discussed next in Section E, followed by design examples in Section F.

***Example* 6:**

The 'trial and error' method of making the moment check will be further illustrated for the frame shown in Fig. 38. Assuming that mechanisms 8-9-10 is the one to form, the ultimate load is given by:

$$P_u = \frac{4M_p}{L}$$

The remaining redundancies from this equation are

$$I = X-(M-1) = 6-(3-1) = 4$$

which shows that it is not possible to obtain four moments by statics. (There are a total of 7 unknown moments for which only 3 independent equilibrium equations are available).

The next step is to make a 'guess' as to the magnitude of moment at 4 hinge locations, and then to solve for the remaining values. If $M = M_p$ then the correct mechanism (and P_u-value) has been determined. The following 'trial' values are taken, using the sign conventions that the moment is positive if tension occurs on the 'dotted' side of the member:

$$M_4 = -M_p$$
$$M_6 = -M_p$$
$$M_8 = +M_p$$
$$M_2 = +M_p$$

*See Eq 29.

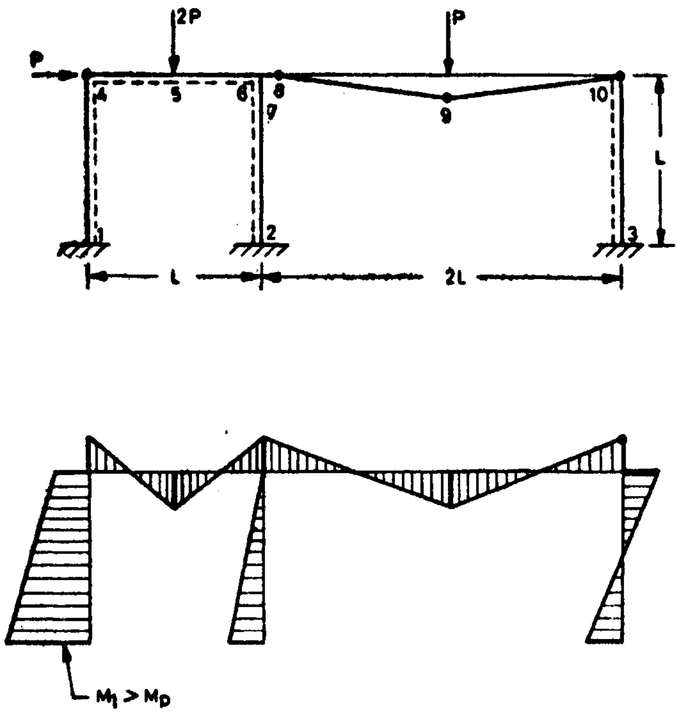


FIG. 38 MOMENT CHECK USING 'TRIAL AND ERROR' METHOD

For span 4-6

$$M_5 = \frac{M_4}{2} + \frac{M_6}{2} + \frac{2PL}{4} = \frac{M_p}{2} - \frac{M_p}{2} + \frac{2\left(4\,\frac{M_p}{L}\right)}{4}$$

$$M_5 = -M_p + 2M_p = +M_p$$

For joint 6-7-8

$$M_6 = M_8 - M_7 = 0$$
$$M_7 = M_6 - M_8 = -M_p + M_p = 0$$

From the sway equilibrium equation,

$$M_1 - M_2 - M_3 - M_4 + M_7 + M_{10} + PL = 0$$

$$M_1 = M_p + M_p - M_p + 0 + M_p - \left(\frac{4M_p}{L}\right)(L)$$

$$M_1 = -2M_p$$

Since $M_1 > M_p$, the plastic moment condition is violated and an incorrect assumption was made. The moment diagram based on the above calculation is shown in Fig. 38.

Example 7:

A moment check for the 2-storey, 2-span structure shown in Fig. 39 will now be made. The 'trial and error' method will again be employed. The plastic analysis gives:

$$P_u = \frac{2M_p}{L}$$

It is not possible to determine the number of redundants for this frame by Eq 38 because that relationship does not apply when 'simultaneous' mechanisms occur. We can, however, determine the number of redundants for this special case by noting that the number of remaining redundancies is equal to the number of remaining unknown moments minus the number of independent equilibrium equations (number of mechanisms) that were not used in the analysis.

The number of unknown moments is 10 (M_1, M_2, M_3, M_4, M_5, M_9, M_{10}, M_{14}, M_{15}, M_{19}). Out of the 10 original equilibrium equations, 4 have been used. Thus, the frame is redundant to the fourth degree. Accordingly, it should be possible to solve for the remaining moments by assuming the value of four of the unknowns.

Assume $M_{14} = +M_p$

$$M_5 = +\frac{M_p}{2}$$

$$M_3 = -\frac{M_p}{2}$$

$$M_2 = 0$$

For joint 18-19-20

$$M_{18} - M_{20} - M_{19} = 0$$
$$M_{19} = M_{18} - M_{20} = -M_p + M_p = 0 \ldots\text{OK}$$

For joint 13-14-15

$$M_{14} + M_{13} - M_{15} = 0$$
$$M_{15} = M_{13} + M_{14} = -M_p + M_p = 0 \ldots\text{OK}$$

For joint 8-11

$$M_8 + M_9 - M_{10} - M_{11} = 0$$
$$M_9 - M_{10} - M_{11} - M_8 = -M_p + M_p = 0$$
$$M_9 = M_{10}$$

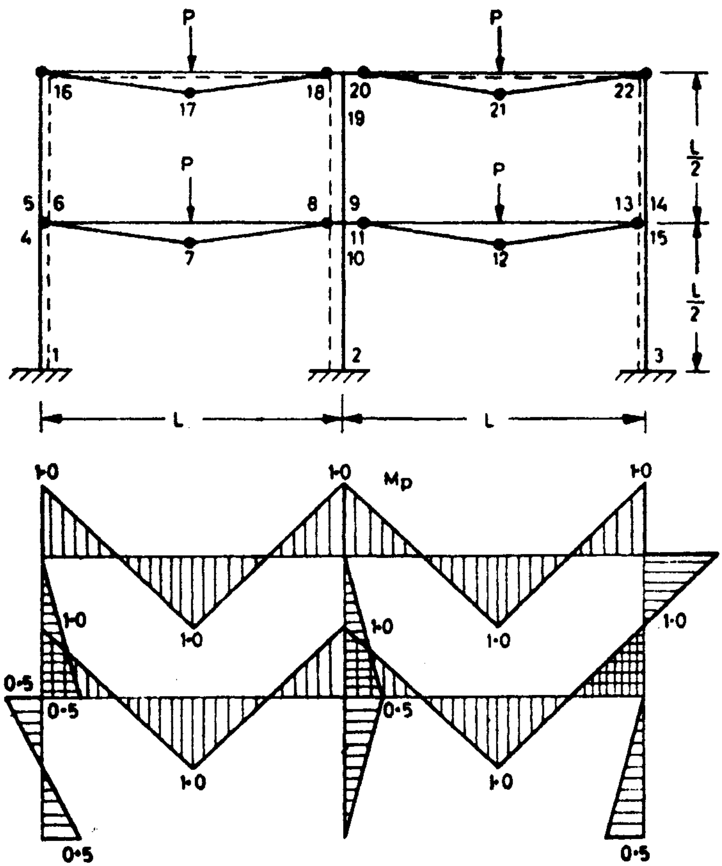


FIG. 39 MOMENT CHECK USING 'TRIAL AND ERROR' METHOD FOR TWO-STOREY TWO-SPAN STRUCTURE

For joint 4-6

$$M_4 - M_5 - M_6 = 0$$

$$M_4 = M_5 + M_6 = \frac{M_p}{2} - M_p = \frac{-M_p}{2} \ldots\ldots \text{OK}$$

From the sway equation for the top storey

$$M_5 - M_9 - M_{14} - M_{16} + M_{19} + M_{22} = 0$$

$$M_9 = M_5 - M_{14} - M_{16} + M_{19} + M_{22} = \frac{M_p}{2} - M_p + M_p + 0 - M_p$$

$$M_9 = \frac{-M_p}{2} \; \ldots . \text{OK}$$

Thus from Eq (a), $M_{10} = \dfrac{-M_p}{2} \ldots . \text{OK}$

From the sway equation for the bottom storey

$$M_1 - M_2 - M_3 - M_4 + M_{10} + M_{15} = 0$$

$$M_1 = M_2 + M_3 + M_4 - M_{10} - M_{15}$$

$$= 0 + \frac{M_p}{2} - \frac{M_p}{2} + \frac{M_p}{2} - 0$$

$$= + \frac{M_p}{2} \ldots . \text{OK}$$

The final moment diagram is shown in Fig. 39 and it is evident that $M \leqslant M_p$ throughout. Therefore the ultimate load is, in fact, equal to $\dfrac{2M_p}{L}$.

SECTION E

APPLICATION TO DESIGN

20. GENERAL

20.1 Thus far the methods of plastic analysis have been presented. The purpose of this section is to consider certain features involved in the application of these methods to actual design. A question that arises first concerns the relative strength of the different members. Next, a discussion of the general design procedure will outline the steps involved in a plastic design.

Finally, the principle content of this section will concern the 'secondary design considerations'. In arriving at the plastic methods of structural analysis certain assumptions were made with regard to the effect of axial force, shear, buckling, etc. Unless attention is given to such factors, the structure may not perform its intended function due to 'premature' failure.

21. PRELIMINARY DESIGN

21.1 On what basis is the first choice of relative plastic moment values made? In the various examples used to illustrate methods of analysis, the problem was to find the ultimate load for a given structure with known plastic moment values of its members. In design, the problem is reversed. Given a certain set of loads the problem is to select suitable members. Since 'uniform section throughout' may not be the most economical solution, some guide is needed for selecting the ratio or ratios of plastic moment strength of the various members.

21.2 Of course, this problem exists in elastic design, so it is not a matter that is unique to design on the basis of ultimate load. However, a few simple techniques will occur to the designer which, coupled with his experience, will enable him to make a preliminary economic choice of relative moment strength without too many trials. Some general principles are as follows:

a) In the event the critical mechanism is an 'independent' one, the rest of the material in the frame is not being used to full capacity. This suggests that a more efficient choice of moment ratios may be made such that the critical mechanism is a 'composite mechanism' involving plastic hinges in several different members.

b) Adjacent spans of continuous beams will often be most economically proportioned when the independent mechanisms for each span form simultaneously. This is illustrated in Design Example 2. Numerous examples of the design of continuous beams are given in Ref 26.

c) The formation of mechanisms simultaneously in different spans of continuous beams or the creation of composite mechanisms will not necessarily result in minimum weight. Examination of alternate possibilities is desirable. Often it will be found that the span involving the greatest determinate moment (M_s) should be given the greatest possible restraint (generally by supply equivalent Z of adjoining members). Thus the best design in this instance will usually result when the solution commences with uniform section for both the rafter and the stanchion. Design Example 7 illustrates this.

d) The absolute minimum beam section for vertical load is obtained if the joints provide complete plastic restraint (that is, restraining members supply a restraining plastic moment equal to that of the beam). Similarly, the minimum column sections are obtained under the action of sway forces when ends are subject to complete plastic restraint. This, therefore, suggests that, if the important loads are the vertical loads, the design might well be commenced on the basis that all joints are restrained as described, the ratio of beam sections be determined on this basis, and that the columns be proportioned to provide the needed joint moment balance and resistance to side load[24]. Design Example 7 is an illustration of this. Alternatively, if the important loads were side loads, the design could start, instead, with the columns.

e) Finally, it should be kept in mind that maximum overall economy is not necessarily associated with the most efficient choice of section for each span. It is necessary to consider fabrication conditions which may dictate uniform section where, theoretically, sections of different weight might be used.

22. GENERAL DESIGN PROCEDURE

22.1 Although there will be variations as to specific procedure and detail, the following six steps will be a part of practically every design:

a) Determine possible loading conditions,

b) Compute the ultimate load(s),

c) Estimate the plastic moment ratios of frame members,

d) Analyse each loading condition for maximum M_p,

e) Select the section, and

f) Check the result according to 'secondary design rules'.

These steps will now be discussed briefly.

The design commences with a determination of the possible loading conditions. There is no change here from conventional practice, except that at this stage it is decided whether to treat distributed loads as such or to consider them as concentrated (*see* 19).

The step (b), 'compute the ultimate load', represents a departure from conventional methods. The loads determined in (a) are multiplied by the appropriate load factor to assure the needed margin of safety. This load factor is selected in such a way that the real factor of safety for any structure is at least as great as that afforded in the conventional design of a simple beam. In the latter case, F is equal to the conventional 'factor of safety' (1·65) multiplied by the shape factor, f. As already noted, this shape factor varies for different WF beams from about 1·09 to about 1·23. The average for all shapes is 1·14 and the most common value is 1·12. The actual load factor selected thus depends upon the concept of safety; that is, if the present design of a beam with the smallest shape factor (1·09) is satisfactory, then a load factor of (1·65) (1·09) = 1·80 would be adequate. Alternatively, average values may be preferable. The following table summarizes the possibilities:

*Factor of Safety**	*Shape Factor*	*Load Factor*
1·65	1·09	1·80
1·65	1·12	1·85
1·65	1·14	1·88
1·65	1·23	2·03

The value 1·85 is selected instead of 1·88 because wide flange shapes with a factor of 1·12 occur more frequently and, further, the number 1·88 implies an accuracy in our knowledge of safety that is not justified. In the case of wind, earthquake, and other forces, specifications normally allow a one-third increase in stresses. Following this same philosophy the value of F for combined dead, live, and wind loading would be $3/4 \times 1{\cdot}85 = 1{\cdot}40$. In summary, then, the load factors are:

$$\left.\begin{array}{ll} \text{Dead load plus live load,} & F = 1{\cdot}85 \\ \text{Dead load plus live load,} & F = 1{\cdot}40 \\ \quad\text{plus wind, earthquake,} & \\ \quad\text{or other forces} & \end{array}\right\} \qquad \ldots(40)$$

As was suggested above, the load factor of safety should be selected in such a way that an indeterminate structure is as safe as a simple beam

*Yield stress divided by working stress in flexure.

designed elastically. There is certainly no point in making a rigid structure any *more* safe. There is no departure from present practice insofar as the *necessary* or minimum factor of safety. Plastic design simply makes it possible to design structures with a more nearly constant factor of safety, no matter what the loading and geometry.

The load factor of a safety of a simple beam according to elastic design is equal to the ratio of the ultimate load, P_u, divided by the working load, P_w. Since for a simple beam, the bending moment varies linearly with the load, the expression for the load factor may be written as:

$$F = \frac{P_u}{P_w} = \frac{M_p}{M_w}$$

From Eq 16 and using the relationship, $M_w = \sigma_w S$, the value of F may be expressed as:

$$F = \frac{\sigma_y Z}{\sigma_w S}$$

from which

$$F = \frac{\sigma_y}{\sigma_w} f$$

where σ_y is the yield stress level, and σ_w is the allowable or working stress according to 'elastic' specifications and f the shape factor. The load factor is thus a function of the ratio between yield stress and allowable stress and of the shape factor.

According to IS: 800-1962* the ratio σ_y/σ_w is $\frac{2\,320}{1\,500} = 1{\cdot}55$. The average shape factor is 1·15. Thus:

$$F = (1{\cdot}55)(1{\cdot}15) = 1{\cdot}78$$

A reasonable figure for the load factor for gravity loads figured according to IS: 800-1962* is thus 1·85.

Since section **12.2.1.1** of IS: 800-1962* permits a one-third increase in stresses when wind is acting, then the corresponding load factor for plastic design may be taken as:

$$F_w = 1{\cdot}85 \times 3/4 = 1{\cdot}40$$

It will be noted that the problems worked in the later portion of this chapter are developed on the basis of Eq 40. The only effect of a change in load factors to the values for use in designs according to Indian Standards is that the required section sizes would be reduced somewhat.

*Code of practice for use of structural steel in general building construction (*revised*).

The Step (c) is to make an estimate of the plastic moment ratio of the frame members. This has been discussed in 21. In outline the procedure would be as follows:

A) Determine the absolute plastic moment values for separate loading conditions. (Assume that all joints are fixed against rotation, but frame free to sway.) For beams, solve beam mechanism equation and for columns, solve the panel mechanism equation. The actual section will be greater than or at least equal to these values.

B) Now select plastic moment ratios using the following guides:

a) Beams: Use ratio determined in step (a)

b) Columns: At corner connections $M_p(\text{col}) = M_p(\text{beam})$

c) Joints: Establish equilibrium

We are then ready to proceed to step (d). In some cases it will be desireable prior to final selection of sections to examine the frame for further economy as may be apparent from a consideration of relative beam and sway moments.

In the step (d) each loading condition is analyzed for the maximum required M_p. Either the statical method (17) or the mechanism method (18) of analysis may be used. Alternatively the simplified procedures of Section G may be used for 'standard' geometrical and loading conditions for which charts and graphs are developed. The only difference in this step and in the analysis procedures of Section D is that the *lowest failure load* was sought in the latter, whereas now we are looking for the *Maximum required plastic moment* as a basis for selecting the section.

The step (e) is to select the section. The equation $M_p = \sigma_y Z$ is solved for Z and the section selected from an economy table arranged according to Z-values.

The step (f) (and a most important one) is to check the design to see that it satisfies the 'secondary design considerations', making sure that premature failure does not occur. This is the subject of the discussion which now follows in 23.

23. SECONDARY DESIGN CONSIDERATIONS

23.0 General — In all of the tests presented in 9 the results confirm in a satisfactory manner the predictions of the 'simple plastic theory'. This theory neglects such things as axial force, shear, and buckling, and yet the engineer knows they are present in most structures and he is accustomed to taking them into account. Those factors that are neglected or are not included in the 'simple' theory (and for which revision of that theory is sometimes needed) are the following:

a) Reductions in the plastic moment (axial force and shear force);

b) Instability (local buckling, lateral buckling, column buckling);
c) Brittle fracture;
d) Repeated loading; and
e) Deflections.

In addition, proper proportions of connections are needed in order that the plastic moment will be developed. In the following paragraphs the effect and characteristics of these factors will be indicated. Where apropriate, the results of theoretical analysis and of tests will be indicated, followed by a suggested 'rule' to serve as a guide for checking the suitability of the original design. Liberal reference is made to other sources in order to condense this article as much as possible.

It should be kept in mind that this situation is no different in principle from that encountered in elastic design. The design should always be checked for direct stress, shear, and so on. It simply means that modifications or limitations in the form of 'rules of design' are necessary as a guide to the suitability of a design based on the simple theory that neglects these factors.

23.1 Influence of Axial Force on the Plastic Moment — The presence of axial force tends to reduce the magnitude of the plastic moment. However, the design procedure may be modified easily to account for its influence because the important 'plastic hinge' characteristic is still retained. This influence has been discussed[9]. The stresss distribution in a beam at various stages of deformation caused by thrust and moment is shown in Fig. 40. Due to the axial force, yielding on the compression side proceeds that on the tension side. Eventually plastification occurs, but since part of the area must withstand the axial

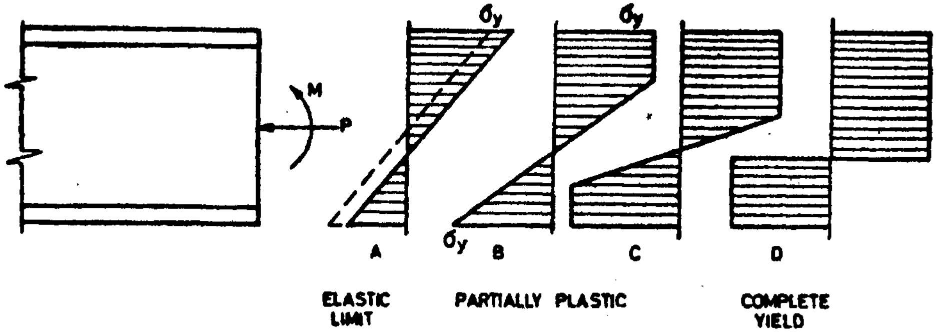


FIG. 40 DISTRIBUTION OF STRESS AT VARIOUS STAGES OF YIELDING FOR A MEMBER SUBJECTED TO BENDING AND AXIAL FORCES

force, the stress block no longer divides the cross-section into equal areas (as was the case of pure moment). Thus, as shown in Fig. 41 the total stress distribution may be divided into two parts — a stress due to axial load and a stress due to bending moment.

For the situation shown in Fig. 41 in which the neutral axis is in the web, the axial force P is given by:

$$P = 2\sigma_y y_o w \quad \ldots \quad \ldots \quad \ldots \quad \ldots(41)$$

where

σ_y = the yield stress,

y_o = the distance from the mid-height to the neutral axis, and

w = the web thickness.

The bending moment M_{pc} is given by the following expression and represents the plastic hinge moment modified to include the effect of axial compression:

$$M_{pc} = \sigma_y \left(Z - w y_o^2\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots(42)$$

where

Z = the plastic modulus. By substituting the value of y_o obtained from Eq 41 into Eq 43, the bending moment may be expressed as a function of the axial force P, or

$$M_{pc} = M_p - \frac{P^2}{4\sigma_y w} \quad \ldots \quad \ldots \quad \ldots \quad \ldots(43)$$

By the same process, an expression for M_{pc} as a function of P could be determined when the neutral axis is in the flange instead of the web.

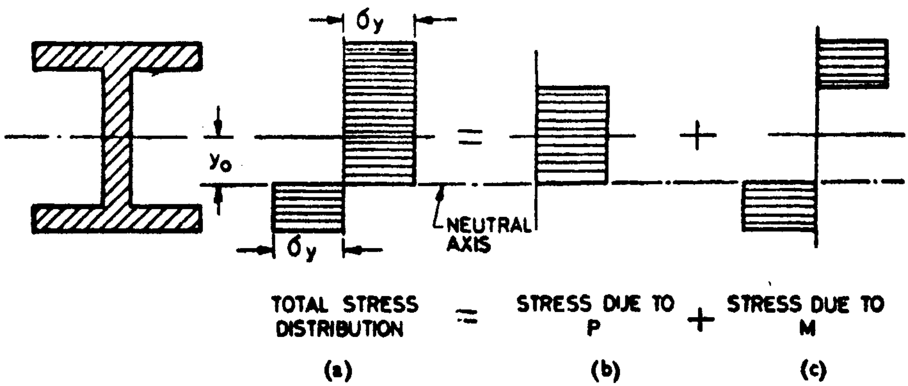


FIG. 41 REPRESENTATION OF STRESS DUE TO AXIAL FORCE AND DUE TO BENDING MOMENT FOR A COMPLETELY PLASTIC CROSS-SECTION SUBJECTED TO BENDING AND AXIAL FORCES

The resulting equation for a wide flange shape is:

$$M_{pc} = \frac{\sigma_y}{Z}\left[d\left(A-\frac{P}{\sigma_y}\right)-\frac{1}{2b}\left(A-\frac{P}{\sigma_y}\right)^2\right] \quad \ldots \quad \ldots(44)$$

For a wide flange section the 'interaction' curve that results from this analysis is shown in Fig. 42. When the axial force is zero, $M = M_p$. When the axial force reaches the value $P = yA$, then the moment capacity is zero. Between these limits the relationship is computed as described and the desired influence of axial force on the plastic moment has thus been obtained.

In design, in order to account for the influence of direct stress either curves such as Fig. 42 could be used, or since most wide flange shapes have a similar curve (when plotted on a non-dimensional basis)[9] the simple approximation of Fig. 43 could be used.

Summarizing, the following 'design rule' may be stated:

Rules for Beams

Rule 1 Axial Force — Neglect the effect of axial force on the plastic moment unless $P > 0{\cdot}15P_y$. If P is greater than 15 percent of P_y the modified plastic moment is given by:

$$M_{pc} = 1{\cdot}18\left(1-\frac{P}{P_y}\right)M_p \quad \ldots \quad \ldots \quad \ldots(45)$$

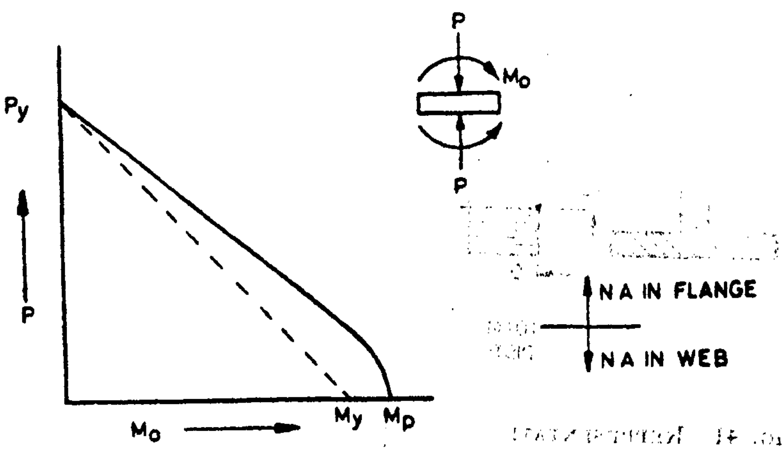


FIG. 42 INTERACTION CURVE FOR A WIDE FLANGE BEAM

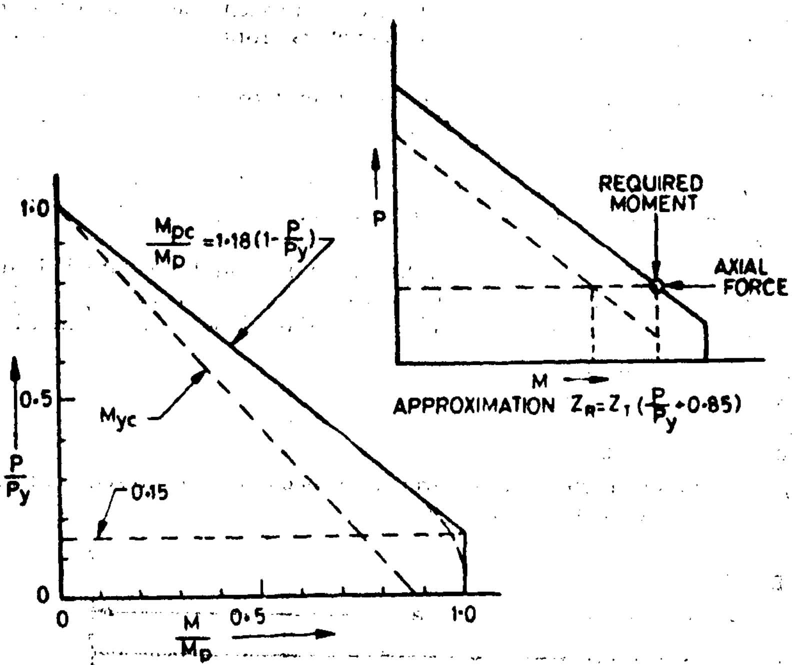


FIG. 43 DESIGN APPROXIMATION FOR LOAD *v* MOMENT INTERACTION CURVE

The required design value of Z for a member is determined by multiplying the value of Z found in the initial design by the ratio M_p/M_{pc} or

$$Z_{req} = \frac{0.85Z^*}{1 - P/P_y} \quad \ldots \quad \ldots \quad \ldots \quad \ldots(46)$$

An illustration of the use of this 'rule' is given in Design Example 7.

Equation 45 may also be expressed in the form:

$$\frac{M_{pc}}{1.18M_p} + \frac{P}{P_y} \leq 1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots(45a)$$

*Actually this gives a value of Z that is too great. As illustrated by the upper portion of Fig. 43, the P/P_y ratio will be less in the re-design and thus the reduction in M_p will be less than first computed. The equation

$$Z_R = Z_T(P/P_y + 0.85) \quad \ldots(47)$$

is an approximation to account for this effect, P/P_y being the ratio obtained in the first design. The final selection should be checked by the use of Eq 46.

Depending on the particular problem and approach the designer wishes to use, either Eq 45, 45a or 47 whichever is appropriate as explained below:

a) Equation 45 is appropriate if one wants to know the magnitude of moment that a given shape will transmit in the presence of axial force P.

b) Equation 47 is suitable if the problem is to obtain the required plastic modulus for $P/P_y > 0{\cdot}15$ in one step without trial and error procedures.

c) Equation 45a gives the condition that should be satisfied at a given cross-section and intimates a 'cut and try' procedure. The precautions of Rules R5 to R8 should be borne in mind.

23.2 The Influence of Shear Force — The effect of shear force is somewhat similar to that of axial force — it reduces the magnitude of the plastic moment. Two possibilities of premature 'failure' due to the presence of shear exist:

a) General shear yield of the web may occur in the presence of high shear-to-moment ratios. (sections at A and B of Fig. 44).

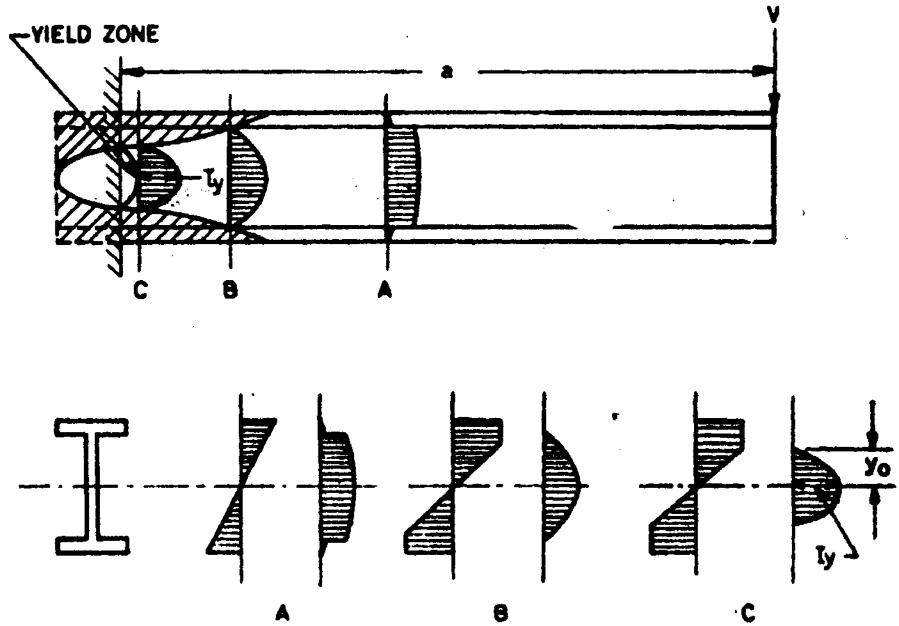


FIG. 44 SHEAR AND FLEXURAL STRESS DISTRIBUTION IN A CANTILEVER BEAM THAT HAS PARTIALLY YIELDED IN BENDING

b) After the beam has become partially plastic at a critical section due to flexural yielding, the intensity of shear stress at the centre-line may reach the yield condition (section at C of Fig. 44)[12,27].

Recent studies have shown that for structural steel with marked strain-hardening properties, behaviour '*b*' need not be considered and it is only necessary to guard against the possibility of complete shear yielding of the web.

For case (a) the maximum possible shear as given by:

$$V = T_y . A_w \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots (48)$$

$$T_y = \frac{\sigma_y}{\sqrt{3}} \text{ and } A_w = w(d-2t), \text{ then}$$

$$V = \frac{\sigma_y}{\sqrt{3}} \, w(d-2t)$$

Since for wide flange shapes $\frac{d}{d-2t} = 1{\cdot}05$, and using $\sigma_y = 2\,520$ kg/cm^2 then the following design guide may be formulated:

Rule 2 Shear Force — The maximum allowable shear force (in kg) in a beam at ultimate load is to be computed from:

$$V_{max} = 1\,265wd \qquad \ldots \qquad \ldots \qquad \ldots (49)$$

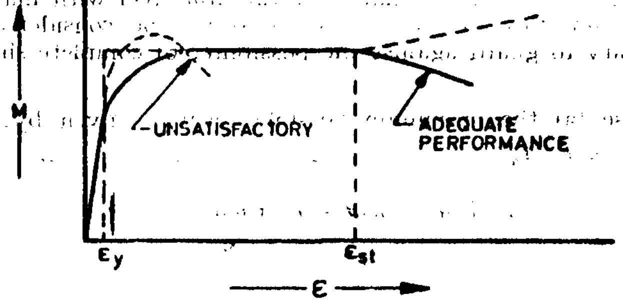
where

w = web thickness in cm and d is the section depth in cm.

23.3 Local Buckling of Flanges and Webs — As a wide flange beam is strained beyond the elastic limit eventually the flange or the web will buckle. Although stocky sections could be expected to retain their cross-sectional form through considerable plastic strain, with thin sections local buckling might occur soon after the plastic moment was first reached. Due to failure of a beam to retain its cross-sectional shape, the moment capacity would drop off; thus the rotation capacity would be inadequate. Therefore, in order to meet the requirements of deformation capacity (adequate rotation at M_p values) compression elements should have width-thickness ratios adequate to insure against premature plastic buckling.

A solution to this complicated plate buckling problem has been achieved by requiring that the section will exhibit a rotation capacity that corresponds to a compression strain equal to the strain-hardening value, ϵ_{st} (Fig. 2). At this point the material properties may be more accurately and specifically defined than in the region between ϵ_y and ϵ_{st}.

The result of this analysis for flanges of wide flange shapes is shown in Fig. 45 together with the results of tests. From these curves and



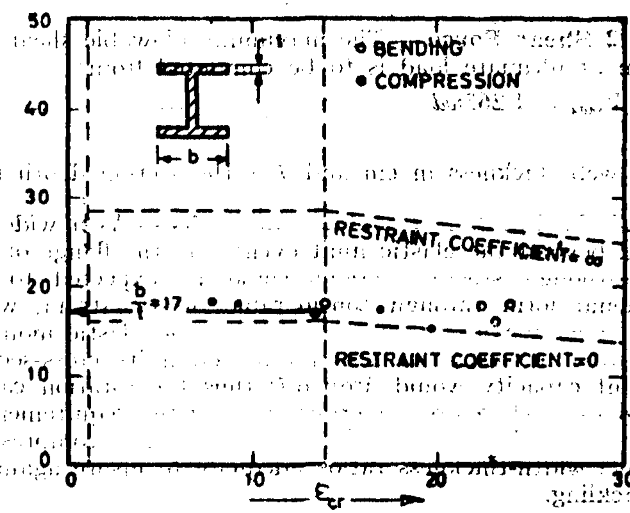


FIG. 45 RESULTS OF TESTS AND THEORY SHOWING A CRITERION FOR ASSUMING THAT THE HINGE MOMENT WILL BE MAINTAINED UNTIL STRAIN HARDENING IS REACHED

from similar relationships established for webs, the following design guide may be established to assure that the compressive strains may reach ϵ_{st} without buckling.

Rule 3 Compression Members — Compression elements, that would be subjected to plastic bending and hinge rotation under ultimate loading, shall have width-thickness ratios no greater than the following:

Flanges of rolled shapes and flange plates of similar built-up shapes, 17; for rolled shapes an upward variation of 3 percent may be tolerated. The thickness of sloping flanges may be taken as their average thickness.

Stiffeners and that portion of flange plates in box-sections and cover plates included between the free edge and the first longitudinal row of rivets or connecting welds, 8·5.

That portion of flange plates in box-sections and cover plates included between longitudinal lines of rivets or connecting welds, 32.

The width-thickness ratio of beam and girder webs subjected to plastic bending without axial loading shall not exceed 70. The width-thickness ratio for the web of beams, girders and columns designed for combined axial force and plastic bending moment at ultimate loading, shall be limited by the following formula but need not be less than 40:

$$\frac{d}{W} \leqslant 70-100\frac{P}{P_y} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots(50)$$

In Ref 17 is treated the influence of axial force on web buckling. Based upon this work, the adequacy of Rule 3 may be shown approximately.

Stiffening would be used where the requirements of Rule 3 were not met. Fortunately, nearly all Indian Standard beam section (*see* IS: 808-1964*) are satisfactory in this regard for $P/P_y < 0{\cdot}15$.

23.4 Lateral Buckling — The effect of lateral buckling is much like that of local buckling. In fact, in many tests the two frequently occur simultaneously. The problem of specifying the critical length of beam such that premature lateral buckling will be prevented has not been completely solved. Currently, studies are being made somewhat along the lines of those which proved to be successful in the case of local buckling. Although this study is not yet finished the results of tests and analyses to date provide some present guidance for the designer. The problem is to specify bracing requirements to prevent deformation out of the plane of the frame.

Yielding markedly reduces the resistance of a member to lateral buckling. Therefore bracing will be required at these points at which plastic hinges are expected. Intermediate between these critical sections, conventional rules may be followed to protect against elastic lateral buckling. In the event that consideration of the moment diagram

*Specification for rolled steel beams, channel and angle sections (*revised*).

reveals that a considerable length of a beam is strained beyond the elastic limit (such as in a region of pure moment) then additional lateral support at such a hinge may be required. The following guide may be used:

'The maximum laterally unsupported length of members designed on the basis of ultimate loading need not be less than that which would be permitted for the same members designed under the provisions of IS: 800-1962* except at plastic hinge locations associated with the failure mechanism. Furthermore, the following provisions need not apply in the region of the last hinge to form in the failure mechanism assumed as the basis for proportioning a given member, nor in members oriented with their weak axis normal to plane of bending. Other plastic hinge locations shall be adequately braced to resist lateral and torsional displacement.

Rule 4 Lateral Bracing — The laterally unsupported distance l_{cr}, from such braced hinge locations to the nearest adjacent point on the frame similarly braced, need not be less than that given by the formula:

$$l_{cr} = \left(60 - 40\frac{M}{M_p}\right) r_y \qquad \dots \qquad \dots \qquad \dots (51)$$

nor less than 35 r_y, where

r_y = the radius of gyration of the member about its weak axis,

M = the lesser of the moments at the ends of the unbraced segment, and

M/M_p = the end moment ratio, positive when M and M_p have the same sign and negative when they are of opposite sign, signs changing at points of contraflexure.

Members built into a masonry wall and having their web perpendicular to this wall may be assumed to be laterally supported with respect to their weak axis of bending.

The magnitude of the forces required to prevent lateral buckling is small and slenderness ratio requirements will normally govern. Both the compression and the tension flanges must be braced at changes of section. Design examples in **26** illustrate a procedure for checking lateral bracing.

Equation 51 not only assures that the cross-section will be able to plastify (develop the full plastic moment) but also be able to rotate through a sufficient inelastic angle change to assure that all necessary plastic hinges will develop. In deriving this equation, the basis lateral

*Code of practice for use of structural steel in general building construction (*revised*).

buckling equation[48] has been used, the analysis being based on an idealized cross-section that consists of only two flanges separated by the web-distance. Therefore, it already reflects and, in fact, makes use of the parameters L/b and d/t. Using the elastic constants of the material, and considering idealized behaviour as shown in Fig. 21, it may be shown that this procedure leads to a critical slenderness ratio of about 100. (*See* also the footnote in Appendix B.)

While this might be reasonable for a section that was only called upon to support M_p, it is unlikely that the resulting critical bracing would allow much inelastic rotation — a rotation that is ordinarily required at the first plastic hinge. It will be adequate to require only that plastic yield penetrate through the flange[10]. It is quite evident from Fig. 18, however, that the resulting further inelastic hinge rotation thus available is relatively small. One of the important contributions of Ref 18 was that it developed methods of correlating the critical length for lateral buckling with the magnitude of required hinge rotation.

23.5 Columns — The plastic theory assumes that failure of the frame (in the sense that a mechanism is formed) is not preceded by column instability. Although the load at which an isolated column will fail when it is loaded with axial force and bending moment can be predicted with reasonable accuracy, the buckling problem becomes extremely complex when the column is a part of a framework. Since a complete solution to this problem is not in hand, somewhat over-conservative simplifications must be made.

Rule 1 would suggest (and the results of tests confirm) that if the axial load is relatively low and, further, if the moment is maximum at the ends of the member, then the stability problem may be neglected. On the other hand, if an examination of the moment diagram shows that the column is bent in single curvature, then a more serious situation exists and a modification would certainly be necessary to assure a safe design.

The following design guide for columns in industrial frames may be immediately formulated:

Rules for Columns

Rule 5 — In the plane of bending of columns which would develop a plastic hinge at ultimate loading, the slenderness ratio l/r shall not exceed 120, l being taken as the distance centre-to-centre of adjacent members connecting to the column or the distance from such a member to the base of the column. The maximum axial load P on such columns at ultimate loading shall not exceed six-tenths P_y, where P_y is the product of yield point stress times column area.

Rule 6 — Columns in continuous frames where sidesway is not prevented (a) by diagonal bracing, (b) by attachment to an adjacent structure having ample lateral stability, or (c) by floor slabs or roof decks secured horizontally by walls or bracing system parallel to the plane of the continuous frames shall be so proportioned that:

$$\frac{2P}{P_y} + \frac{1}{70r} \leq 1\cdot0$$

Rule 7 — Except as otherwise provided in this section, M_o/M_p, the ratio of allowable end moment to the full plastic bending strength of columns and other axially loaded members, shall not exceed unity nor the value given by the following formulas, where they are applicable:

Case I — For columns bent in double curvature by moments producing plastic hinges at both ends of the columns:

$$1\cdot18 - 1\cdot18\left(\frac{P}{P_y}\right)$$

Case II — For pin-based columns required to develop a hinge at one end only, and double curvature columns required to develop a hinge at one end when the moment at the other end would be less than the hinge value:

$$B - G\left(\frac{P}{P_y}\right)$$

the numerical values for B and G, for any given slenderness ratio in the plane of bending, l/r, being those listed in Table 1.

Case III — For columns bent in single curvature by end moments of opposite sign:

$$1\cdot0 - K\left(\frac{P}{P_y}\right) - J\left(\frac{P}{P_y}\right)^2$$

the numerical values for K and J being those given in Table 2. For Case II columns where l/r in the plane of bending is less than 60, and for Case I columns, the full plastic strength of the member may be used ($M_o = M_p$) when P/P_y would not exceed 0·15.

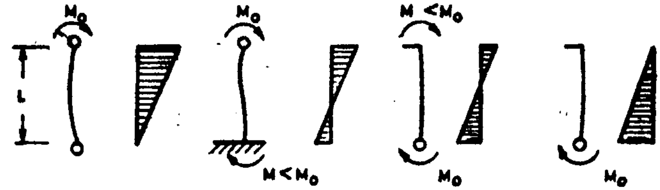
Rule 8 — In no case shall the ratio of axial load to plastic load exceed that given by the following expression:

$$\frac{P}{P_y} = \frac{8\,700}{(l/r)^2} \text{ when } \frac{l}{r} > 120$$

where l and r are the unbraced length and radius of gyration of the column in the plane normal to that of the continuous frame under consideration.

TABLE 1 CASE II PIN BASED COLUMNS, VALUES OF *B* AND *G*

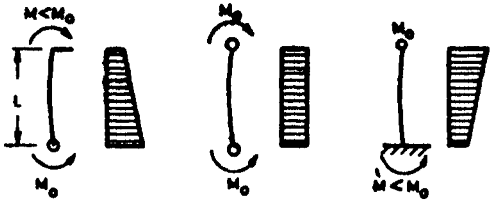
$$\frac{M_o}{M_p} = B - G\frac{P}{P_y}$$



l/r	*B*	*G*	*l/r*	*B*	*G*	*l/r*	*B*	*G*
16	1·140	1·172	51	1·164	1·271	86	1·201	1·616
17	1·140	1·174	52	1·165	1·276	87	1·202	1·633
18	1·141	1·177	53	1·165	1·281	88	1·204	1·651
19	1·141	1·179	54	1·166	1·286	89	1·205	1·669
20	1·142	1·182	55	1·167	1·292	90	1·206	1·688
21	1·142	1·184	56	1·168	1·297	91	1·207	1·707
22	1·143	1·187	57	1·169	1·303	92	1·209	1·726
23	1·143	1·189	58	1·170	1·310	93	1·210	1·746
24	1·144	1·191	59	1·171	1·316	94	1·211	1·767
25	1·145	1·194	60	1·172	1·323	95	1·213	1·788
26	1·145	1·196	61	1·173	1·330	96	1·214	1·810
27	1·146	1·198	62	1·174	1·337	97	1·215	1·832
28	1·146	1·200	63	1·175	1·344	98	1·217	1·855
29	1·147	1·203	64	1·176	1·352	99	1·218	1·879
30	1·148	1·205	65	1·177	1·360	100	1·220	1·903
31	1·148	1·207	66	1·178	1·369	101	1·221	1·928
32	1·149	1·209	67	1·179	1·377	102	1·222	1·953
33	1·150	1·212	68	1·180	1·386	103	1·224	1·979
34	1·150	1·215	69	1·181	1·396	104	1·225	2·006
35	1·151	1·217	70	1·182	1·406	105	1·227	2·033
36	1·152	1·220	71	1·183	1·416	106	1·228	2·061
37	1·152	1·222	72	1·184	1·426	107	1·230	2·090
38	1·153	1·225	73	1·186	1·437	108	1·231	2·119
39	1·154	1·228	74	1·187	1·448	109	1·233	2·149
40	1·155	1·231	75	1·188	1·460	110	1·234	2·179
41	1·155	1·234	76	1·189	1·472	111	1·236	2·211
42	1·156	1·237	77	1·190	1·485	112	1·237	2·243
43	1·157	1·240	78	1·191	1·497	113	1·239	2·275
44	1·158	1·243	79	1·192	1·511	114	1·240	2·309
45	1·159	1·247	80	1·194	1·524	115	1·242	2·343
46	1·159	1·251	81	1·195	1·539	116	1·243	2·378
47	1·160	1·254	82	1·196	1·553	117	1·245	2·414
48	1·161	1·258	83	1·197	1·568	118	1·247	2·450
49	1·162	1·263	84	1·198	1·584	119	1·248	2·487
50	1·163	1·267	85	1·200	1·600	120	1·250	2·525

TABLE 2 CASE III COLUMNS BENT IN SINGLE CURVATURE, VALUES OF *K* AND *J*

$$\frac{M_o}{M_p} = 1{\cdot}0 - K\left(\frac{P}{P_y}\right) - J\left(\frac{P}{P_y}\right)^2$$



l/r	K	J	l/r	K	J	l/r	K	J
1	0·434	0·753	41	1·015	0·149	81	1·824	−0·738
2	0·449	0·736	42	1·032	0·133	82	1·850	−0·769
3	0·463	0·720	43	1·048	0·116	83	1·877	−0·801
4	0·478	0·703	44	1·064	0·099 8	84	1·903	−0·833
5	0·492	0·687	45	1·081	0·083 2	85	1·930	−0·866
6	0·506	0·671	46	1·097	0·066 3	86	1·958	−0·900
7	0·520	0·655	47	1·114	0·049 2	87	1·986	−0·984
8	0·534	0·640	48	1·131	0·031 8	88	2·014	−0·969
9	0·548	0·624	49	1·148	0·014 3	89	2·042	−1·004
10	0·562	0·609	50	1·166	−0·003 6	90	2·071	−1·041
11	0·576	0·594	51	1·183	−0·021 7	91	2·101	−1·077
12	0·590	0·579	52	1·201	−0·040 1	92	2·130	−1·115
13	0·604	0·564	53	1·219	−0·058 8	93	2·161	−1·153
14	0·619	0·549	54	1·237	−0·077 7	94	2·191	−1·192
15	0·633	0·534	55	1·256	−0·097 0	95	2·222	−1·231
16	0·647	0·519	56	1·274	−0·117	96	2·254	−1·272
17	0·661	0·504	57	1·293	−0·137	97	2·286	−1·313
18	0·675	0·490	58	1·312	−0·157	98	2·318	−1·354
19	0·689	0·475	59	1·332	−0·177	99	2·350	−1·397
20	0·703	0·461	60	1·351	−0·198	100	2·384	−1·440
21	0·717	0·447	61	1·371	−0·220	101	2·417	−1·484
22	0·731	0·432	62	1·391	−0·241	102	2·451	−1·529
23	0·746	0·418	63	1·411	−0·263	103	2·486	−1·575
24	0·760	0·403	64	1·432	−0·286	104	2·521	−1·621
25	0·774	0·389	65	1·452	−0·309	105	2·556	−1·668
26	0·789	0·374	66	1·473	−0·332	106	2·592	−1·716
27	0·803	0·360	67	1·495	−0·356	107	2·628	−1·765
28	0·818	0·345	68	1·516	−0·380	108	2·665	−1·814
29	0·832	0·331	69	1·538	−0·404	109	2·703	−1·865
30	0·847	0·316	70	1·560	−0·429	110	2·741	−1·916
31	0·862	0·301	71	1·583	−0·455	111	2·779	−1·968
32	0·877	0·287	72	1·605	−0·481	112	2·816	−2·021
33	0·892	0·272	73	1·628	−0·507	113	2·857	−2·057
34	0·907	0·257	74	1·652	−0·534	114	2·897	−2·123
35	0·922	0·242	75	1·675	−0·562	115	2·937	−2·185
36	0·937	0·227	76	1·699	−0·590	116	2·978	−2·242
37	0·953	0·211	77	1·724	−0·618	117	3·020	−2·300
38	0·968	0·196	78	1·748	−0·647	118	3·062	−2·358
39	0·984	0·180	79	1·773	−0·677	119	3·104	−2·147
40	1·000	0·165	80	1·799	−0·707	120	3·147	−2·478

As already implied, the failure load of a column and its ability to transmit plastic moments are dependent upon the loading conditions. These are as follows:

a) Double curvature with plastic hinges at both ends,
b) Double curvature with plastic hinge at one end and opposite end intermediate between pinned and at plastic hinge value,
c) Single curvature with one end pinned and moment applied at the opposite end,
d) Single curvature with unequal end moments, and
e) Single curvature with equal end moments.

Ref 33 treats these cases and develops formulas that will assist the designer. However, most of the column problems that arise in the structures considered in this handbook will not require the corresponding refinements.

Whenever it is found that conditions for the preceding 'rules' are not met, it will be conservative to use the solution for (c) above, as given in Rule 7 for Case III.

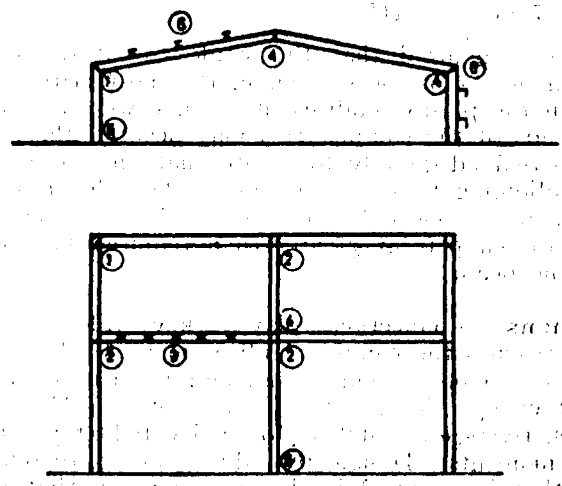
It will be recognized that the single curvature loading condition places the mid height of the column in the most critical loading condition. Thus, in the plastic analysis, if a hinge were assumed to form in one or both ends of the column, this may occur if the column strength has been increased adequately to assure that any necessary hinges will form in the adjoining beams. Therefore, when the design is complete, the column should be selected so that it will have an actual end moment capacity five to ten percent greater than required for the development of hinges in the beam.

23.6 Connections — Connections play a key role in assuring that the structure reaches the computed ultimate load. Points of maximum moment usually occur at connections; and further, at corners the connections must change the direction of the forces. Also, the connecting devices (welds, rivets, or bolts) are often located at points subjected to the greatest moments. Design procedures must, therefore, assure the performance that is assumed in design — namely, that connection will develop and subsequently maintain the required moment.

The ability of fabricaters to successfully connect members by welding has lent impetus in recent years to the application of plastic design methods; because by welding it is possible to join members with sufficient strength that the full plastic moment may be transmitted from one member to another. However, this is but one of the methods of fabrication for which plastic design is suitable. Plastic design is also applicable to structures with partially welded (top plate) or with riveted or bolted connections whenever demonstrated that they will allow the formation of hinges.

The various types of connections that might be encountered in steel frame structures are shown in Fig. 46 and are as follows: corner connections (straight, haunched), beam-column connections, beam-to-girder connections, splices (beam, column, roof), column anchorages, miscellaneous connections (purlins, girts, bracing). Primary attention is given to corner connections and to beam-column connections, but similar approaches may be used when considering the other connection types.

23.6.1 *Requirements for Connections* — The design requirements for connections are introduced by considering the general behaviour of different corner connection types as observed under load. This has been done in Ref 19 and 9 and it is thus possible to formulate four principal design requirements — requirements that in principle are common to all connections. These are (a) strength, (b) stiffness, (c) rotation capacity,



TYPES

1. Corner
2. Beam-column
3. Beam-girder
4. Splice
5. Column Base
6. Miscellaneous

FIG. 46 TYPES OF CONNECTIONS IN BUILDING FRAMES ACCORDING TO THEIR FUNCTION and (d) economy. They are now discussed in the light of the behaviour of corner and interior connections:

a) *Strength* — The connection should be designed in such a way that the plastic moment (M_p) of the members (or the weaker of the two members) will be developed. For straight connections the critical or 'hinge' section is assumed at point H in Fig. 47(a). As will be seen below, for haunched connections, the critical sections are assumed at R_1 and R_2, Fig. 47(b).

b) *Stiffness* — Although it is not essential to the development of adequate strength of the completed structure, it is desirable that average unit rotation of the connecting materials does not exceed that of an equivalent length of the rolled beam being joined. It would be an unusual situation, in which deflections of the structure were extremely critical, that this requirement would be applicable. The equivalent length is the length of the connection or haunch measured along the frame line. Thus in Fig. 47(a):

$$\Delta L = r_1 + r_2 \qquad \ldots \qquad \ldots \qquad \ldots(53)$$

This requirement reduces to the following:

$$\theta_h \leq \frac{M_p}{EI} \Delta L \qquad \ldots \qquad \ldots \qquad \ldots(54)$$

which states that the change in angle between sections R_1 and R_2 as computed shall not be greater than the curvature (rotation per unit of length) times the equivalent length of the knee.

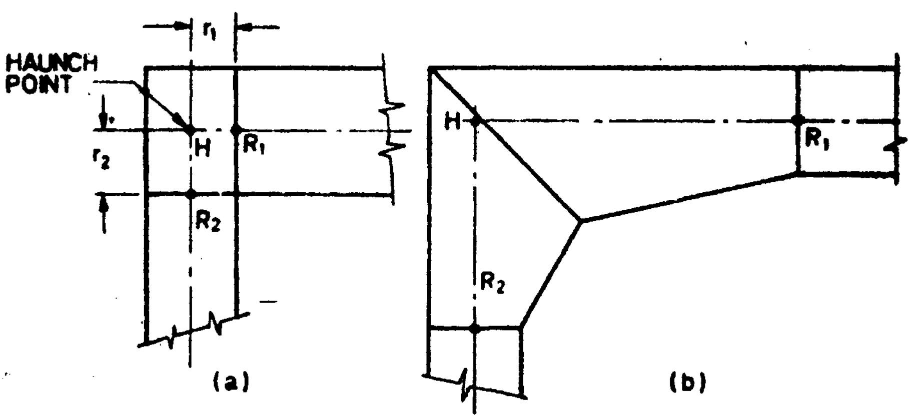


Fig. 47 Designation of Critical Sections in Straight and Haunched Sections

Normally an examination of the design to see whether or not it meets the stiffness requirement will not be necessary. Compared with the total length of the frame line, the length of the connection is small. Therefore, if it is a bit more flexible than the beams which it joins, the general overall effect will not be very great[19].

c) *Rotation Capacity* — Of much greater importance than sufficient elastic stiffness is an adequate reserve of ductility after the plastic moment value has been reached. This rotation is necessary to assure that all necessary plastic hinges will form throughout structure. Thus all connections must be proportioned to develop adequate rotation at plastic hinges. This subject is discussed later in further detail.

d) *Economy* — Obviously, extra connecting materials should be kept to a minimum. Wasteful joint details will result in loss of overall economy.

On the basis of the above requirements, we are now in a position to analyze the behaviour of various connection types.

23.6.2 *Straight Corner Connections* — The strength of unstiffened corner connections will be considered first; the connection and loading is shown in Fig. 48. The design objective is to prevent yielding of the web due to shear force at low load. This leads immediately to the following strength requirement: The moment at which yielding commences due to shear force, M_h (τ), should not be less than the plastic moment, M_p.

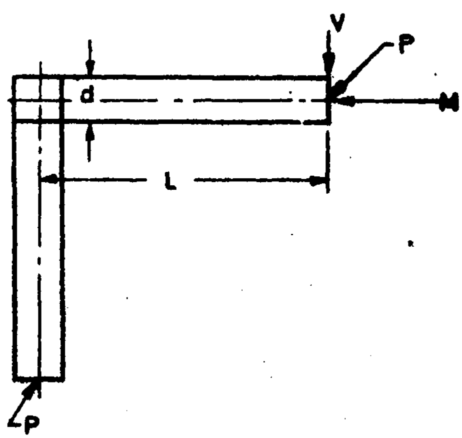


FIG. 48 IDEALIZED LOADING ON STRAIGHT CORNER CONNECTION

Using the maximum shear stress yield condition ($\tau_y = \sigma_y/Z$), and assuming that the shear stress is uniformly distributed in the web of knee, and that the flange carries all of the flexural stress, we can obtain a value of $M_h(\tau)$ which may then be equated to M_p. Using these assumptions (*see* stress-distribution and forces in Fig. 49), it may be shown that:

$$M_{h(\tau)} = \frac{wd^2\sigma_y}{\sqrt{3}\left(1-\frac{d}{L}\right)} \quad \ldots \quad \ldots \quad \ldots (55)$$

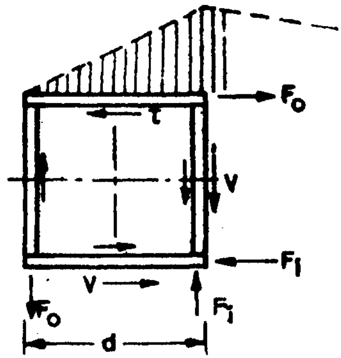


FIG. 49 FORCES AND STRESSES ASSUMED TO ACT ON UNSTIFFENED STRAIGHT CORNER CONNECTION

Equation 55 is equated to $M_p = \sigma_y Z$ to obtain the required web thickness:

$$t_w = \frac{\sqrt{3}fS}{d^2}\left(1-\frac{d}{L}\right) \quad \ldots \quad \ldots \quad \ldots (56)$$

Summarizing, the following design guide may be given:

Rule 9 Straight Corner Connections — Connections are to be proportioned to develop the full strength of the members joined. The critical section is to be taken at the haunch, '*H*'. The required web thickness is given by:

$$t_w \geqslant \sqrt{3}\frac{S}{d^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (56a)$$

Examination of rolled shapes (using Eq 56) shows that many of them require stiffening to realize the design objective for straight connections. When such stiffening is required, Rule 10 should be followed. Alternatively if doublers are suitable they would be proportioned according to

$W_d = \frac{\sqrt{3}S}{d^2} - t_w$.

Assuming, now, that the knee web is deficient as regards to its ability to resist the shear force, a diagonal stiffener may be used. A 'limit' approach may be used to analyze such a connection as sketched in Fig. 50. The force F_o is made up of two parts, a force carried by the web in shear and a force transmitted at the end by the diagonal stiffener, that is, $F_o = F_{web} + F_{stiffener}$

when both web and diagonal stiffener have reached the yield condition:

$$F_o = \frac{\sigma_y w d}{\sqrt{3}} + \frac{\sigma_y b_s t_s}{\sqrt{2}} \quad \ldots \quad \ldots \quad \ldots (57)$$

where b_s and t_s are the width and thickness of stiffener.

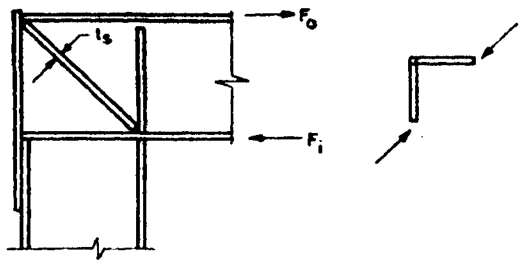


FIG. 50 CORNER CONNECTION WITH DIAGONAL STIFFENER

The available moment capacity of this connection type is thus given by:

$$M_k = \left(\frac{\sigma_y d}{1 - \frac{d}{L}}\right)\left[w\frac{d}{\sqrt{3}} + \frac{b_s t_s}{\sqrt{2}}\right] \quad \ldots \quad \ldots \quad \ldots (58)$$

Equating this moment to the plastic moment ($\sigma_y Z$), the following guide is obtained in which similar approximations have been made as before:

Rule 10 Diagonal Stiffeners in Connections — The required thickness of diagonal stiffeners in corner connections that would otherwise be deficient in shear resistance is give by:

$$t_s = \frac{\sqrt{2}}{b}\left(\frac{S}{d} - \frac{wd}{2}\right) \quad \ldots \quad \ldots \quad \ldots (59)$$

Instead of using the maximum shear stress theory of yielding, Eq 56 and 59 could have been derived using the Mises-Hencky yield criterion. The result will be a more liberal rule, and the above-mentioned equations become:

$$t_w \geqslant \frac{\sqrt{3}S}{d^2} \quad \ldots \quad \ldots \quad \ldots (56a)$$

and

$$t_s = \frac{\sqrt{2}}{b}\left(\frac{S}{d} - \frac{wd}{\sqrt{3}}\right) \qquad \ldots \qquad \ldots \qquad \ldots(59a)$$

Design Examples will be found in Section F. Generally the use of a diagonal stiffener with a thickness equal to that of the rolled section will be adequate and not unduly wasteful of material.

23.6.3 *Haunched Connections* — Haunched connections are the product of the elastic design concept by which material is placed in conformity with the moment diagram to achieve greatest possible economy. On the other hand, in plastic design (through redistribution of moment) material is used to full capacity without necessity for use of haunches.

Since the use of a haunch will automatically cut down on the span length, than a smaller rolled shape should be possible in a plastically-designed structure. If a haunch is to be specified for architectural considerations, the designer might just as well realize the additional savings in the material. Further the use of haunches in long span frames might make possible the use of rolled sections, whereas built-up members would otherwise be needed.

Four types of haunched connections are shown in Fig. 51. Analysis and test have shown all of them to be suitable in design, although the designer may find more frequent demand for the types shown in the figure.

It is difficult to generalize with regard to comparative deflections as between a frame designed with haunches and one without them. A frame with straight connections will have larger rolled members, tending to decrease frame flexibility. On the other hand, a frame with haunches is more flexible on the one hand because of the lighter members, but is stiffer on the other hand because of the deeper haunched knees. In one comparison[49] a plastically designed frame with straight connections was actually stiffener than the corresponding elastic design in which haunches were used.

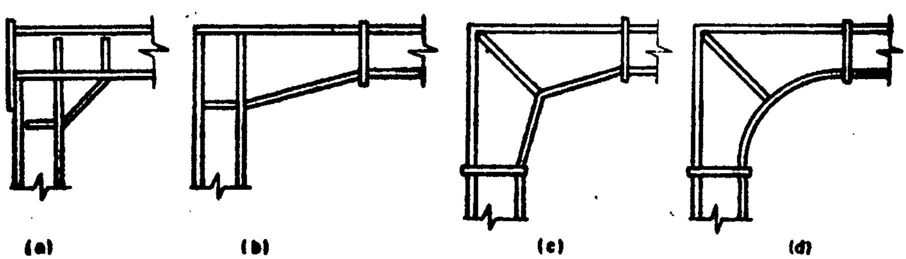


FIG. 51 TYPICAL HAUNCHED CORNER CONNECTIONS

The analysis of a frame with haunched connections involves no new principles. The effect of the haunches is to increase the number of sections at which plastic hinges may form, but otherwise the procedures are the same as before.

Similarly, the methods for computing deflections would embody the same principles as those described in 23.9.

The design requirements will generally be quite similar to those for straight corners. Haunched knees may exhibit poor rotation capacity[14].

This is due to inelastic local and/or lateral buckling. The solution is to force the formation of the plastic hinge to occur at the end of the haunch. This is accomplished by requiring that the haunch proper remain elastic throughout. Thus the flange thickness should be increased to meet the demands of the applied plastic moment. Stiffness is automatically provided in a great majority of cases; and no rotation capacity is required because all plastic deformation occurs in the rolled sections joined.

Adequate bending strength in the strong direction is only one of the strength requirements. The other is that it does not 'kick out' or buckle laterally prior to reaching the design condition. The tendency for this mode of failure is greater than in the straight connections because in the haunched knees the stress distribution is more nearly uniform along the compression flange, it cannot be laterally supported along the full length, and, therefore, a larger amount of energy can be released by buckling. The requirement that the connection remain elastic is, therefore, of considerable advantage.

For tapered haunches the design problem will be to find the required thickness of inner flange of the haunch to assure hinge formation at the extremities (locations *A* and *C* of Fig. 52). Also the knee web should have adequate thickness to prevent general plastic shear*.

Therefore, the analysis problem is to have a method for predicting the maximum flange stress due to the applied loading; secondly, a method of suitable simplicity should be available for computing the maximum shear stress. In a recently completed report available methods are compared, and in so far as normal stresses are concerned, it was found that the method of Olander was quite reasonable. The report also compares the results with tests.

Curved knees have been treated in Ref 32 and the results of this work have been applied to conventional design procedures in Ref 29. It is still necessary to force hinge formation at the extremeties of the haunch and thus a further increase of flange thickness appears necessary.

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*Normally this check is required only for type shown in Fig. 51(b).

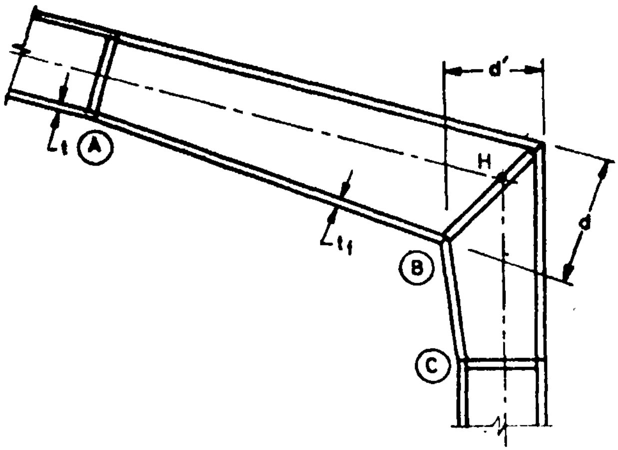

FIG. 52 EXAMPLE OF TAPERED HAUNCH

Although studies to date have not been completed to the point where the required flange thicknesses may be picked from a chart, the results suggest that an increase of 50 percent in flange thickness requirements should lead to a safe design.

In summary, then:

Rule 11 Haunched Connections — Haunched connections are to be proportioned to develop plastic moment at the end of rolled section joined.

In order to force formation of hinge at the end of a ***tapered haunch***, make flange thickness 50 percent greater than that of section joined. For ***curved knees*** the inner flange thickness is to be 50 percent greater than required by the rules of Ref 29.

Use Rule 9 to check web thickness (adequate to resist shear forces). The distance '*d*' is to be that as shown in Fig. 52.

Current research has extended and systematized the procedures with regard to the actual proportioning of haunched connections. A theoretical study and experimental investigation are nearing completion on this aspect of the problem.

23.6.4 ***Analysis of Interior Beam-Column Connections*** — The interior beam-to-column connections are those shown as '2' in Fig. 46 and in further detail in Fig. 53. The function of the 'Top' and the 'Interior' connections is to transmit moment from the left to the right beam, the

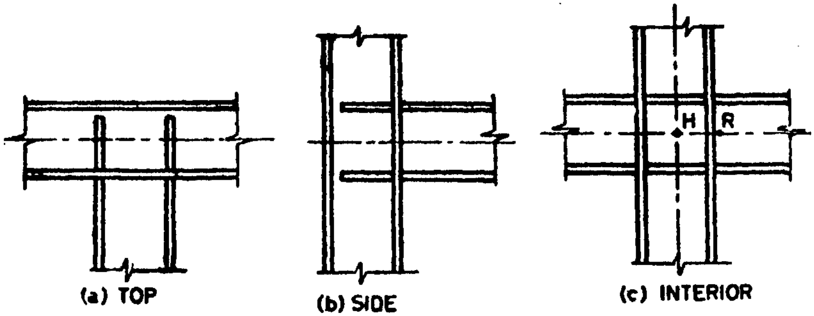

FIG. 53 BEAM TO COLUMN CONNECTIONS OF (a) TOP, (b) SIDE, AND (c) INTERIOR TYPE

column carrying any unbalanced moment. The 'Side' connection transmits beam moment to upper and lower columns. The design problem is to provide sufficient stiffening material so that the connection will transmit the desired moment (usually the plastic moment M_p). Therefore, methods should be available for analyzing the joint to predict the resisting moment of unstiffened and stiffened columns.

The moment capacity of unstiffened beam-to-column connections [Fig. 54(a)] may be computed on a somewhat similar basis as that adopted usually in conventional (elastic) design practice. In the limit, the force

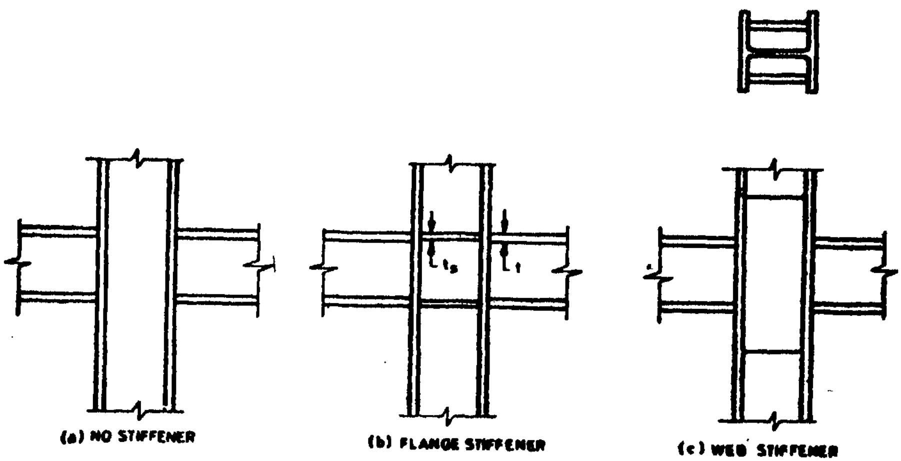

FIG. 54 METHODS OF STIFFENING AN INTERIOR BEAM TO COLUMN CONNECTIONS

which the column web can sustain is equal to the area available to carry the reaction times the yield-point stress. Referring to Fig. 55, the force which should be transmitted is known $\left(T = \sigma_y \frac{A_{beam}}{2}\right)$. The reaction width is equal to the column web thickness, w_c. As a conservative approximation it can be assumed that the length of reaction zone is half of the beam depth plus three times the k-distance of the column. Therefore one may write:

$$T = (\text{Reaction area}) \times (\sigma_y) \qquad \ldots \qquad \ldots \qquad \ldots(60)$$

or

$$\frac{\sigma_y A b}{2} = \left[w_c \left(\frac{db}{2} + 3\,kc \right) \right] (\sigma_y) \qquad \ldots \qquad \ldots \qquad \ldots(61)$$

From Eq 61 a direct design check may be formulated,

$$w_c \geq \frac{A_b}{d_b + 6k_c} \qquad \ldots \qquad \ldots \qquad \ldots(62)$$

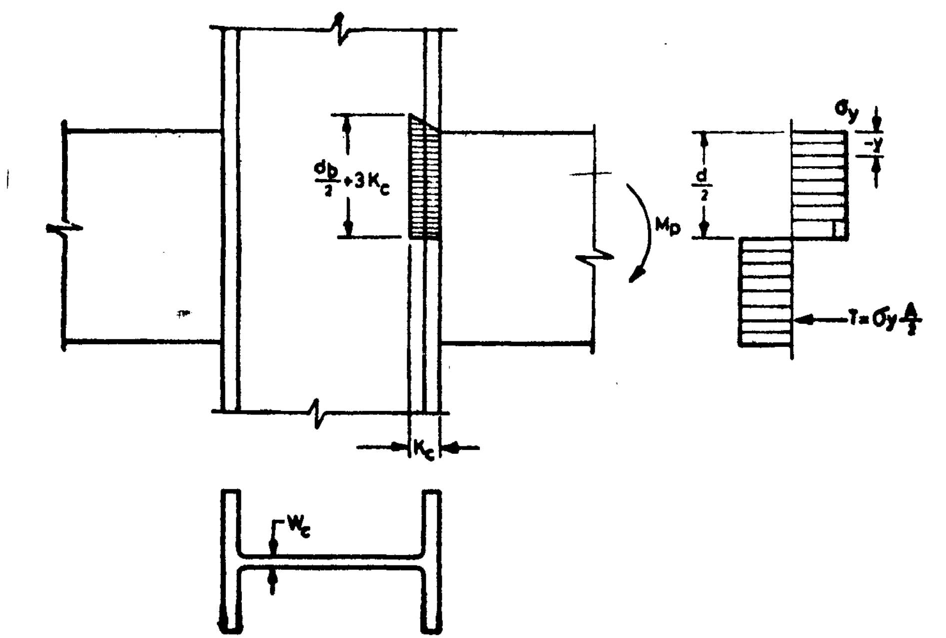

FIG. 55 ASSUMED STRESS DISTRIBUTION IN BEAM COLUMN CONNECTION WITH NO STIFFENER

which gives the required column web thickness, w_c to assure that the plastic moment will be developed in the beam. Except for those cases where the columns are relatively heavy in comparison to the beams, the test by Eq 62 will often show inadequate strength of the column. Recourse is then made to flange or web stiffeners of the type shown in Fig. 54.

A 'limit' analysis of connections with flange stiffeners may be used which results in a direct design procedure for determining the required thickness of stiffener, t_s. Assume that a stiffener is required and that it will adequately brace the column web against buckling. Then, referring to Fig. 57 in which the plastic moment (M_p) is acting at the end of the beam, the thrust T should be balanced by the strength of the web (T_w) and of the flange plate (T_s) or

$$T = T_w + T_s \qquad \ldots \qquad \ldots \qquad \ldots (63)$$

with T_w = force resisted by the web

$$= \sigma_y w_c \left(\frac{d_b}{2} + 3k_c\right)$$

and T_s = force resisted by stiffener plate

$$= \sigma_y \, t_s b$$

and T $= \sigma_y \dfrac{A_b}{2}$,

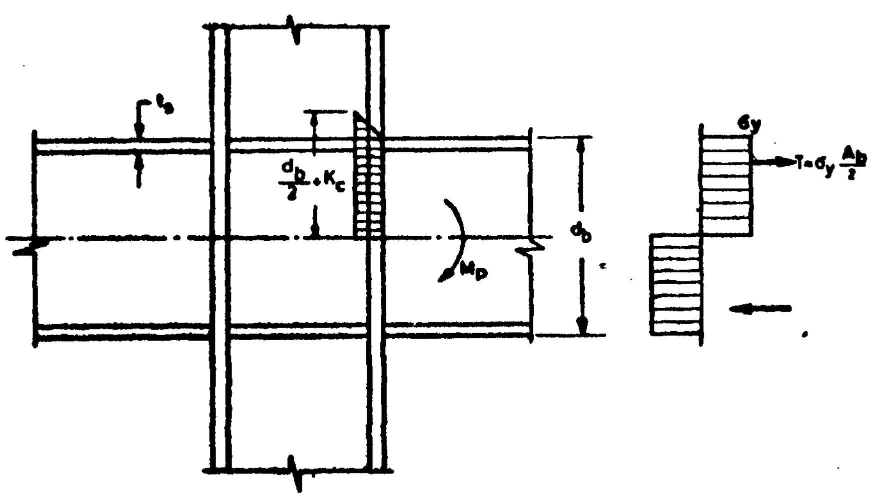

FIG. 56 ASSUMED STRESS DISTRIBUTION IN BEAM TO COLUMN CONNECTION WITH FLANGE TYPE STIFFENER

a direct solution for required stiffener thickness is:

$$t_s = \frac{1}{2b}\left[A_b - w_c(d_b + 6k_c)\right] \quad \ldots \quad \ldots \quad \ldots(64)$$

The results of tests show that this approach is conservative.

Web stiffeners may be proportioned on a similar basis to that described for unstiffened connections. For use in Eq 60 the reaction area is made up of the area supplied by the column web and the two inserted auxiliary webs [Fig. 54(c)]. This is given by:

$$\text{Reaction area} = w_c\left(\frac{d_b}{2} + 3k_c\right) + 2w_s\left(t_b + 3k_c\right) \quad \ldots \quad \ldots(65)$$

where

w_s = the thickness of the web-type stiffener, and

t_b = the stress of the beam flange.

Adequate information is thus available for obtaining its required value.

The second general type of stiffener that might be needed is that necessary to assist in transmitting shear forces. 'Side' connections [Fig. 53(b)] or interior connections with large unbalanced moments may require 'shear stiffening' if the column does not carry much direct stress. In such a case the column web at the joint is called upon to transmit forces such like those of Fig. 49. An examination similar to that leading to Eq 56 would, therefore, be desirable in this infrequently encountered case.

In summary the following design guide is suggested:

Rule 12 Interior Beam-Column Connections — To assure that an unstiffened column will transmit the plastic moment of the adjoining beam, its web thickness should be governed by:

$$W_c \geqslant \frac{A_b}{d_b + 6k_c} \quad \ldots \quad \ldots \quad \ldots(62)$$

If 'flange' stiffeners are used for reinforcement, their required thickness is given by:

$$t_s = \frac{1}{2b}\left[A_b - w_c(d_b + 6k_c)\right] \quad \ldots \quad \ldots \quad \ldots(65)$$

Alternatively, if 'web' type stiffeners are used:

$$w_s = \frac{A_b - w_c(d_b + 6k_c)}{4(t_b + 6k_c)} \quad \ldots \quad \ldots \quad \ldots(66)$$

The thickness w_s should not be less than that of the columns.

In exterior columns or in other cases of large unbalanced moment, examine adequacy of web to transmit shear force.

Example 8:

Illustration of the application of the equations in Rule 12 will now be given. Joints that are typical of interior beam-column connections are shown in Fig. 53 and an example of their occurrence in design is shown in Sheet 1 of Design Example 9, sketch (a). The designs of three types of connections will now be considered.

The connection shown in Fig. 57(a) should transmit moment from the beam to the columns above and below. The first question is, 'are stiffeners required?'

From Eq 62 using properties of the ISLB 600 and ISLB 550:

$$A_b = 126{\cdot}69 \text{ cm}^2$$
$$d_b = 60{\cdot}0 \text{ cm}$$
$$K_c = 3{\cdot}70 \text{ cm}$$
$$W_c = 0{\cdot}99 \text{ cm}$$

The required thickness of column web is:

$$t_w \geqslant = \frac{A_b}{d_b+6k_c} = \frac{126{\cdot}69}{(60+22{\cdot}2)} = 1{\cdot}54 \text{ cm} > 0{\cdot}99 \text{ cm}$$

Therefore stiffeners are required. Using horizontal 'flange' stiffeners, the required thickness is given by Eq 64:

$$t_s = \frac{1}{2b}\left[A_b - w_c(d_b+6k_c)\right]$$

$$= \frac{1}{2(21)}\left[126{\cdot}69 - 0{\cdot}99(60+22{\cdot}2)\right]$$

$$= 1{\cdot}09 \text{ cm}$$

Use 12 mm thick stiffeners.

Next, the web should be examined to see if it is adequate to resist the shear force introduced by the column moment. Very recently it has been shown that an extension of Eq 58 leads to the following relationship for a 3- or 4-way connection:

$$t_w \geqslant \frac{71M}{A} \qquad \ldots \qquad \ldots \qquad \ldots (66a)$$

where

t_w = column web thickness in cm,

M = unbalanced moment on the connection in m.t, and

A = planar area of connection in cm².

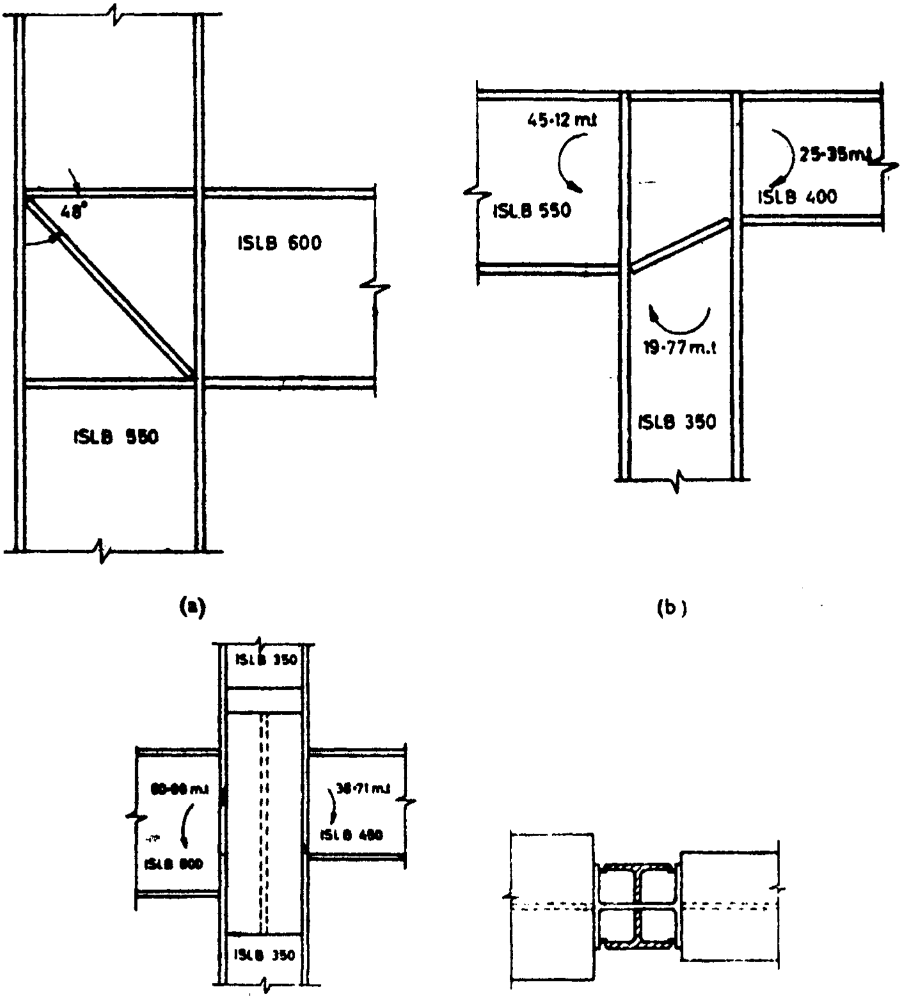

FIG. 57 BEAM TO COLUMN CONNECTION WITH STIFFENERS — *Contd*

In an actual design, the moment computed in the frame analysis would be used. In this problem, the moment will be taken as the maximum possible value, namely, M_p of the section. Thus

$$t_w \geqslant \frac{71 \times 70 \cdot 52}{60 \times 55} = 1 \cdot 57 > 0 \cdot 99$$

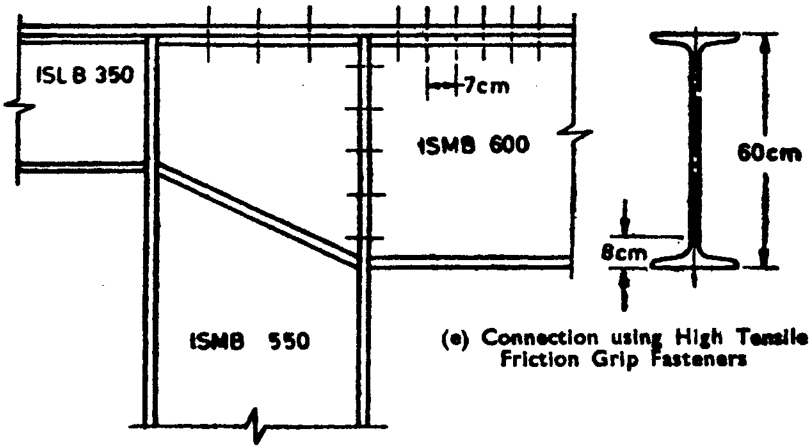

FIG. 57 BEAM TO COLUMN CONNECTION WITH STIFFENERS

Thus shear stiffening is required. A diagonal stiffener as shown in Fig. 57(a) will be specified. Assuming that all of the unbalanced moment is carried by the flanges, the area of stiffener may be computed from:

$$\sigma_y \; A_s \; \cos 48^\circ = \frac{M_u}{d_b}$$

$$A_s = \frac{M_u}{\sigma_y \; \cos 48^\circ \; d_b} = \frac{Z_u \; (1{\cdot}57 - 0{\cdot}99)}{(1{\cdot}57 \; \cos 48^\circ) d_b}$$

$$A_s = \frac{2\;798{\cdot}6 \times 0{\cdot}57}{1{\cdot}57(0{\cdot}67)60} = 25{\cdot}3 \; \text{cm}^2$$

Use 2 stiffeners of size 100×15 mm.

The connection shown in Fig. 57(b) is next examined to see if the web is adequate for shear.

From Eq 66a:

$$t_w \geqslant \frac{71M}{A}$$

$$= \frac{71(45{\cdot}12 - 25{\cdot}35)}{55 \times 35}$$

$$= 0{\cdot}726 > 0{\cdot}74$$

No additional stiffening is considered necessary.

The connection shown in Fig. 57(c) should be examined for adequacy with regard to moment and shear stiffeners. Having in mind that a connection of this type will frequently have a beam framing into it at right angles to the plane of the joint, a web type stiffener should be used if the calculations suggest that one is required. Further, the two beams framing into the column are of different depths. Using Eq 62:

$$W_c \geqslant \frac{A_b}{d_b+6k_c}$$
$$= \frac{126 \cdot 69}{60 \cdot 0+6(3 \cdot 085)}$$
$$= 1 \cdot 43 \text{ cm} > 0 \cdot 74 \text{ cm}$$

Stiffening is required

Using Eq 66:

$$W_s = \frac{A_b - W_c(d_b+6k_c)}{4(t_b+6k_c)}$$
$$= \frac{126 \cdot 69 - 0 \cdot 74[60+6 \times 3 \cdot 085]}{4(1 \cdot 55+3 \times 3 \cdot 085)}$$
$$= 1 \cdot 58 \text{ cm}$$

The stiffener shown in Fig. 57(d) may either be fabricated from plate stock or by splitting ISLB 350, trimming the flanges to suit the purpose. There is a fabrication advantage in using the latter since it would only involve the procurement of a short additional length of the column section already specified.

In checking for shear,

$$t_w \geqslant \frac{71M}{A} = \frac{71(60 \cdot 08 - 33 \cdot 71)}{60 \times 35}$$
$$= 0 \cdot 89 \text{ cm.}$$

More than an adequate amount of material is thus available to transmit the applied shear force.

22.6.5 *Connections Using High-Strength Bolts* — High-strength bolts (*see* IS: 4000-1967*) may be used to join members [*see* Fig. 57(e)] in one of two ways. Either they may be considered as splices in regions of negligible moment or they may be used at positions at which plastic hinges are expected to form. In the latter case at ultimate load the design may be based upon tension values equal to the guaranteed minimum proof load and shear values equal to the normal area of the bolt times 1 760 kgf/cm².

---

*Code of practice for assembly of structural joints using high tensile friction grip fasteners.

An illustration of the design of a moment connection in the vicinity of a plastic hinge will now be given. The example chosen is joint 8 of sheet 5 of Design Example 7, sketch (h), shows the welded detail. The problem is to join the girder to the column at section 8 with high strength bolts to transmit the necessary moment. The design calculations follow. Bolts of dia 14 mm will be specified with a proof load of 7 900 kg.

For vertical shear, $\dfrac{33{\cdot}7\times10^3}{7\,900} = 4{\cdot}3$ bolts

Minimum number of bolts = 5

Bolts required to develop top flange load

$$n = \frac{A_{net}\times 2\,540}{7\,900} = \frac{[2{\cdot}08\times21{\cdot}0-2(2{\cdot}08)(2{\cdot}5)]2\,540}{7\,900} = 10{\cdot}7$$

Top plate design:

$$\text{Thickness} = \frac{7\,900\times12}{2\,540[20-(2\times2{\cdot}5)]} = 2{\cdot}48 \text{ cm}$$

Use a top plate of 200×25 mm connected by 12 bolts to the top flange. Try 6 bolts in tension. Moment capacity of connection should be greater than plastic moment of ISMB 600.

$$M_p = \sigma_y Z = 88{\cdot}46 \text{ m.t}$$

$$M = [12\times7\,900\times(60-8{\cdot}0)+16\,300(2)(37{\cdot}5+30{\cdot}5+23{\cdot}0)]$$

$$= (49{\cdot}40+29{\cdot}6) = 79{\cdot}0$$

This is less than M_p.

NOTE — Vertical plates joined by bolts loaded in tension by the applied moment must be designed adequate to transmit the tension.

The following comments are in explanation of the above steps:

a) The 'tension value' of a 24 mm bolt is taken as the guaranteed minimum proof load of 16 300 kg. The shear value is assumed as 7 900 kg.

b) With a vertical shear of 33·7 t acting upon the joint, a minimum of 5 bolts are needed. The 8 bolts furnished will be adequate.

c) The calculation of the number of bolts required to 'develop' the strength of the top flange when it is plastic results in the number 10·7 bolts are, therefore, furnished, and the top plate is proportioned such that it will actually transmit the force due to 12 bolts loaded in shear. Thus this plate will transmit not only the flange force, but also a portion of the web force.

d) The 6 bolts in tension are also assumed to be working at their guaranteed minimum proof load when the plastic moment comes on the joint. Actually the bolts will be considerably stronger than this minimum value.

e) Bolts in the splice just over the column would be in sufficient number to transmit the necessary shear to the column. The exact number would depend upon the number of bolts that would be required for the ISLB 350, beam joining the left flange of the column.

23.6.6 *Riveted Connections* — Riveted connections could be proportioned in a manner that would make use of similar principles to those involved in **23.6.5**. For this purpose the tension value of a rivet would be computed on the basis of a yield stress of 2 540 kg/cm². For rivets in shear, at ultimate load the shear stress would be limited to 1 760 kgf/cm².

23.7 Brittle Fracture — Since brittle fracture would prevent the formation of a plastic hinge, it is exceedingly important to assure that such failure does not occur. But it is an equally important aspect of conventional elastic design when applied to fully-welded continuous structures. As has already been pointed out in previous sections the assumption of ductility is important in conventional design and numerous design assumptions rely upon it.

In the past years the failures of ships and pressure vessels have focussed attention on the importance of this problem. And although hundreds of articles have been published on the problem of brittle fracture, no single easy rule is available to the designer.

In plastic design the engineer should be guided by the same principles that govern the proper design of an all-welded structure designed by conventional methods, since the problem is of equal importance to both. Thus:

a) The proper material should be specified to meet the appropriate service conditions.

b) The fabrication and workmanship should meet high standards. In this regard, punched holes in tension zones and the use of sheared edges are not permitted. Such severe cold working exhausts the ductility of the material.

c) Design details should be such that the material is as free to deform as possible. The geometry should be examined so that triaxial states of tensile stress will be avoided.

How can we be sure that brittle fracture will not be a problem even if the suggestions mentioned above are followed? While no positive guarantee is possible, experience with tests of rolled members under normal loading conditions (but with many 'adverse circumstances' present that might be expected to lead to failures) has not revealed premature brittle fractures of steel beams. Further, the use of fully continuous welded construction in actual practice today has not resulted in failures, and factors that are otherwise neglected in design have most certainly caused plastic deformations in many parts of such structures. Summarizing, the following guides are suggested:

Rule 13 Structural Ductility — Ordinary structural grade steel for bridges and buildings may be used with modifications, when needed, to insure weldability and toughness at lowest service temperature.

Fabrication processes should be such as to promote ductility. Sheared edges and punched holes in tension flanges are not permitted. Punched and reamed holes for connecting devices would be permitted if the reaming removes the cold-worked material.

In design, triaxial states of tensile stress set up by geometrical restraints should be avoided.

23.8 Repeated Loading — Up to this point the tacit assumption has been made that the ultimate load is independent of the sequence in which the various loads are applied to the structure. One would also suppose that a certain degree of fluctuation in the magnitude of the different loads would be tolerable so long as the number of cycles did not approach values normally associated with fatigue.

In the large majority of practical cases this is true. For ordinary building design no further consideration of variation in loads is warranted. However, if the major part of the loading may be completely removed from the structure and re-applied at frequent intervals, it may be shown theoretically that a different mode of 'failure' may occur. It is characterized by loss of deflection stability in the sense that under repeated applications of a certain sequence of load, an increment of plastic deformation in the same sense may occur during each cycle of loading. The question is, does the progressive deflection stop after a few cycles (does it 'shake down') or does the deflection continue indefinitely? If it continues, the structure is 'unstable' from a deflection point of view, even though it sustains each application of load.

Loss of deflection stability by progressive deformation is characterized by the behaviour shown in Fig. 58. If the load is variable and repeated and is greater than the stabilizing load, P_s, then the deflections tend to increase for each cycle. On the other hand if the variable load is equal to or less than P_s, then, after a few cycles the deflection will stabilize at a constant maximum value and thereafter the behaviour will be elastic.

In the event that the unusual loading situation is encountered, methods are available for solving for the stabilizing load, P_s, and the

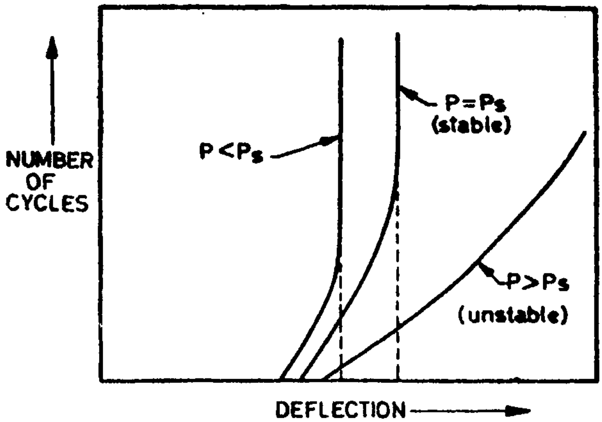

FIG. 58 CURVES EXPLAINING LOSS OF DEFLECTION STABILITY BY PROGRESSIVE DEFORMATION

design may be modified accordingly[11,12*]. As mentioned earlier, however, this will not be necessary in the large majority of cases. In the first place the ratio of live load to dead load should be very large in order that P_s be significantly less than P_u, and this situation is unusual. Secondly, the load factor of safety is made up of many factors other than possible increase in load (such as variation in material properties and dimensions, errors in fabrication and erection, etc). Variation in live load, alone, could not be assumed to exhaust the full value of the factor of safety; and thus the live plus dead load would probably never reach P_s. Further, as pointed out by Neal[11], failure in this sense is accompanied by a very definite warning that loss of deflection stability is imminent. This implies that a lower load factor would be appropriate as regards P_s than as regards P_u.

Rule 14 Repeated Loading — Plastic design is intended for cases normally considered as 'static' loading. For such cases the problem of repeated loading may be disregarded.

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*Another repeated loading effect is called 'alternating plasticity' or 'plastic fatigue'. It is characterized by an actual reversal of stress of a magnitude sufficient to cause plastic deformation during each cycle. Unless the design criterion is required to be controlled by fatigue, the discussion which follows in this section applies equally well to 'plastic fatigue' as well as to 'deflection stability'.

Where the full magnitude of the principle load(s) is expected to vary, the ultimate load may be modified according to analysis of deflection stability.

23.9 Deflections — Methods for computing the deflection at ultimate load and at working load have been summarized in Ref 9, and these will be outlined herein. However, the problem of deflections is not a serious one to plastic design, because in most cases a structure designed for ultimate loading by the plastic method will actually deflect no more at working loads (which are nearly always in the elastic range) than a structure designed according to 'elastic' specifications. For example, Fig. 59 shows three different designs of a beam of 10 m span to carry a working load of 10 t. Curve I is the simple beam design. Curve III is the plastic design. The deflections at working load for the plastic design are significantly less than those of the simply-supported beam, albeit slightly greater than the elastic design of the restrained beam (Curve II).

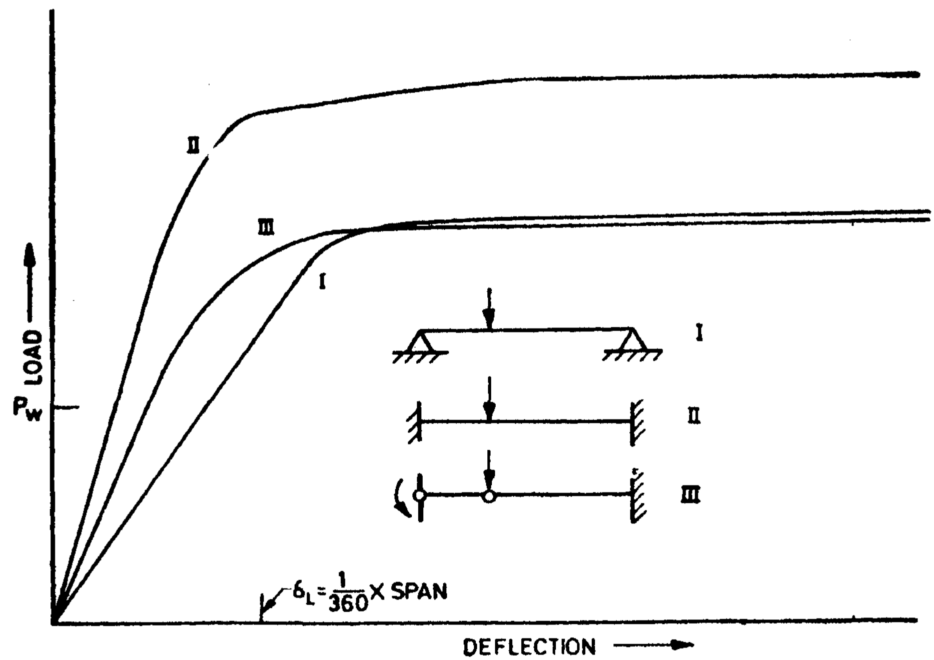


FIG. 59 LOAD DEFLECTION RELATIONSHIP FOR THREE DESIGN BEAMS FOR SUPPORTING THE SAME LOAD

The primary design requirement is that the structure should carry the assumed load. The deflection requirement is a secondary one — the structure should not deform too much out of shape. Therefore, our needs involving deflection computation may be satisfied with approximations and they fall into two categories:

a) *Determination of approximate magnitude of deflection at ultimate load* — The load factor of safety does not preclude the rare overload, and an estimate of the corresponding deflections would be of value.

b) *Estimate of deflection at working load* — In certain cases, the design requirements may limit deflection at this load.

Fortunately, even though such calculations will rarely be required, methods are available for making these computations that approach in simplicity the methods for analysing for ultimate load. The analysis neglects catenary forces (which tend to decrease deflection and increase strength) and second-order effects (which tend to increase deflection and decrease strength). Also ignored are any factors that influence the moment-curvature relationship. (In Ref 9, 15, and 30 may be found discussion of these and other factors.)

23.9.1 *Deflection at Ultimate Load* — The so-called 'hinge method' (discussed in the references mentioned above) gives a reasonably precise approximation to the load-deflection curve and affords a means for estimating the deflection at ultimate load. This method is based on the idealized $M-\phi$ relationship (Fig. 17) which means that each span retains its elastic flexural rigidity (EI) for the whole segment between sections at which plastic hinges are located. Further, although 'kinks' form at the other hinge sections, just as the structure attains the computed ultimate load, there is still continuity at that section at which the *last* plastic hinge forms.

As a consequence, the slope-deflection equations may be used to solve for relative deflection of segments of the structure. The moments having been determined from the plastic analysis. The following form of these equations will be used, the nomenclature being as shown in Fig. 60 with clockwise moment and angle change being positive:

$$\theta_A = \theta\frac{l}{A} + \frac{\Delta}{l} + \frac{l}{3El}\left(M_{AB} - \frac{M_{BA}}{2}\right)$$

The only remaining question is: which hinge is the last to form? An elasti-plastic analysis could be carried out to determine the sequence of formation of hinges, and thus the last hinge. However, a few examples will demonstrate that a simpler method is available: calculate the deflection on the assumption that each hinge, in turn, is the last to form. The correct deflection at ultimate load is the maximum value obtained from the various trials.

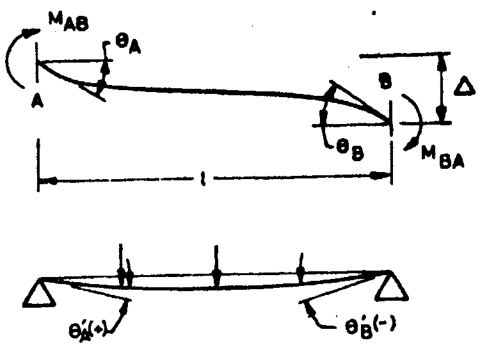


FIG. 60 SIGN CONVENTION AND NOMENCLATURE FOR USE IN THE SLOPE-DEFLECTION EQUATIONS

In outline, the following summarizes the procedure for computing deflections at ultimate load:

Rule 15 Deflection at Ultimate Load

a) Obtain the ultimate load, the corresponding moment diagram and the mechanism (from the plastic analysis).

b) Compute the deflection of the various frame segments assuming, in turn, that each hinge is the last to form:

   i) Draw free-body diagram of segment, and

   ii) Solve slope-deflection equation for assumed condition of continuity.

c) Correct deflection is the largest value (corresponds to last plastic hinge).

d) A check: From a deflection calculation based on an arbitrary assumption, compute the 'kinks' formed due to the incorrect assumption. Remove the 'kinks' by mechanism motion and obtain correct deflection. (This is also an alternate procedure.)

The procedure is now illustrated in examples 6 and 7 which follow:

Example 6:

(Fixed-ended beam, uniform vertical load)

a) *Ultimate load* (Eq 27)

$$W_u = \frac{16M_p}{L}$$

b) *Moment Diagram and Mechanism* — Fig. 61(a)
c) *Computation of Vertical Deflection*

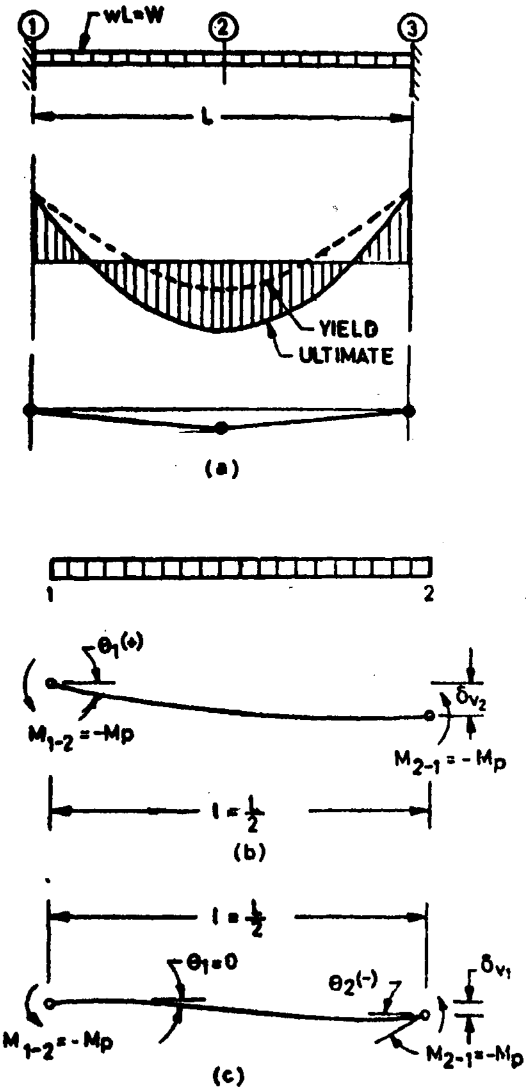


FIG. 61 DEFLECTION ANALYSIS OF FIXED-ENDED, UNIFORMLY LOADED BEAM

TRIAL AT LOCATION 2 (Location 2 assumed as last hinge to form)

Free-body diagram — Fig. 61(b)

Slope-deflection equation for member 2-1 using the condition that:

$$\theta_2 = 0$$

$$\theta_2 = \theta_2' + \frac{\delta v_2}{l} + \frac{l}{3EI}\left(M_{21} - \frac{M_{12}}{2}\right)$$

θ_2' = Simple beam end rotation $= -\dfrac{M_p L}{12EI}$

δv_2 = Vertical deflection with continuity assumed at Section 2

$$\therefore 0 = -\frac{M_p L}{12EI} + \frac{\delta v_2}{L/2} + \frac{L/2}{3EI}\left(-M_p + \frac{M_p}{2}\right)$$

$$\delta v_2 = +\frac{M_p L^2}{12EI}$$

TRIAL AT LOCATION 1

Even though it is obvious that last hinge forms at '2', what is the effect of incorrect assumption?

Free body — Fig. 61(c)

Slope-deflection equation for sequent 1-2 using the condition that:

$$\theta_1 = 0$$

$$\theta_1 = \theta_1' + \frac{\delta v_1}{l} + \frac{l}{3EI}\left(M_{12} - \frac{M_{21}}{2}\right)$$

$$0 = +\frac{M_p L}{12EI} + \frac{\delta v_1}{L/2} + \frac{L/2}{3EI}\left(-M_p + \frac{M_p}{2}\right)$$

$$\therefore \delta v_1 = 0$$

Thus the correct answer is $\delta V = \dfrac{M_p L^2}{12EI}$

***Example* 7:**

(Rectangular portal frame, fixed bases) — Fig. 62

a) *Ultimate Load* (by plastic analysis)

$$P_u = \frac{6M_p}{L}$$

b) ***Moment Diagram and Mechanism*** Fig. 62(b) and (c)

c) ***Free-body Diagrams*** — Fig. 62(d)

d) ***Computation of Vertical Deflection***

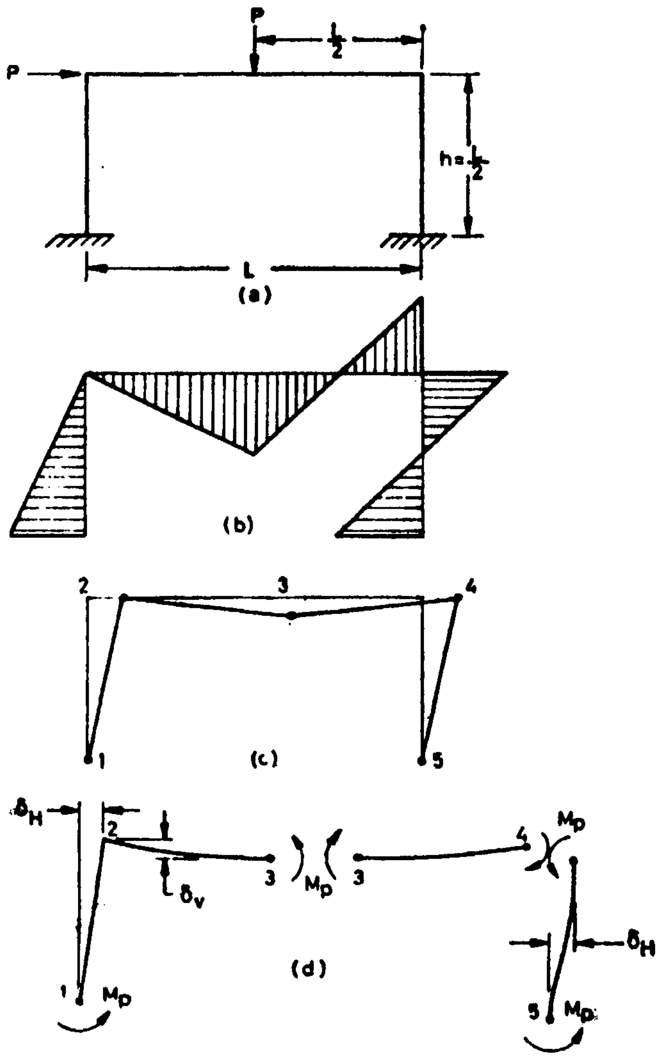


FIG. 62 DEFLECTION ANALYSIS OF RECTANGULAR FRAME WITH FIXED BASES

RATIO OF δ_H AND δ_V:
From the condition of continuity at Location 2, $\theta_{23} = \theta_{21}$:

$$\theta_A = \theta'_A + \frac{\delta v_2}{l} + \frac{l}{3EI}\left(M_{AB} - \frac{M_{BA}}{2}\right)$$

$$\theta_{23} = 0 + \frac{\delta v_2}{L/2} + \frac{L/2}{3EI}\left(0 + \frac{M_p}{2}\right) = \frac{2\delta v_2}{L} + \frac{M_pL}{12EI}$$

$$\theta_{21} = 0 + \frac{\delta_{H_2}}{L/2} + \frac{L/2}{3EI}\left(0 + \frac{M_p}{2}\right) = \frac{2\delta_{H_2}}{L} + \frac{M_pL}{12EI}$$

$$\frac{2\delta v_2}{L} + \frac{M_pL}{12EI} = \frac{2\delta_{H_2}}{L} + \frac{M_pL}{12EI}$$

$$\therefore \delta_V = \delta_H$$

TRIAL AT LOCATION 1: Member 1-2, $\theta_1 = 0$

$$0 = 0 + \frac{\delta_{H_1}}{L/2} + \frac{L/2}{3EI}(-M_p + 0)$$

$$\delta_{H_1} = + \frac{M_pL^2}{12EI}$$

$$\therefore \delta v_1 = \frac{M_pL^2}{12EI}$$

TRIAL AT LOCATION 3: $\theta_{32} = \theta_{34}$

$$\theta_{32} = 0 + \frac{\delta v_3}{L/2} + \frac{L/2}{3EI}(-M_p + 0) = \frac{2\delta v_3}{L} - \frac{M_pL}{6EI}$$

$$\theta_{34} = 0 - \frac{\delta v_3}{L/2} + \frac{L/2}{3EI}\left(M_p - \frac{M_p}{2}\right) = \frac{2\delta v_3}{L} + \frac{M_pL}{12EI}$$

$$\theta_{32} = \theta_{34}$$

$$\therefore \delta v_3 = \frac{M_pL^2}{16EI}$$

TRIAL AT LOCATION 4: Similar procedure using $\theta_{43} = \theta_{45}$

$$\delta v_4 = \frac{M_pL^2}{24EI}$$

TRIAL AT LOCATION 5: Similar procedure using $\theta_5 = 0$

$$\delta v_5 = \frac{M_pL^2}{24EI}$$

Correct answer is: $\delta_V = \delta_{max} = \delta v_1 = \dfrac{M_pL^2}{12EI}$ (Last hinge at location 1)

23.9.2 *Deflection at Working Load* — Usually the structure will be elastic at working load, implying a need for an elastic analysis of the structure. But it is desirable to avoid such an elastic analysis, if at all possible. For certain standard cases of loading and restraint, solutions are already available in handbooks. For such cases one would divide the computed ultimate load by F, the load factor of safety, and solve for working load deflection from tables. Taking the fixed-ended beam of Example 6, for instance (Fig. 61), it is found that:

$$\delta_w = \frac{W \delta L^3}{384EI} = 0{\cdot}022 \frac{M_p L^2}{EI}$$

When end restraint conditions are not known, often they may be estimated and the above technique employed.

As an indication as to whether or not an actual calculation of deflection at working load should be made, recourse may be had to the methods of the previous section. The deflection at ultimate load (δ_u) may be computed by the hinge method, and a value that will be *greater* than the true deflection at working load may be obtained from:

$$\delta_{w'} = \frac{\delta_u}{F} \qquad \dots \qquad \dots \qquad \dots(68)$$

This is illustrated by the dashed line in Fig. 63 for the uniformly-loaded, fixed-ended beam. The error may often be greater than 100 percent,

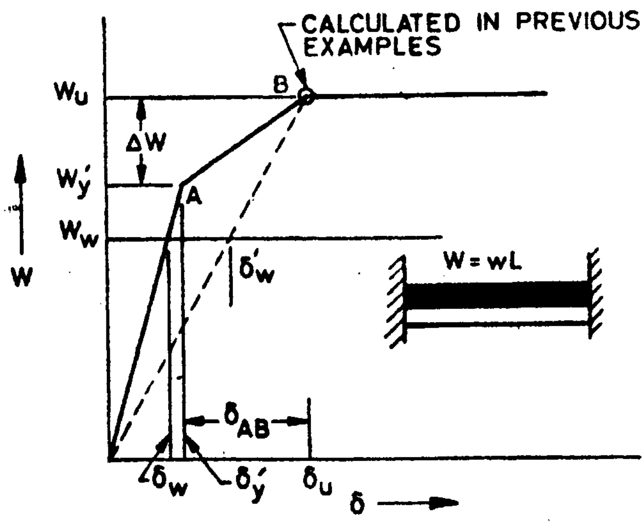


FIG. 63 IDEALIZED LOAD DEFLECTION RELATIONSHIP FOR FIXED-ENDED BEAM WITH UNIFORMLY DISTRIBUTED LOAD

but it gives an upper limit to δ_u and indicates when more refined calculations are necessary.

Rule 16 Deflection at Working Load — If computation of beam deflection at working load is required, this may be done by reference to handbook tables.

An upper limit of the deflection of a frame at working load is obtained from $\delta_{w'} = \delta_u/F$.

In Section F will be found several additional examples of the estimation of deflections for design purposes.

23.9.3 *Rotation Capacity* — In order that a structure attains the computed ultimate load, it is necessary for redistribution of moment to occur. As pointed out in **15** this is only possible if the plastic moment is maintained at the *first* hinge to form while hinges are developing *elsewhere* in the structure. The term 'rotation capacity' characterizes this ability of a structural member to absorb rotations at near-maximum (plastic) moment. It is evident that certain factors such as instability and fracture may limit the rotation capacity of a section; and one might anticipate having to calculate the amount of *required* rotation in any given problem to meet the particular limitation. This would seriously complicate plastic design.

However, computations of the required rotation angle (called 'hinge rotation') are normally not required in design, since the foregoing rules of practice will assure that structural joints possess it in adequate measure. In setting up the procedure for safeguarding against local buckling (Rule 3) it was specified that the section should not buckle until the extreme fibre strain had reached ϵ_{st}. The hinge rotation supplied in this case is about 12 ($\epsilon_{st}/\epsilon_y = 12$); this value is sufficient to meet most practical structural requirements.

The procedure for computing the hinge rotation at a plastic hinge in a given structure is based directly on the methods for computing deflections at ultimate load. It has been illustrated in Ref 9 and the problem has been treated in Ref 31.

SECTION F

DESIGN EXAMPLES

24. INTRODUCTION

24.1 This section will treat actual design problems for the purpose of illustrating the principles of plastic design. In addition to obtaining the required section following the general procedures laid down in **21** and **22**, each design will be examined in the light of the 'secondary design considerations' (Section E). In the process of analyzing each structure, 'short cut' methods will not be used. Instead, each problem will be worked by a direct and complete plastic analysis. The experienced designer will, of course, want to use all possible techniques to shorten the design time, but at this stage the objective is to illustrate the principles.

The available 'short-cuts' for speeding up the design process will be treated in Section G. Just as in conventional elastic design where the engineer has available various formulas, tables and charts with which to analyze standard cases, so also it has been possible to arrange convenient design aids for the rapid selection of member sizes.

In arriving at a final section size it will be noticed that a table of Z-values has been used: when the required M_p-value has been determined, Z is computed and Table 4 is used to select the section. An alternate procedure that would save a step in the calculations is to arrange the sections according to M_p-values instead of Z-values. This limits the use of the table, however, to a single value of the yield stress level σ_y. Still another method would be to use the presently available tables of section modulus, S. This would involve a guess as to the proper value of the shape factor, f, a value that would be corrected, if necessary, in the final step.

The load factor of safety has been discussed in **22**. A value of 1·85 is used for dead load plus live load and a value of 1·40 for these loads plus wind or earthquake forces.

As a convenience for later reference, the examples are all worked in figures or 'plates', the discussion of the steps being included in the text.

25. DESIGN EXAMPLES ON CONTINUOUS BEAMS

25.1 Design Example 1 — A design example is worked out in the following two sheets to illustrate the design of a beam of uniform section throughout. It develops that the end span is critical and, therefore, it is not possible to determine by statics alone the moments and reactions for the three central spans. The semi-graphical construction demonstrates that the plastic moment is not exceeded so the selection of the ISLB 600, will provide adequate bending strength.

A *precise* determination of the reactions at ultimate load would require an elastic analysis. They are computed in this problem, however, on the assumption that the load on the interior span 3-5 is divided evenly between the two supports 3 and 5, 30·06 t being distributed to each. Actually the shear in span 3 to 5 does not vary too much and should fall somewhere between values that would correspond to the two limiting conditions indicated by Cases I and II in the portion of the moment diagram re-plotted. Thus, V_{35} may vary between the assumed value of 30·06 t (Case I in the sketch) and which would be obtained from Condition II $\left(30{\cdot}06+\frac{36{\cdot}42}{13}=32{\cdot}86\ \text{t}\right)$.

The maximum shear (35·22 t to the left of support 3) is well within the permitted value of 79·7 t for this shape. But when the cross-sectional proportions are checked it is found that $d/w = 57{\cdot}14 > 55$ (permissible). Hence it is recommended that an ISMB 550 be used. It is checked that the cross-sectional proportions for this profile is adequate.

With regard to bracing, the structure is assumed to be enclosed. Thus the top flange is continuously braced. Vertical plates are supplied at section 2 to provide some torsional restraint to the beam.

Splices for shear only will be adequate at the indicated sections. At a distance of 2·5 m from the support (at the indicated points), a small variation from the actual point of inflection is not of serious consequence to load-carrying capacity.

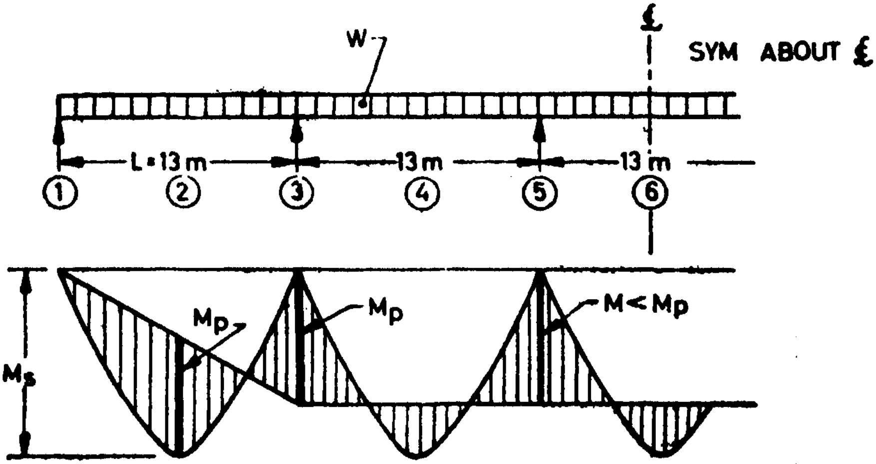
Whether or not the deflection calculation would be made depends on the design conditions. The greatest deflection will be in the end spans and will probably not be far from the value 2·9 cm for the case of the indicated approximation.

DESIGN EXAMPLE 1 BEAM WITH UNIFORM SECTION

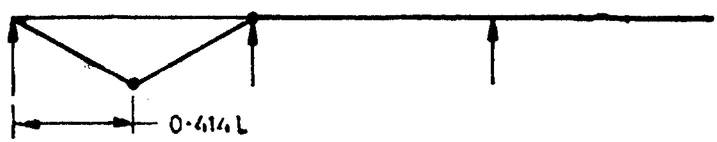
Structure and Loading

$L = 13$ m
$W = 2{\cdot}5$ t/m

Uniform Section Throughout



DETERMINATE MOMENT DIAGRAM



MECHANISM

$$M_s = \frac{W_u L^2}{8} = \frac{4{\cdot}625 \times 13^2}{8} = 97{\cdot}7 \text{ m.t}$$

$$M_p = 0{\cdot}686\ M_s = 0{\cdot}085\,8\ W_u L^2 = 0{\cdot}085\,8 \times 13 \times 13 \times 4{\cdot}625$$

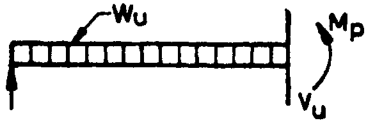
$$= 67{\cdot}06 \text{ m.t}\quad Z = \frac{M_f}{\sigma_y} = 67{\cdot}06 \left(\frac{100\,000}{2\,520}\right)^* = 2\,661{\cdot}2 \text{ cm}^3$$

*For later calculation use $39{\cdot}68 = \frac{100\,000}{2\,520}$

(Continued)

DESIGN EXAMPLE 1 BEAM WITH UNIFORM SECTION — *Contd*

Reactions **(at Ultimate load)**



$$R_1(0{\cdot}414L) - W_u \frac{(0{\cdot}414L)^2}{Z} = M_p$$

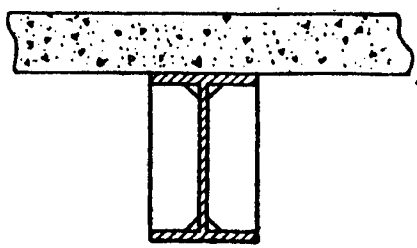
$R_1 = 24{\cdot}91$ t
$R_3 = V_{32} + V_{34} = (-24{\cdot}91 + 60{\cdot}125) + 30{\cdot}06$
$R_3 = 65{\cdot}275$ t, $R_6 = W_u L$ $R_6 = 60{\cdot}125$ t

Try ISLB 600
$W = 10{\cdot}5$ cm
$d = 60$ cm
$I = 72\,867{\cdot}6$ cm^4

Max Shear

$V_{max} = V_{32} = 35{\cdot}22$ t
V_{max} (allowable) (Rule 2) $= 1\,265\ wd = 796{\cdot}95$ kg or $79{\cdot}7$ t$<35{\cdot}22$ t .....OK

Cross-Section Proportions **(Rule 3)**



$b/t = \frac{210}{15{\cdot}5} = 13{\cdot}55 < 17$ ... OK

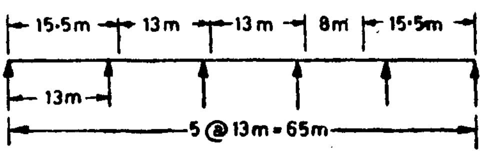
$d/W = \frac{600}{10{\cdot}5} = 57{\cdot}14 > 55$

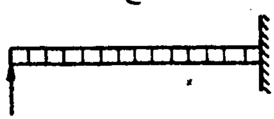
∴ adopt ISMB 550,
check as follows
$t = 1{\cdot}93$ cm
$b = 19{\cdot}0$ cm
$w = 1{\cdot}12$ cm
$d = 55{\cdot}0$ cm

V_{max} (allowable) $= 1\,265\ wd = 77{\cdot}92\ t > 35{\cdot}22$ ..... OK
$b/t = 9{\cdot}8 < 17$ ..... OK
$d/w = 49{\cdot}1 < 55$ ..... OK

Bracing Requirements **(Rule 4)**

Note — Beam supports concrete slab, bottom flange exposed.





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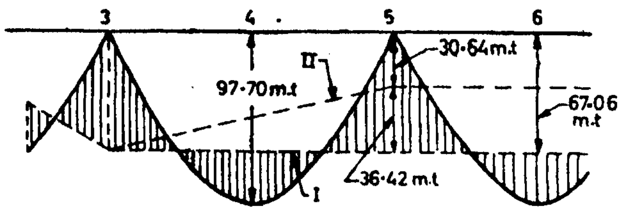
DESIGN EXAMPLE 1 BEAM WITH UNIFORM SECTION — *Contd*

Provide shear splices at points indicated above. Alternatively splice at convenient locations for moment indicated in diagram.

Provide welded vertical plates at section 2.

Deflection at Working Load P (Rule 16)

$$\delta \cong \frac{WL^4}{185EI} = \frac{2\cdot5\times10^9\times13^4}{185\times EI} = 2\cdot9 \text{ cm}$$



25.2 Design Example 2 — This is the design example of a 3-span continuous beam, with dissimilar sections are to meet the needs of the different spans. The simple span moment diagram is first laid out to scale to facilitate the semi-graphical solution. The centre span is the critical one and requires an ISLB 450 shape.

Since it is only planned to splice for shear, the ISLB 450 member will extend into the side spans to the points of inflection. The required moment capacity of these two spans will thus be determined by the moments at Sections 2 and 6. The magnitude of these moments (17·3 m.t and 16·0 m.t) are either calculated as shown or picked off graphically as are the distances to hinge points 2 and 6.

All sections are satisfactory with regard to shear force. Since the smallest beam carries the largest shear, the ISMB 300 need not be checked.

Splices are located at points of inflection and need be designed for shear only. Alternatively, if full moment splices were desirable, then the length of the heavier ISLB 450 beam could be decreased 1·5 m on the left and 1·0 m on the right. The position of the splices are indicated by the dotted ordinates in the moment diagram. It is doubtful if the additional fabrication cost warrants the savings in weight of main material unless the latter is of paramount importance.

No additional bracing at Section 4 was specified because in this configuration, this hinge will not form prior to that at Sections 3 and 5.

DESIGN EXAMPLE 2 CONTINUOUS BEAM

Structure and Loading

3-Span Continuous Beam

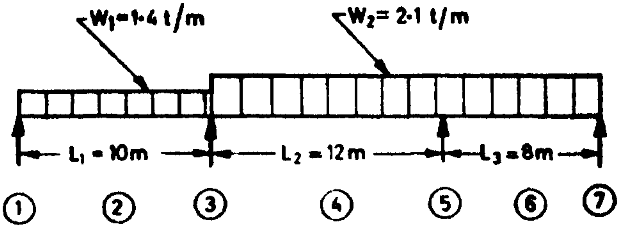
Dissimilar sections will be specified to suit the moment diagram.

$L = 8$ m

$\therefore\ L_1 = 1{\cdot}25\ L$

$L_2 = 1{\cdot}5\ L$

$L_3 = 1{\cdot}0\ L$



$W_u = (1{\cdot}4)(1{\cdot}35) = 2{\cdot}59$ t/m $\therefore\ W_{1u} = W_u$

$W_{2u} = 1{\cdot}5\ W_u$

Design the beam for single loading condition.

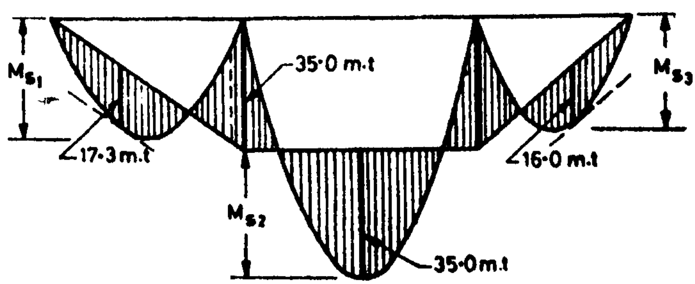
Moment Diagram

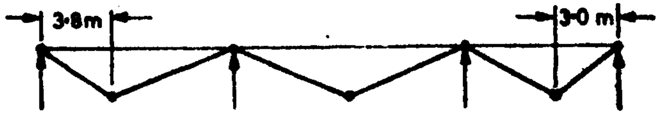
$$M_{s1} = \frac{W_u L_1^2}{8} = \frac{2{\cdot}59 \times 10^2}{8} = 32{\cdot}375 \text{ m.t}$$

$$M_{s2} = \frac{1{\cdot}5 \times 2{\cdot}59 \times 12^2}{8} = 69{\cdot}93 \text{ m.t}$$

$$M_{s3} = \frac{1{\cdot}5 \times 2{\cdot}59 \times 8^2}{8} = 31{\cdot}08 \text{ m.t}$$

$$M_{p1} = \frac{1{\cdot}5\ W_u L_2^2}{16} = \frac{M_{s2}}{2} = 35{\cdot}0 \text{ m.t}$$





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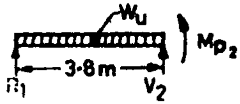
DESIGN EXAMPLE 2 CONTINUOUS BEAM — *Contd*

M_{p2} (determined by scale) = 17·3 m.t
M_{p6} (determined by scale) = 16·0 m.t

Mechanism — *See* moment diagram below

$Z_6 = \frac{M_p}{\sigma_y} = 34{\cdot}965 \times 39{\cdot}68 = 1\,387{\cdot}41\ \text{cm}^3$

Reactions (Ultimate load)



Use ISLB 450

$R_1(3{\cdot}8) - \frac{W_u(3{\cdot}8)^2}{2} = M_{p2}$ $\quad R_1 = 9{\cdot}47$ t

$R_3 = V_{32} + V_{34} = (W_u L_1 - R_1) + \frac{(1{\cdot}5\ W_u L_2)}{2}$ $\quad R_3 = 39{\cdot}74$ t **Use ISLB 325** $\quad$ **$Z = 687{\cdot}8$ cm³**

$R_7(3{\cdot}0) - (1{\cdot}5) = \frac{W_u(3{\cdot}0)}{2} = M_{p6}$ $\quad R_7 = 11{\cdot}16$ t

$R_5 = V_{56} + V_{54} = (1{\cdot}5\ W_u L_3 - R_7) + \frac{(1{\cdot}5\ W_u L_2)}{2}$ $\quad R_5 = 43{\cdot}23$ t **Use ISLB 325**

Note — Total R = 103·6 = Total load applied . . . . . OK

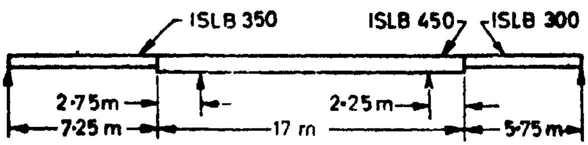
Cross-Section Proportions (Rule 3)

$(b/t < 17;\ \frac{d}{w} < 55{\cdot}0)$

Section	*b/t*	*d/w*
ISLB 450	17/1·34 = 12·69	450/0·806 = 52·3
ISLB 325	16·5/0·98 = 16·84	32·5/0·7 = 46·4
ISMB 300	14·0/102·4 = 11·3	30·0/0·67 = 44·8

Splices

Provide shear splices at points indicated



For Right End Span

$V_{Max} = V$ (in this case) at point of splice $= 1{\cdot}5 \times 2{\cdot}59 \times (8 - 2{\cdot}25) - B_7 = 11{\cdot}17$ t
From Rule 3 V_c of ISMB 300 = 1 265 wd = 25·4 > 11·17 t

In Middle Span

V_c of ISMB 450 = 1 265 wd = 48·95 t
$V_{Max} = V_{34} = 23{\cdot}31$ t < 48·95 t

The *right end* span need not be checked as the smaller section used in right end span is found adequate.

Bracing Requirements (Rule 4)

Top flange continuously supported by concrete slab as in **Design Example 1.** Provide welded vertical plates at sections 2 and 6 as in **Design Example 1.**

25.3 Design Example 3 — This design example is the same as Design Example 2 except that a uniform section is used throughout, reinforced where necessary with cover plates on top and bottom flanges. The left hand span controls the selection of the uniform section, a section which turns out to be adequate for the right-hand span as well without being wasteful of material.

To carry the moment at location 4, cover plates are required with a moment capacity of 25·51 m.t ($M_s - 2M_p$). Two plates 12·0×2·5 cm will be adequate. They should extend somewhat beyond the point at which $M_s = M_p$. This distance is selected as about 0·2 m and the plates should, therefore, be 8 m long.

Two local (beam) mechanisms result. The reactions were not computed in this example, the same procedures being used as in Design Example 2.

The shear force begins to approach (but does not reach) a critical value in this problem. Had it exceeded 35·9 t, then local stiffening of the web would have been required in the region in which $V > V_{Max}$.

The position of the splice(s) is controlled in this problem by transport requirements. A single splice (for shear) is shown at the point of inflection in span 3-4. The cover plates are to be fillet-welded to the beam flanges.

Comparing the weights of three designs (uniform section, design Examples 2 and 3), the following is obtained:

Design	*Shapes*	*Unit Weight*	*Length*	*Weight*
Uniform section	ISLB 450	65·3 kg	30 m =	1 959 kg
Dissimilar section	ISLB 450	65·3 kg	17 m =	1 113·1 kg
	ISLB 325	43·1 kg	7·25 m =	312·5 kg
	ISLB 325	43·1 kg	5·75 m =	247·8 kg
				1 673·4 kg
Uniform section with cover plates	ISMB 350, 12·0×2·5 cm plate	52·4 kg	18 m =	1 572 kg
				188·4 kg
				1 760·4 kg

The lightest design is, therefore, the one in which dissimilar sections are used (Design Example 2). However, local fabrication conditions would dictate whether or not the extra splice in this design would be a more economical choice than the fillet welding of the cover plates of Design Example 3.

DESIGN EXAMPLE 3 CONTINUOUS BEAM WITH COVER PLATES

(Same as Design Example 2 except for cover plates being used on a uniform section)

Structure and Loading

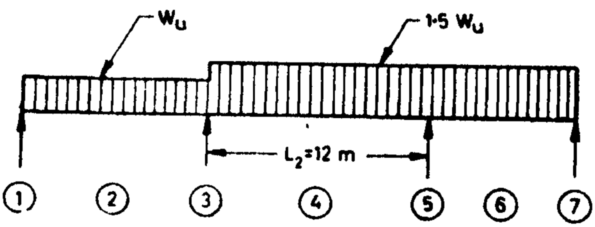
Same as in Design Example 2

Moment Diagram

Left span controls the design.

$M_{p_1} = 0{\cdot}686\ M_s$
$= 22{\cdot}21$ m.t

$Z = 881{\cdot}46\ \text{cm}^3$

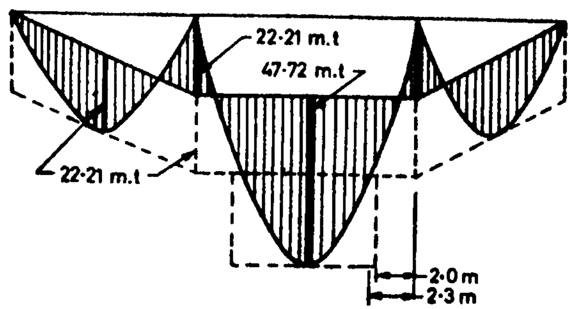


Selection of Section

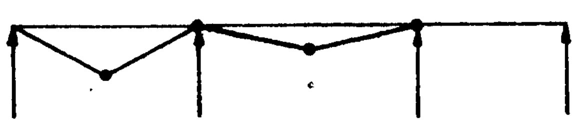
Use ISMB 350

$Z = 889{\cdot}57\ \text{cm}^3$

$M_p = 22{\cdot}42$ m.t



Reinforcing Plates



$\Delta M_p = 69{\cdot}93 - 2(22{\cdot}21) = 25{\cdot}51$ m.t

$$A_{pl} = \frac{\Delta M_p}{\sigma_y d} = 28{\cdot}9\ \text{cm}^2$$

Use 120 × 25 mm plate

$\Delta M_{pc} = 26{\cdot}46$ m.t

(Continued)

DESIGN EXAMPLE 3 CONTINUOUS BEAM WITH COVER PLATES — *Contd*

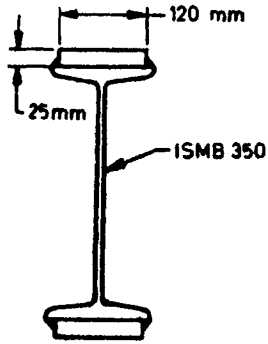
Mechanism — 2 beam mechanisms

Reactions — Compute by statics. *See* previous examples.

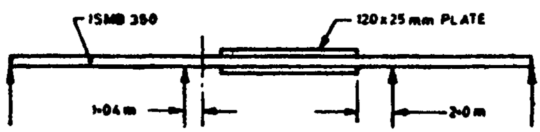
Shear Force (Rule 2) $V_{Max} = V_{34} = 23{\cdot}31$ t*

$V_{Max} = (1{\cdot}265)(0{\cdot}81)(350) = 35{\cdot}9 \text{ t} > 23{\cdot}31 \text{ t}$ .... OK

Cross-Section (Rule 3) $b/t = \dfrac{14{\cdot}0}{1{\cdot}42} = 9{\cdot}8,\ d/w = \dfrac{35{\cdot}0}{0{\cdot}81} = 43 = 14{\cdot}0$ cm .... OK



Splices



Provide splice as indicated in the sketch above.

**See* Design Example 2 $V = 1{\cdot}5\ W_x L_1/2$.

26. DESIGN EXAMPLES ON INDUSTRIAL BUILDING FRAMES

26.0 The designs of rigid steel building frames of the 'industrial' type are illustrated in this clause. This includes single-storey structures only. Single-span and multiple-span frames are treated. The problems include flat and gabled roofs, pinned and fixed bases, and the use of haunches is illustrated.

26.1 Design Example 4 (Single Span, Flat Roof, Hinged Base)— A single span, flat roof, pin-based frame to withstand vertical and horizontal load is worked out in this example.

All applicable 'rules' will be checked. In the absence of wind, $F = 1{\cdot}85$. For the second loading condition a load factor of 1·40 is applied against all loads (live+dead+wind). The uniform vertical load is replaced by concentrated loads at the quarter points (sketch *d*), since in the previous examples we have already seen how to analyze a problem in which the loading was actually assumed as distributed. The distributed horizontal load is replaced by a single load acting at the eaves line.

In arriving at the preliminary choice of member sizes, it is assumed that a uniform section will be used. The important load is the vertical load and thus maximum restraining moments at the ends are desirable (21).

The analysis is carried out by the statical method (17). The redundant is selected as H_6, the horizontal reaction at 6. The fixing line 1-a-b-6 is drawn such that a mechanism forms as shown in sketch *d*. This is a case where, necessarily, the frame is overdeterminate at failure with hinges forming at locations 2, 5 and along 3-4. The required plastic moment is 29·97 m.t.

Case II is now analyzed and it is found that the mechanism is the same as that for Case I, being shown in sketch *f*. The composite moment diagram is that shown by the shaded portion of sketch *e*. In that sketch, the solid line is the determinate moment, the dashed line the moment due to loading by the redundant H_6. The required plastic moment for Case II is 39·69 m.t. This case, therefore, controls the design.

In selecting the section, a plastic modulus of 1 764 cm³ would be required. ISLB 500 supplies a Z of 1 774 cm³. In view of the fact that the analysis was carried out on the basis of concentrated loads (sketch *b*) the lighter section would certainly be adequate. The dotted parabolic moment diagram reveals, in fact, the required M_p is reduced from 39·69 m.t to about 38·0 m.t as determined by scale.

After the reactions are computed, the next step is to check the secondary design considerations. In checking the axial force (Rule 1) it is found that $P/P_y = 0{\cdot}14$; the full value of M_p is thus available.

In checking for lateral bracing, it will be assumed that the purlin spacing is 1·5 m. Between bracing points the ratio L/r_y is 44·5. Now, this value is greater than the value $L/r_y = 30$ given in Eq 51, but this value also assumes that some plastic rotation is required. In the rafter all that is required is that the section reach M_p since the last hinge forms there. So the 1·5 m spacing is adequate.

Turning attention, now, to the columns, the girt spacing is also assumed as 1·5 m. Since the first hinge forms at location 5, the selected spacing may or may not be adequate depending on how much plastic rotation is required to develop the last hinge in the rafter, and how much the adjoining beam lengths restrain the 'critical' segment. While one could calculate the required plastic rotation at point 5*, it is quicker to check the restraint coefficient according to Appendix C. This has been done in the problem; the restraint coefficient, C_f, turns out to be 1·3, increasing the critical bracing slenderness ratio to 39·0. This is close enough to 39·5 for the selected 1·5 m spacing to be adequate.

Bracing details are suggested in the example.

None of the columns are loaded in single curvature and, since $P/P_y < 0{\cdot}15$ the full plastic moments will be transmitted.

The connection detail is sketched h, the thickness of diagonal stiffener being determined from Rule 10.

Although the deflection of such a structure would probably not be computed, an 'estimate' by Rules 15 and 16 shows that the deflection is less than 2·27 cm. This is undoubtedly satisfactory since the crude limitation, $L/360$, gives 2·5 cm as the limit.

*The rotation angle has been calculated and found to give a value of $\frac{HR}{\phi_p LB} < 1{\cdot}0$ indicating a small rotation angle requirement.

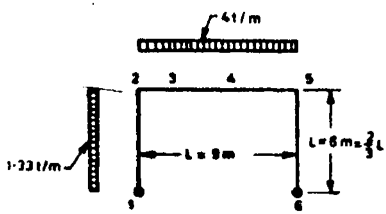
DESIGN EXAMPLE 4 SINGLE SPAN FRAME, FLAT ROOF WITH PIN BASES

Structure and Loading

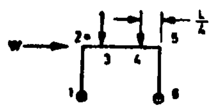
Loading Conditions

Case I — $(DL+LL)$ $F = 1{\cdot}85$
$W_u = 4\times1{\cdot}25=7{\cdot}40$ t/m

Case II — $(DL+LL+\text{Wind}) = 1{\cdot}40$
$W_v = 4{\cdot}0\times1{\cdot}40=5{\cdot}6$ t/m $= W_u$
$W_h = 0{\cdot}95\times1{\cdot}40=1{\cdot}33$ t/m $=0{\cdot}3\ W_u$



(a)



(b)

Replace uniform vertical load by concentrated loads at quarter point $P = \dfrac{W_u L}{2}$

Replace horizontal load with concentrated load with equal overturning moment

$$W = \frac{0{\cdot}3\ W_u L}{3} = 0{\cdot}1\ W_u L$$

Plastic Moment Ratios — Uniform section throughout

Case I — *Analysis*

Moment Diagram (Redundant $= H_6$)

$$M_s = \frac{PL}{4} = \frac{W_u L^2}{8} = \frac{7{\cdot}4\,(9)^2}{8}$$

$$M_s = 59{\cdot}94 \text{ m.t}$$

$$M_p = \frac{M_s}{2} = 29{\cdot}97 \text{ m.t}$$

Mechanism — *See* sketch *d*

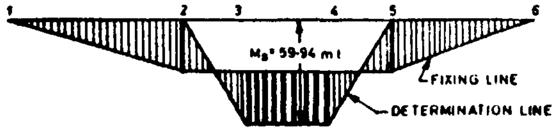
Reactions

$$H_6 = H_1 = \frac{M_p}{h} = \frac{29{\cdot}97}{6} = 5 \text{ t}$$

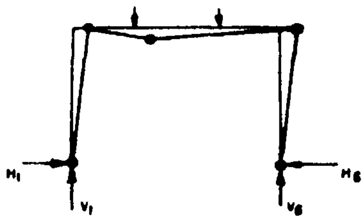
$$V_1 = V_6 = P = 7{\cdot}4\times4{\cdot}5 = 33{\cdot}3 \text{ t}$$

(Continued)

DESIGN EXAMPLE 4 SINGLE SPAN FRAME, FLAT ROOF WITH PIN BASES — *Contd*



(c)



(d)

NOTE — The values given above are to be compared with values for Case II and maximum figures used.

Case II — Analysis

Moment Diagram (Redundant = H_2)

$$M'_2 \text{ (Det)} = Wh = W_vL/10(2L/3) = \frac{W_vL^2}{15}$$

$$= 5{\cdot}6 \times 9^2/15 = 30{\cdot}24 \text{ m.t}$$

$$M_3 = \frac{PL}{4} = \frac{W_uL^2}{8} = 56{\cdot}7 \text{ m.t}$$

Equilibrium at 3 — $M_3 + \frac{3}{4} M'_2 = 2\,M_p$ **Case II (with wind) is critical**

$$\therefore \quad M_p = \frac{56{\cdot}7 + 22{\cdot}68}{2} = 39{\cdot}69 \text{ m.t}$$

Selection of Section **Try ISLB 500**

$$Z = M_p\sigma_y = \frac{39{\cdot}69 \times 100\,000}{20\,250} = 1\,764$$

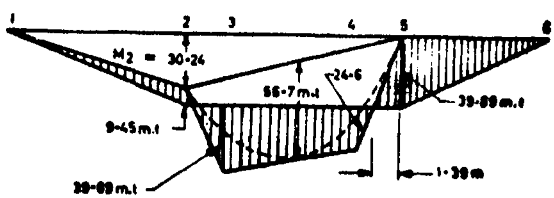
$Z = 1\,773{\cdot}7$ cm^3

Mechanism — Sketch *f*

Reactions (ultimate load)

$$H_6 = \frac{M_p}{h} = \frac{39{\cdot}69}{6} = 6{\cdot}6 \text{ t}$$

$$H_1 = H_6 - W = 6{\cdot}6 - 5{\cdot}04 = 1{\cdot}56 \text{ t}$$



(e)

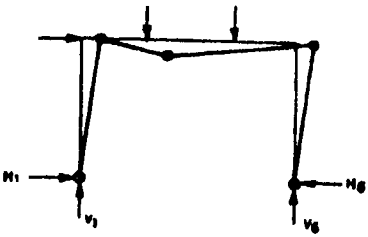
(Continued)

DESIGN EXAMPLE 4 SINGLE SPAN FRAME, FLAT ROOF WITH PIN BASES — *Contd*

$$V_1 = \frac{H_1 h + M_p}{L/4} = \frac{1{\cdot}56 \times 6 + 39{\cdot}69}{2{\cdot}25}$$

$= 21{\cdot}8$ t

$V_6 = 2P - V_1 = 47{\cdot}44 = 50{\cdot}4 - 21{\cdot}8 = 28{\cdot}6$ t



(f)

Moment Check

$M_3 = M_p - M_2 = 39{\cdot}69 - 30{\cdot}24 = 9{\cdot}45$ m.t

$M_4 = V_6(2{\cdot}25) - H_6(6) = 24{\cdot}66$ m.t . . . . OK

Axial Force (Rule 1) (Right-hand column critical)

$$\frac{P}{P_y} = \frac{V_6}{\sigma_y A} = \frac{33{\cdot}300}{2\,520 \times 95{\cdot}5} = 0{\cdot}14 < 0{\cdot}15$$

Shear Force (Rule 2)

$V_{Max} = V_{45} = V_6 = 33{\cdot}3$ t

V_{Max} (ISLB 500) $= 1{\cdot}265\ wd = 58{\cdot}2 > 33{\cdot}3$ . . . . . OK

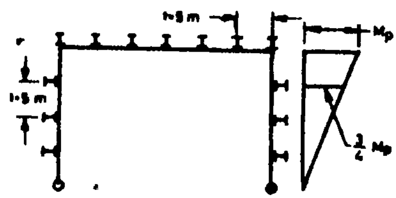
Cross-Section Proportions (Rule 3)

$$b/t = \frac{18}{1{\cdot}41} = 1{\cdot}27,\ d/w = \frac{50{\cdot}0}{9{\cdot}2} = 54 \ . \ . \ . \ . \text{OK}$$

Lateral Bracing (Rule 4)

Spacing

$$\frac{L_B}{r_y} = \frac{1{\cdot}5 \times 100}{3{\cdot}34} = 44{\cdot}9$$



(g)

Rafter hinge OK. Last hinge forms in rafter

Column hinge $\frac{M}{M_p} = \frac{3/4 M_p}{M_p} = 0{\cdot}75$

$\left(60 - 40\frac{M}{M_p}\right) = 30 < 39{\cdot}5$

More refined check is necessary as per Appendix C.

$L_R = 1{\cdot}5$ m

$L_L = 1{\cdot}5$ m

(Continued)

DESIGN EXAMPLE 4 SINGLE SPAN FRAME, FLAT ROOF WITH PIN BASES — *Contd*

$L_{R_{cr}}$ = critical length of elastic segment in column between the first and second girt down from the roof.

$\because\ L_{R_{cr}} = L_u = 3{\cdot}6^{*}$ m

$L_{L_{cr}}$ = critical length of partially plastic segment in girder adjacent to section 5

$$L_{L_{cr}} = (60-40\ M/M_p)r_y = \left(60-40\ \frac{3{\cdot}15}{39{\cdot}69}\right)3{\cdot}34$$

$$= 190 \text{ cm}$$

Evolution of restraint coefficient:

$$f = \tfrac{1}{2}\left(\frac{L_R}{L_{Ru}} + \frac{L_L}{L_{Lcr}}\right) = \tfrac{1}{2}\left(\frac{1{\cdot}5}{3{\cdot}6} + \frac{1{\cdot}5}{1{\cdot}9}\right) = 0{\cdot}6$$

$$Cf = 1{\cdot}3\left(\frac{L_B}{r_y C^r}\right) = Cf\ \frac{L_B}{r_y} = (1{\cdot}3)(30{\cdot}0) = 39{\cdot}0 < 39{\cdot}5 \text{ (Adequate)}$$

Bracing Details

1) Provide welded vertical plates at the three central purlins.
2) At sections 2 and 5 brace to inner (compression) cover from purlin.

Columns (Rule 5) (Rule 7) (Right-hand column critical)

$$\frac{P}{P_y} = 0{\cdot}14 < 0{\cdot}15$$

$\therefore$ Full M_p is available

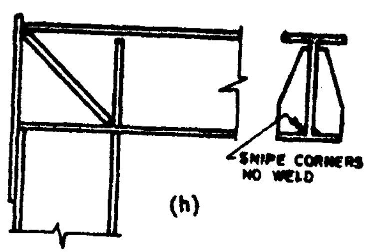
$$\text{weak axis: } 1 - \frac{L/r_y}{330} = 1 - \frac{39{\cdot}5}{330} = 0{\cdot}88 > 0{\cdot}125$$

Connection Detail (Rule 10)

$$t_s = \frac{\sqrt{2}}{b}\left(\frac{s}{d} - \frac{Wd}{\sqrt{3}}\right) = \frac{\sqrt{2}}{18}\left(\frac{1\,543{\cdot}2}{50} - \frac{0{\cdot}92\times 50}{\sqrt{3}}\right)$$

$$= 0{\cdot}50 \text{ cm}$$

See Sketch *h*



(h)

Note — 0·7 cm thick plate required to meet b/t use 9·0 × 0·7 cm plates as stiffeners.

Splices: Provided as part of corner connection detail

Note — Snap the corners (no weld).

$^{*}\frac{d}{tf}$ (ISLB 500) = $\frac{50{\cdot}0}{1{\cdot}41}$ = 35·5 corresponding to this ratio d/t, if l/b = 20 (*Max*) the permissible bending stress F_b = 1 575 kg/cm² (*see* Table 1), l = 20 × 18 = 360 cm or 3·6 m.

(*Continued*)

DESIGN EXAMPLE 4 SINGLE SPAN FRAME, FLAT ROOF WITH PIN BASES — *Contd*

Deflection at Working Load (Rules 15, 16)

Ultimate load	$P_u = 25{\cdot}2$ t
Moment diagram	Sketch *c*
Mechanism	Sketch *f*
Free body diagram	Sketch *i*

Slope deflection Eq: $(\theta_{32} = \theta_{34})$

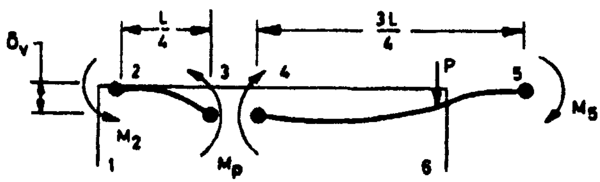
$$\theta_{32} = \frac{\delta v_3}{l} + \frac{1}{3EI}\left(M_3 - \frac{M_2}{2}\right) = \frac{\delta v_3}{L/4} + \frac{L/4}{3EI}\left(-39{\cdot}69 + \frac{9{\cdot}45}{2}\right)$$

$$\theta_{34} = \theta'_{34} + \frac{\delta v_3}{3L/4} + \frac{3L/4}{3EI}\left(39{\cdot}69 + \frac{39{\cdot}69}{2}\right)$$

$$\theta'_{34} = 0{\cdot}028\,\frac{PL^2}{EI}$$

$$\therefore\ \delta v_3 \geqslant \delta u = 4{\cdot}2 \text{ cm}$$

$$\delta w < \frac{\delta u}{F} = \frac{4{\cdot}2}{1{\cdot}85} = 2{\cdot}27 \text{ cm}$$



(i)

$M_2 = 9{\cdot}45$ m.t
$M_3 = 39{\cdot}69$ m.t
$M_5 = 39{\cdot}69$ m.t

26.2 Design Example 5 Single Span Frame with Flat Roof and Fixed Base — In this design example a similar frame to that of Design Example 4 will be designed except that the bases will be fixed. The frame will have a span of 16 m, a column height of 5·33 m, a uniform distributed vertical load of 1·7 t/m and a side (wind) load of 0·7 t/m.

Considerable expense is involved in providing sufficient rigidity to resist the overturning moments at column bases, and this factor should be considered carefully to see that the additional expense is warranted. There is less advantage to fixed column bases if the side loads are small. At the opposite extreme is a structure designed to withstand blast load. In one instance the capacity of a structure to resist externally-applied side load was increased nine-fold simply by fixing the column bases and without changing member sizes whatsoever. Quite evidently there are areas where the additional construction expense would be warranted in view of the improved load-carrying capacity of the structure. Since plastic design makes maximum possible use of the material, it extends the applicability of fixed column bases. Tall buildings, and industrial frames carrying relatively large cranes which might otherwise be sensitive to lateral deflections would constitute two other cases where fixed bases would be considered.

As in the previous example, the most economical design results from using a uniform member throughout.

Of the various methods for handling distributed loads, one which has been discussed (*see* **19**) but not yet illustrated is to assume the purlin spacing at the outset and to analyze the frame on the basis of the purlin loads. This method will be used here; the purlin load is found to be 6·29 t for Case I and 5·76 t for Case II.

The mechanism method of analysis is used in this problem in view of the greater redundancy of the structure when compared with Design Example 4. For Case I with no side load, mechanism 1 will control and it is found that $M_p = 50{\cdot}32$ m.t. Note that if the actual distributed load had been used, then $M_p = W_u L^2/16 = 50{\cdot}32$ m.t. This is the same value as for concentrated load, and is contrary to expectation. The reason is that the end purlin reacts directly on the column. Although the frame is redundant at failure $[I = X-(M-1) = 3-(3-1) = 1]$ the moment check is easily made by remembering that the elastic carry-over factor is one-half for cases such as members 3-1 and 4-5.

Analyzing Case II it is found that mechanism 1 still controls with a required M_p of 46·08 m.t. In this part of the problem two approaches are possible: (a) try the two most 'likely' mechanisms, namely, Mechanisms 1 and 3; or (b), try Mechanism 1 and make a moment check. The former was done in this case, the moment check following for mechanism 1 when it was discovered that Mechanism 1 controlled.

Case I is found to be critical with a required M_p of 50·32 m.t consequently an ISLB 550 shape is specified ($Z = 2\,228{\cdot}2$ cm^3).

The check (according to Eq 51) to determine the adequacy of the selected girt spacing again indicates that more refined calculation is needed. This is made according to Appendix C and it is found to be adequate.

The work axis check for the column is not needed, the axial load ratio being a very low value.

A deflection analysis was not made in this example. If desired, the procedure is as outlined in **23.9.1**. Since Case I is the controlling condition, the deflection would be calculated on the basis that $\theta_{s_3} = 0$ (last hinge forms at the centre of beam).

DESIGN EXAMPLE 5 SINGLE SPAN, FLAT ROOF, FIXED BASE

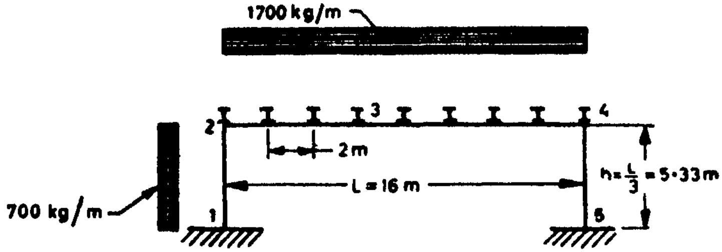
Structure and Loading — Sketch *a*

Loading Conditions (Purlins at 2 m spacing)

Case I — $(DL+LL)$ $\quad F=1{\cdot}85$

$W_u = (1\,700)(1{\cdot}85) = 3\,145$ kg/m

$P_u = 2\ W_u = 6{\cdot}29$ t



(a)

Case II — $(DL+LL=\text{Wind})$ $F = 1{\cdot}40$

$P_u = 2\ W_u = 5{\cdot}96$ t

$W_v = (1{\cdot}7)(1{\cdot}40) = 2{\cdot}38$ t/m $= W_u$

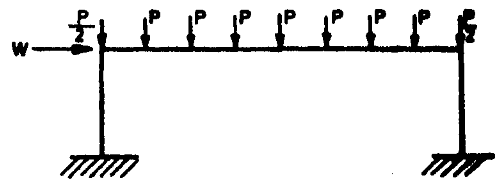
$W_h = (0{\cdot}7)(1{\cdot}40) = 0{\cdot}98$ t/m $= 0{\cdot}412\ W_u$

$$W = \frac{W_h h^2}{2h}$$

$$= 0{\cdot}412\ W_u(2{\cdot}67)$$

$$= 1{\cdot}1\ W_u = 2{\cdot}62 \text{ t}$$

Plastic Moment Ratio—Uniform section throughout



(b)

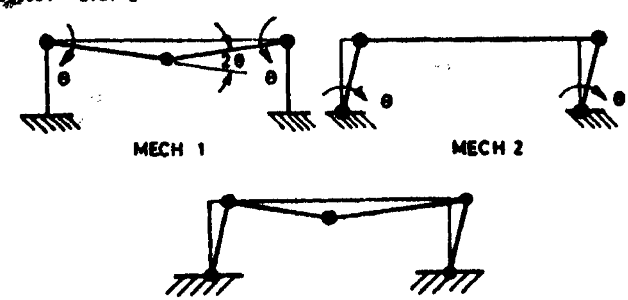
Case I — ***Analysis*** (Mechanism method)

Possible plastic hinges $= 5$ (sections 1, 2, 3, 4, 5)

Possible mechanisms $= 2$ $(n = N - X = 5 - 3 = 2)$

Elementary: No. 1 and 2

Composite: No. 3



(c)

(Continued)

DESIGN EXAMPLE 5 SINGLE SPAN, FLAT ROOF, FIXED BASE — *Contd*

Solution for Mechanism 1

$$M_p(\theta+2\theta+\theta) = \frac{PLA}{8}\left(1+2+3+\frac{4}{2}\right)2 = 2PL\theta$$

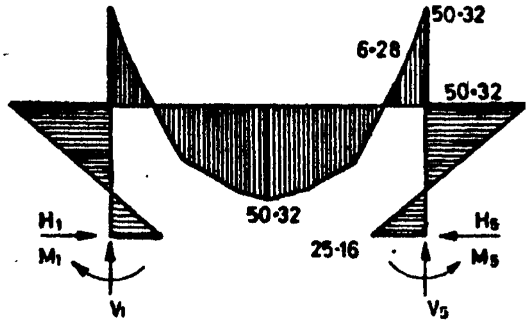
$$M_p = \frac{PL}{2} = \frac{6{\cdot}29\times16}{2} = 50{\cdot}32 \text{ m.t}$$

Moment Check — Sketch *d*

$$M_1 = \frac{M_p}{2} = \frac{50{\cdot}32}{2} = 25{\cdot}16 \text{ m.t}$$

sway equilibrium

$$M_5 = M_1 = 25{\cdot}16 < M_p \text{ ..OK}$$



(d)

Reactions at Ultimate Load

$$V_1 = V_5 = 4P = 25{\cdot}16 \text{ t}$$

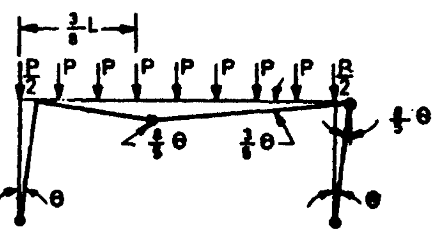
$$H_1 = H_5 = \frac{M_5+M_1}{h} = \frac{75{\cdot}48}{5{\cdot}33} = 14{\cdot}16 \text{ t}$$

Case II — Analysis

Hinges and Mechanism (*See* Case I)

Solutions for Mechanism 1 — Sketch *c*

$$M_p = \frac{PL}{2} = 5{\cdot}76\times8 = 46{\cdot}08 \text{ t}$$



(e)

Solution for Mechanism 3 — Sketch *e*

$$M_p\theta\left(1+\frac{8}{5}+\frac{8}{5}+1\right) = W\theta h+\frac{PL\theta}{8}\left[1+2+3+\frac{3}{5}(1+2+3+4)\right]$$

$$5{\cdot}2\,M_p = 2{\cdot}62(5{\cdot}33)+\frac{3}{2}(5{\cdot}76)16$$

$$M_p = 29{\cdot}27 \text{ m.t}$$

NOTE — Moment at the centre of beam $< 50{\cdot}32$ m.t.

(*Continued*)

DESIGN EXAMPLE 5 SINGLE SPAN, FLAT ROOF, FIXED BASE — *Contd*

Moment Check for Mechanism 1

Assume $M_5 = M_p$ and use trial and error method.

$\therefore \; M_1 - M_2 + Wh = 0$

$M_1 = M_2 - Wh = M_p - (2{\cdot}62)(5{\cdot}33) = 46{\cdot}08 - 13{\cdot}96$

$= 32{\cdot}12 < 46{\cdot}08$ mt $= M_p$ . . . . OK

Selection of Section — Case I (without wind) is critical

Use ISLB 550

$A = 109{\cdot}97$ cm² $t = 9{\cdot}90$ mm
$d = 550$ mm; $w = 15{\cdot}0$ mm
$b = 190$ mm; $S = 1\,933{\cdot}2$ cm³
$r_y = 3{\cdot}48$; $I = 53\,161{\cdot}6$ cm⁴

Axial Force (Rule 1)

$$\frac{P}{P_y} = \frac{V_1}{\sigma_y A} = \frac{25{\cdot}16}{2\,520 \times 109{\cdot}97} = 0{\cdot}091 < 0{\cdot}15 \ldots\ldots \text{OK}$$

Shear Force (Rule 2)

$V_{Max} = V_{31} = V_5 = 25{\cdot}16$

Allowable $V = 1{\cdot}265\ wd = 68{\cdot}9 > 25{\cdot}16$ . . .OK

Cross-Section (Rule 3)

$b/t = 12{\cdot}67 < 17$

$d/w = 55{\cdot}56$ Slightly > 55 . . . . . OK

Lateral Bracing (Rule 4) — *Spacing Check*

Rafter (purlin spacing = 2 m)

$$\frac{L_B}{r_y} = \frac{2 \times 100}{3{\cdot}48} = 57{\cdot}47 > 35 \ldots\ldots \text{OK as last hinge in the rafter}$$

Column (girt spacing = 1·5 m from top)

$$\frac{L_B}{r_y} = \frac{1{\cdot}5 \times 100}{3{\cdot}48} = 43{\cdot}1$$

$$\frac{M}{M_p} = \frac{M_p - \frac{5}{16}(M_p + M_2)}{M_p} = \left[50{\cdot}32 - \frac{5}{16}\left(\frac{50{\cdot}32 + 25{\cdot}16}{50{\cdot}32}\right)\right] = 0{\cdot}53$$

From Eq 51, $\left(\frac{L_B}{r_y}\right)_{cr} = 60 - 40\frac{M}{M_p} = 38{\cdot}8 < 43{\cdot}1$

$\therefore$ A more refined check is necessary

Evaluation of restraint coefficient

$$f = \tfrac{1}{2}\left(\frac{L_R}{L_{R_{cr}}} + \frac{L_L}{L_{L_{cr}}}\right) = \tfrac{1}{2}\left(\frac{1{\cdot}5}{L_u} + \frac{2{\cdot}0}{L_B'}\right) = \tfrac{1}{2}\left(\frac{1{\cdot}5}{3{\cdot}8^*} + \frac{2{\cdot}0}{1{\cdot}93}\right) = 0{\cdot}715$$

$C_f = 1{\cdot}2$

$$\therefore \left(\frac{L_B}{r_y}\right) = C_f\frac{L_B}{r_y} = 1{\cdot}2 + 38{\cdot}8 \times 46{\cdot}56 > 43{\cdot}1 \ldots\ldots \text{OK}$$

Bracing Details

1) Provide vertical welded plates at centre as in Design Example 1.

2) At Section 2 and 4 brace to inner (compression) corners.

*$\frac{d}{t} = \frac{550}{15} = 36{\cdot}6$, $l = 20 \times 19 = 3{\cdot}8$ m.

(Continued)

DESIGN EXAMPLE 5 SINGLE SPAN, FLAT ROOF, FIXED BASE — *Contd*

Columns (Rule 5) (Rule 7)

$$\frac{P}{P_y} = 0\cdot092 < 0\cdot15$$

$\therefore$ Full M_p is available

Connection Detail (Rule 10)

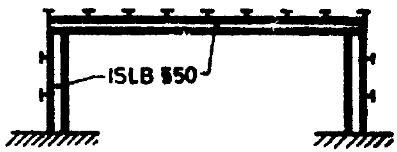
$$t_s = \frac{\sqrt{2}}{b}\left(\frac{s}{d} - \frac{Wd}{\sqrt{3}}\right) = \frac{\sqrt{2}}{19\cdot0}\left(\frac{1\,933\cdot2}{55\cdot0} - \frac{0\cdot99\times55}{\sqrt{3}}\right) = 0\cdot3 \text{ cm}$$

Use $8\times0\cdot6$ cm plates as splices

Splices

See detail *h* in Design Example 4. Provide as part of corner connection (Beam is continuous across column top).

Frame Layout



26.3 Design Example 6 (Single-span Portal Frame with Gabled Roof)— In this design example a single-span portal frame with gabled roof will be designed to resist vertical and side load. The frame has a span of 30 m, the column height is 6 m and the rafter rise is 4·5 m. Greatest economy of steel will be realized if haunches are used at the corners.

The vertical distributed load is replaced by concentrated loads applied at the purlins (1·5 m spacing). The horizontal distributed load is replaced by a single concentrated load, acting at the eaves, which produces the same moment about point 1. In other words, it is a concentrated load which produces an over-turning moment equal to that of the uniformly distributed load.

Since the frame is only redundant to the first degree, the equilibrium method of analysis is used. It may thus be determined that the hinge forms under the second rafter from the crown. The problem is to find the required plastic moment of the girder and then to proportion the column for the required moment at location 2. The required M_p-value is determined by equating the moments at these locations. The required plastic moment for this case is 59·9 m.t.

Instead of using the statical method of analysis, the mechanism of sketch *c* could have been used as the basis for satisfying the equilibrium condition. The position of the instantaneous centre is first located (*see* sketch *a* below), the coordinate being 9·6 m vertically and 4·17 m horizontally from column base number 1. The mechanism angle at hinge 3 is, therefore, $(2{\cdot}7/6{\cdot}9+1)\theta - 1{\cdot}39\ \theta$. The angle at section 4 is equal to θ. The vertual work equation may next be written. The external work for one-half of the frame is:

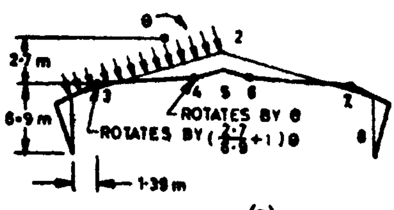
$$W_E = P\theta(0{\cdot}33+1{\cdot}83+3{\cdot}33+4{\cdot}83+6{\cdot}33+7{\cdot}83+7{\cdot}83+7{\cdot}83/2)$$
$$-P(2{\cdot}70/6{\cdot}9)(1{\cdot}5+3{\cdot}0)$$

$$W_E = 34{\cdot}46\ P\theta$$

The internal work is:

$$W_I = M_p\theta(1{\cdot}39+1) = 2{\cdot}39 M_p\theta$$

thus



(a)

$$M_p = 34{\cdot}46\ P/2{\cdot}39 = \frac{34{\cdot}46\times4{\cdot}16}{2{\cdot}39} = 59{\cdot}0 \text{ m.t}$$

which is, of course, identical with the answer obtained before by the statical method.

Analyzing for Case II, the redundant is selected as H_8 (sketch *d*). The moment diagram for the determinate structure is shown by the solid line in sketch *c*. Rather than work Case II as a new problem, it will only be determined whether or not the member selected for Case I is adequate for the Case II loading. Therefore, the composite moment diagram is drawn such that the girder moment is 59·9 m.t, the value obtained for Case I. The mechanism is shown in sketch *f* with hinges forming at Sections 4 and 8. The problem is to find the moment at 8 and observe whether or not it is greater than the 101·1 m.t found for Case I. From equilibrium at Section 4, H_9 is determined (6·95 P); hence by equilibrium at Section 8 it is found that the required M_p—88·7 m.t. Since this is less than the value of 101·1 m.t, the Case I appears to control.

The moment check should still be made, and while satisfactory, the moment at Section 7 is very close to the maximum available M_p(59·22 as compared with 59·9). The design of details should therefore be carried out on the basis that a plastic hinge could form at Sections 4, 7, and 8 (and by symmetry Sections 2, 3, and 6).

Case I is thus found to be the critical case; the reactions for this case are also the greatest.

In checking for axial force, it is found that the P/P_y ratios are greater than in the previous problems. As a matter of fact, the axial force ratio is higher in the girder than in the column because the member is lighter and due to the sloping roof, both the horizontal and vertical reactions at the column base produce a thrust component in the rafter.

With a purlin spacing of 1·5 m the L_B/r_y turns out to be 39·5. In the centre hinge positions (locations 4 and 6) this slenderness ratio is satisfactory even though the moment diagram is 'flat' because the corresponding plastic hinges will be the last to form. With regard to the hinges that form in the rafter and adjacent to the haunch, a consideration of the moment ratio ($M/M_p = 0{\cdot}406$) shows that the resulting allowable slenderness ratio is 47·8 which is greater than the value of 39·5 supplied. No further check is therefore necessary. Concerning the column, the member was proportioned simply to provide strength and not to participate in mechanism action. A single brace between the end of the haunch and the column base would, therefore, be adequate.

With regard to the bracing details, support is required on the inner (compression) side at all points on the haunch where the flange force changes in direction. It is considered desirable to provide similar bracing at the peak. Concerning the bracing at plastic hinges that form near the peak, it was pointed out above that the two loading conditions required very nearly the same plastic modulus. In the one case the plastic hinge forms at the second purlin (3 m) from the crown and in the other at the third purlin (4·5 m). Therefore it is desirable to brace at all four locations.

In calling for a 1·0 cm plate at the peak (sketch *j*), it is assumed that the web will carry no thrust, and a plastic analysis is carried out to proportion the vertical plate stiffener.

The 'design' of the haunch details is controlled in part by the initial choice of dimensions. At the outset it was decided that the haunch would extend 3 m into the girder span and 2·0 m down the column as shown in sketch *b*. The remaining geometry is open to choice, to a degree at least. For a geometry similar to that selected in this example, it has been shown that the angle shown in sketch *j* should be greater than 11°[28]. An angle of about 13° gives a depth d_1 of 118 cm providing a reasonable appearance, and is therefore selected. As required by Rule 11, the flange thickness is made 2·5 cm which is 50 percent greater than that of the ISLB 600. The width of this flange is made uniform at 21·0 cm along the girder portion of the haunch and then is gradually tapered to meet the 25·0 cm width of the ISWB 600 flange. The web is selected at 1·2 cm which is about the same as that of the rolled members joined.

If, for some other problem the geometry were much different from that shown in sketch *k*, a further check on adequate strength of the haunch could be made either by the elastic method of Ref 28 or by an approximate plastic analysis. In the latter case, we would check to see that the plastic modulus supplied at the critical section exceeds the required value by the same margin as that which exists at Section 7. This calculation may be carried out using as a value for Z the expression given in Eq 69:

$$Z = bt(d-t)+\frac{w}{4}(d-2t)^2 \qquad \ldots \qquad \ldots \qquad \ldots(69)$$

Such a calculation for this particular problem also shows the design to be adequate.

As far as the remaining details of the haunch are concerned, the end plates need only be of reasonable thickness, and on the basis of the calculations made for the peak, are selected as 1·0 cm. The diagonal stiffener thickness should be equal to that of the rolled section flange. A 1·6 cm plate is therefore selected.

This completes the design of the frame which is shown in sketch *l*. With regard to splices, the columns and haunches could be shop-assembled with a field splice at Section 7. Alternatively, a splice for full moment using high-strength bolts could be made at Section 7 or bolted splices for less than full moment strength could be supplied at a section near the point of inflection (haunch in sketch *l*).

It is of interest to compare the results of this design with the elastic solution and with the plastic solution for the case where no haunch has been used. Not only is there considerable savings in each case of the plastic over the elastic design, but it may be shown that possible weight savings may be achieved in plastic design as well as elastic design whenever a haunch is specified. Of course, the haunch fabrication expense should be borne in mind when making comparison of overall costs.

DESIGN EXAMPLE 6 SINGLE SPAN GABLED FRAME

Structure and Loading — Sketch *b*

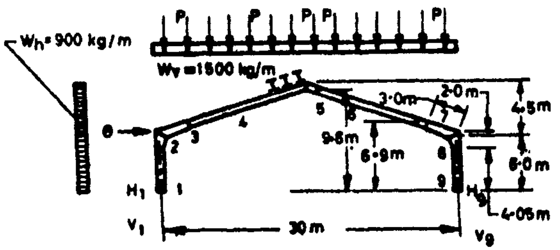
Loading Conditions

Purlin spacing assumed = 1·6 m

Case I — $(DL+LL)$ $F = 1{\cdot}85$

$W_u = (1{\cdot}5)(1{\cdot}85) = 2{\cdot}775$ t/m

$P_u = 1{\cdot}5\ W_u = 4{\cdot}16$ t



(b)

Case II — $(DL+LL+\text{Wind})$ $F = 1{\cdot}40$

$P_u = 1{\cdot}5\ W_u = 1{\cdot}5 \times 2{\cdot}1 = 3{\cdot}15$ t

$W_v = (1{\cdot}5)(1{\cdot}40) = 2{\cdot}10$ t/m $= W_u$

$W_h = (0{\cdot}9)(1{\cdot}40) = 1{\cdot}26$ t/ m $= 0{\cdot}6\ W_u$

$$Q = \frac{W_h\left(\frac{h+f}{2}\right)^2}{h} = \frac{1{\cdot}26\ (10{\cdot}5)^2}{2(6)} = 11{\cdot}58 = 3{\cdot}67\ P$$

Plastic Moments Ratios

Adjust M_p to suit the Moment Diagram — *See* below:

Case I — *Analysis* (Statical Method)

Moment Diagram

(Redundant = H_9)

$$M_s = \frac{W_u L^2}{8} = 2{\cdot}775(30)^2\frac{1}{8} = 312 \text{ m.t}$$

Equilibrium — $H_1(8) - M_8$ (Determinate) $= M_4$ (Determinate) $- H_1(9{\cdot}6) = M_p$

$$M_8 = (10P)(3{\cdot}0) - P\left(1{\cdot}5 + \frac{3}{2}\right) = 27\ P$$

$$M_4\ (10P)(12{\cdot}0) - P\left(1{\cdot}5+3+4{\cdot}5+6+7{\cdot}5+9+10{\cdot}5+\frac{12}{2}\right) = 72\ P$$

$$H_1 = \frac{M_4+M_8}{9{\cdot}6+6{\cdot}9} = \frac{99P}{16{\cdot}5} = 6\ P$$

$$M_p = 72P - 6P\ (9{\cdot}6) = 14{\cdot}4\ P = 59{\cdot}9 \text{ m.t}$$

Column: M_p (Col) $= (H_1)(4{\cdot}05) = 6P(4{\cdot}05)\ 24{\cdot}3P = 101{\cdot}1$ m.t

(Continued)

DESIGN EXAMPLE 6 SINGLE SPAN GABLED FRAME — *Contd*

Mechanism — Sketch *c*

Moment Check — All $M < M_p$

Reactions

$H_1 = H_9 = \ 6P = 24{\cdot}96$ t
$V_1 = V_9 = 10P = 41{\cdot}6$ t

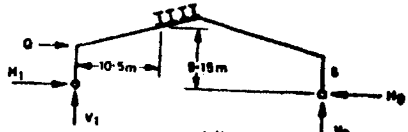
(c)

Case II — Analysis:

Moment Diagram — (Redundant $= H_9$)

$M_s = 75\ P_u$

M corner (det) $= Q(6) = 22{\cdot}02\ P$



(d)

Mechanism — Sketch *f*

Equilibrium at Section 4

$M_4 + H_9(9{\cdot}15) =$ Determinate Moment at 4

Determinate Moment at $4 = (0{\cdot}65)(22{\cdot}04) + M_a$

$$M_a = (10P)(10{\cdot}5) - P\left(1{\cdot}5 + 3 + 4{\cdot}5 + 6{\cdot}0 + 7{\cdot}5 + 9{\cdot}0 + \frac{10{\cdot}5}{2}\right) \quad \text{(e)}$$

$M_a = 68{\cdot}25\ P$

M_4 should be adjusted as $M_4 = M_p = \dfrac{59{\cdot}9}{3{\cdot}15} = 19{\cdot}01\ P$

$H_9(9{\cdot}15) = (6{\cdot}5)(22{\cdot}02\ P) + 68{\cdot}25\ P - 19{\cdot}01\ P$

$H_9 = \dfrac{63{\cdot}5}{9{\cdot}15} = 6{\cdot}95\ P$

(f)

Equilibrium at Section 8

$M_8 = H_9(4{\cdot}05)$

$M_8 = 88{\cdot}7$ m.t $< 101{\cdot}1$ (Case I)

Moment Check — Diagram plotted — sketch *e*

$$M_7 = H_9(6{\cdot}9) - M_7(\text{Determinate}) = 6{\cdot}95\ P(6{\cdot}9) - \left[10\ P(3) - P\left(1{\cdot}5 + \frac{3{\cdot}0}{2}\right)\right] - \frac{22{\cdot}02\ P}{10}$$

$M_7 = 18{\cdot}8\ P = 59{\cdot}22$ m. t $< 59{\cdot}9$ m.t

Reactions

$H_9 = 6{\cdot}95\ P = 21{\cdot}9$ t
$H_1 = H_9 - Q = 21{\cdot}9$ t $- 3{\cdot}67\ P$
$H_1 = 10{\cdot}32$ t

(g)

(Continued)

DESIGN EXAMPLE 6 SINGLE SPAN GABLED FRAME — *Contd*

$$V_1 = \frac{18\ P(15) + \frac{P}{2}\ (30) - Q(6)}{30} = 8{\cdot}77\ P$$

$V_1 = 27{\cdot}62$ t
$V_2 = 20\ P - V_1 = 18{\cdot}9 = 35{\cdot}38$ t

NOTE — Case I reactions control the design.

Case I (without wind) is critical.

(h)

Selection of Section

Girder: $Z = \frac{M_p}{\sigma_y} = (59{\cdot}9)\left(\frac{100\ 000}{2\ 520}\right) = 2\ 380\ \text{cm}^3$

Use ISLB 600

A =	126·69 cm²	r_y =	3·79 cm
d =	600 mm	Z =	2 798·6 cm³
b =	210 mm	S =	2 428·9 cm³
t =	15·5 mm	I =	72 867·6 cm⁴
w =	10·5 mm		

Column: $Z = (101{\cdot}1)\left(\frac{100\ 000}{2\ 520}\right) = 4\ 020\ \text{cm}^3$

Use ISWB 600

A =	184·86 cm²	w =	11·8 mm
d =	600 mm	Z =	4 341·6 cm³
b =	250 mm	S =	3 854·2 cm³
t =	23·6 mm	I =	1 15 626·6 cm⁴
r_y =	5·35 cm		

Axial Force

Column: $\frac{P}{P_y} = \frac{V_2}{\sigma_y A} = \frac{41{\cdot}6 \times 1\ 000}{2\ 520\ A} = \frac{41\ 600}{465{\cdot}85} = 0{\cdot}089\ 3 < 0{\cdot}15$ ..... OK

Girder: $\frac{P}{P_y} = \frac{H_2 \cos\theta + (V_2 - 2{\cdot}5\ P)\sin\theta}{2\ 520(126{\cdot}69)}$

$\theta = \tan^{-1}\frac{4{\cdot}5}{15} = 16°40$

$\sin\theta = 0{\cdot}288$
$\cos\theta = 0{\cdot}960$

$$\frac{P}{P_y} = \frac{(24{\cdot}96)(0{\cdot}96) + (41{\cdot}6 - 2{\cdot}5 \times 4{\cdot}16)}{2{\cdot}520 \times 126{\cdot}69} = \frac{0{\cdot}288}{0{\cdot}103} < 0{\cdot}15 \ldots\ldots \text{OK}$$

Shear Force

V_{max} = Shear at end of girder

$V_{max} = V_2 - \frac{P}{2} = 41{\cdot}6 - \frac{4{\cdot}16}{2} = 39{\cdot}52$ t

Permissible shear = 79·7 t > 39·52 t ..... OK

(Continued)

DESIGN EXAMPLE 6 SINGLE SPAN GABLED FRAME — *Contd*

Cross-Section Proportions

	Girder	Column
b/t =	13·55	10·59 < 17 . . . . . OK
d/w =	57·14	50·85 < 55 . . . . . OK

Lateral Bracing — Spacing Check

Girder — (purlin spacing = 1·5 m)

$$\frac{L_B}{r_y} = \frac{1\cdot5\times100}{3\cdot79} = 39\cdot5 > 35$$

'centre' position OK as hinges form there last.

Check moment ratio correction on hinges at haunches.

$M_s = M_p = 59\cdot9$ m.t

$$M_A = -H_1(7\cdot35) + V_1(4\cdot5) - P\left(1\cdot5 + 3 + \frac{4\cdot5}{2}\right)$$

$$M_A = -24\cdot34 \text{ m.t}$$

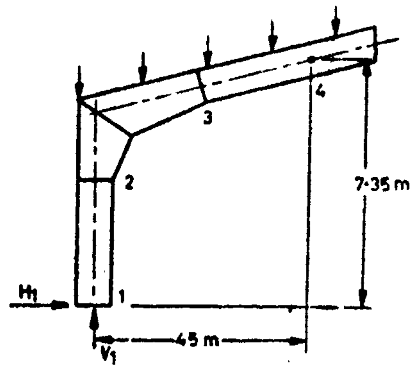
$$\therefore \frac{M_A}{M_P} = 59\cdot9 = +0\cdot406$$

$$\left(\frac{L_B}{r_y}\right)C_r = 60 - 30\frac{M}{M_p} = 60 - 30\times0\cdot406$$

$$= 47\cdot82$$

$$> 39\cdot5 \quad \text{...OK}$$

Provide brace midway between end of haunch (Section 8) and column base.



(I)

$$\frac{L_B}{r_y} = \frac{2\cdot00\times100}{5\cdot35} = 37\cdot4$$

OK because large hinge rotation not required at Location 2.

Bracing Details

Haunches—Provide bracing to inner (compression) flange at each end and at centre.

Peak — Provide bracing to inner flange.

Note — These purlins should be adequately braced in order to provide support to rafters.

Columns

$$\frac{P}{P_y} = 0\cdot098 < 0\cdot15 \text{ . . . . OK}$$

$$L/r = 37\cdot4 < 60 \text{ . . . . . OK (Rule 7)}$$

(Continued)

DESIGN EXAMPLE 6 SINGLE SPAN GABLED FRAME — *Contd*

Connection Details

Peak — Proportion stiffener to transmit flange thrust.

$$\sigma_y A_s = 2\sigma_y A_f \sin\theta$$
$$bt_s = 2bt \sin\theta$$
$$t_s = 2t \sin\theta$$
$$= (2)(1{\cdot}55)(0{\cdot}288)$$
$$= 0{\cdot}89 \text{ cm}$$

Use 10 mm plate

Haunch (Rule 11)

Geometry — Sketch *k*

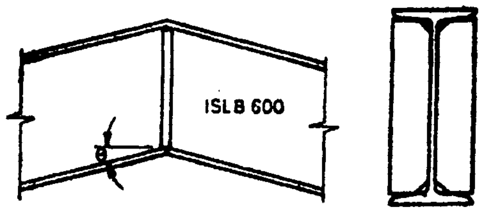
Select $\beta \simeq 13°$ $(>11°)$

Flange

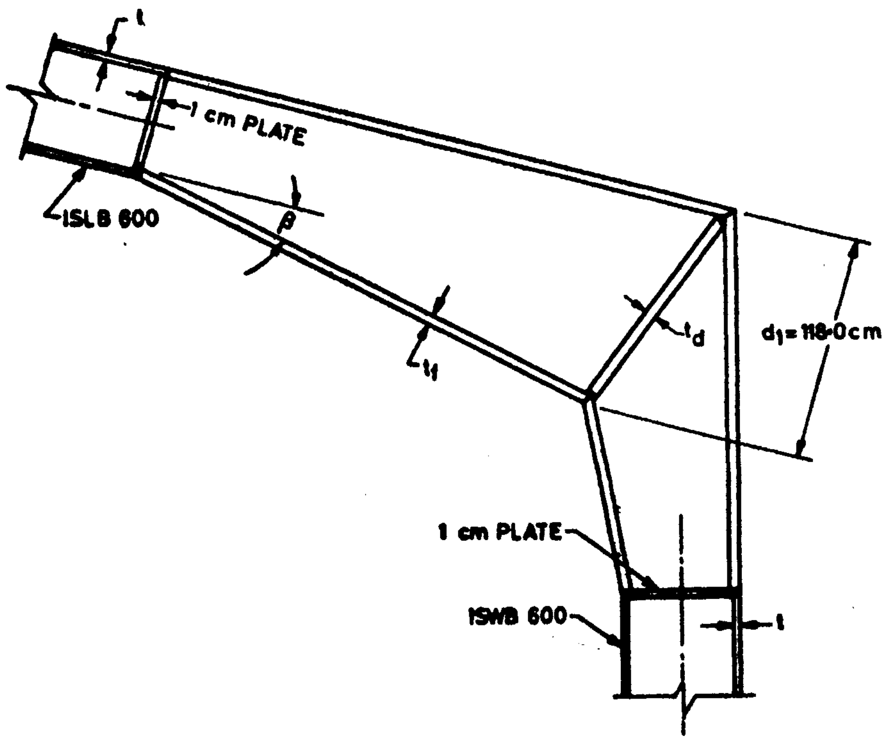
$t_f = 1{\cdot}50\ t$

$t_f = (1{\cdot}50)(1{\cdot}55) = 2{\cdot}33$ cm

Use 25 mm plate



(j)



(k)

(Continued)

DESIGN EXAMPLE 6 SINGLE SPAN GABLED FRAME — *Contd*

End Plates

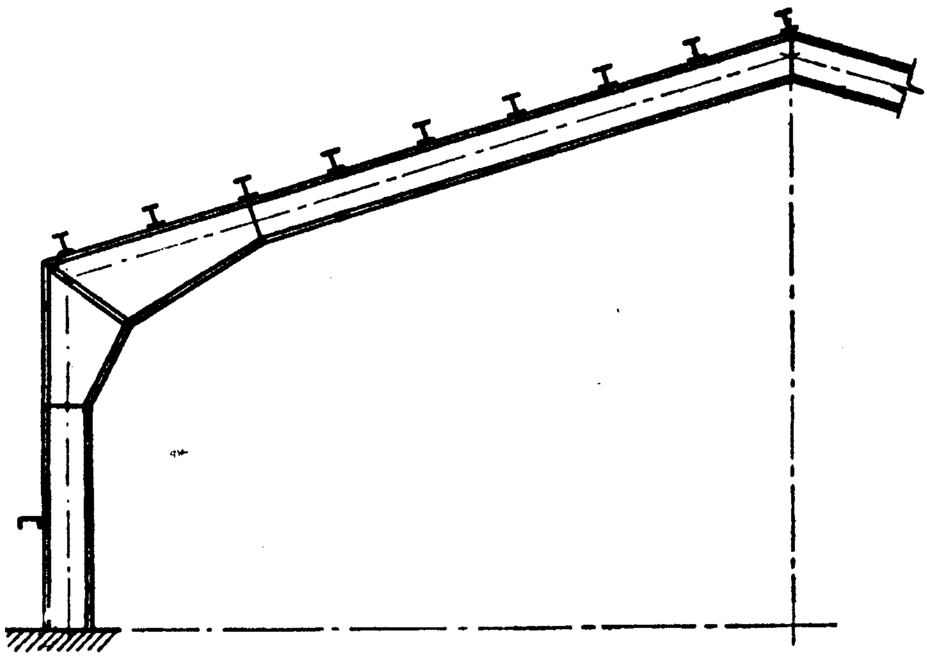
Only plates of 'nominal' thickness are required, as at peak.

Diagonal Stiffener	Use 10 mm plate
$t_d = t = 1{\cdot}55$ cm	Use 16 mm plate
web	Use 12 mm plate

Splices

Provide as part of the haunch and peak detail.

Frame Layout — as shown below:



(I)

26.4 Design Example 7 (Two-Span Industrial Flat Roof, Frame) — In this design example an industrial frame will be designed to carry a vertical load of 1 800 kg/m and a horizontal side load of 900 kg/m. The mechanism method will be used to analyze the various loading conditions. Distributed load will be treated as such, although the loads will actually come to the structure through purlins. Opportunity will be afforded in this two-span frame to illustrate the 'preliminary design' procedures for estimating plastic moment ratios. As shown in sketch *a* the column height is 5 m, the left span is 10 m, and the right span is 30 m.

For the time being the value M_p is assigned to the left rafter, the value k_1M_p to the right rafter and k_2M_p for the interior column. The dotted lines shown in sketch *a* are simply an aid towards keeping the signs straight — positive moment produces tension on the side of the beam next to the dotted lines.

There are two possible loading conditions. For Case I with dead load and live load, the load factor of safety is 1·85. The distributed load becomes 3·33 t/m. For Case II (dead load *plus* live load *plus* wind) the load factor of safety is 1·40 and the vertical load is 2·52 t/m, the horizontal load being half this value.

In order to determine the plastic moment ratio for the rafters, the beams are considered as fixed ended as shown in sketch *b*. The value k_1 is thus determined as 4·0. For this special condition, the minimum possible plastic moment values would be determined, the joints being fixed against rotation but the frame theoretically free to sway. The resulting ratio is therefore the basis for later analysis of the frame. For greatest economy the end columns should provide full restraint to the beams, and therefore the plastic moment values are made equal to the appropriate beam values. The value k_2 for the interior column may be determined by considering equilibrium of joint 6-7-8 (sketch *c*). A value of k_2 equal to 3 is obtained.

The structure is now analyzed for Case I loading. Actually the analysis was completed in the previous step but we will go through the various operations. There are 7 possible plastic hinges. The frame is redundant to the third degree ($X = 3$). Therefore, there are four possible independent mechanisms and these are shown in sketches *b*, *c* and *d*.

The solution for Mechanism 1 is made on the basis that Mechanisms 1 and 2 form simultaneously. Consequently M_p is determined as 20·8 m.t and k_1M_p is 83·2 m.t.

The moment check as shown in sketch *e* reveals that the moment is nowhere greater than M_p and thus the solution is correct for this loading condition. The computation of reactions at ultimate load completes the first analysis.

The analysis for Case II is then performed to see whether or not the plastic moment values determined will be adequate. The same plastic moment ratios, k_1 and k_2 will be used.

The solution for Mechanism 1 may be determined from Case I and is found to be 15·74 m.t. The solution for Mechanism 4 (sketch *c*) shows that M_p is so much less than the value determined for Mechanism 1 that no further consideration of this mechanism is necessary.

The solution for Mechanism 5, which is a combination of Mechanisms 1, 2, 3 and 4, shows a required M_p value of 16·5 m.t. Although it is less than the value of 20·8 m.t for Case I, it is close enough so that the moment check should be made. Incidentally, the solution for this mechanism assumed that hinges formed in mid span. Thus, x_1 in sketch *d* was made equal to $L_1/2$ and $x_2 = L$. The moment check gives a possible equilibrium configuration as shown in sketch *f* (page 165). The plastic moment condition will be violated near Sections 5 and 9. Even so, if the analysis were completed on the basis of a precise determination of the values x_1 and x_2, the required M_p-value would be less than the value determined for Case I. Therefore, no precise consideration of distributed load is necessary.

In further explanation of this moment check, we shall expect in the first place that the plastic moment condition will be violated in each of the rafters because of our initial assumption that plastic hinges formed at mid-span (locations 5 and 9). This is only the correct position when the end moments are equal. The moment check is completed by using the equilibrium equations, and these are shown in sketch *f*. Using the equation for beam 4-6 it is found that M_4 equals 13·5 m.t which is less than M_p, and similarly for span 8-10. Using the joint equilibrium equation for 6-7-8, it is found that the moment in the column top is also less than $k_2 M_p$.

Sufficient information is thus available for drawing the moment diagram, and it is plotted to scale in sketch *f*. As expected, the moment is greater than the plastic moment value near the centre of the two rafters. To the left of Section 5, $M = 16·63$ m.t as compared with $M_p = 16·5$ m.t. To the left of Section 9, $M = 66·4$ m.t as compared with $4M_p = 66·0$ m.t. We may, therefore, conclude that Mechanism 5 is the correct one, M_p being slightly larger than 16·5 m.t. Case I ($M_p = 20·8$ m.t) therefore controls the design.

If it had been desirable to analyze Mechanism 5 and determine the precise location of plastic hinges this could either be done graphically, by trial and error, or by maximizing the required M_p-value expressed in terms of the distances x_1 and x_2. The following equation in terms of x_1 and x_2 would be differentiated partially in respect to x_1 and with respect to x_2, would be set equal to 0, and the resulting two equations solved simultaneously for the x_1 and x_2 value:

$$M_p\theta\left(\frac{L}{L-x_1}+\frac{x_1}{L-x_1}\right)+k_1M_p\theta\left(\frac{2L}{2L-x_2}+\frac{x_2}{2L-x_2}\right)$$
$$=w_u\frac{L_1}{2}\theta x_1+w_u\frac{L_2}{2}\theta x_1+w_k\frac{h}{2}h\theta \qquad \ldots(10)$$

Since Case I (without wind) is the critical condition, the selection of required section will be made on the basis of the M_p values thus determined. In selecting the section for the right-hand beam and column a plastic modulus of 3 311·2 cm³ is required. The ISMB 600, supplies a plastic modulus of 3 510·6 cm³. Even if the moment capacity of the beam section is somewhat less than the required value, it is considered adequate, particularly since the centre column provides a restraining moment that is considerably greater than the required value of 2 183·4 cm³.

In checking Rule 1 for axial force in the members, it is found that the centre column has a P/P_y value of 0·156 which is greater than 0·15. Using the recommended formula (Eq 47) it is found that the original choice was satisfactory since the Z-value actually furnished is greater than the modification factor requires. No check of the right-hand column is necessary since the centre column is satisfactory, and the beams are adequate because the horizontal thrusts are less than the vertical ones.

All members are satisfactory as far as shear force is concerned.

In evaluating the cross-section proportions it is found that the sections have b/t ratio of less than 17 and therefore satisfactory. The ISLB 600 is found to have inadequate d/w ratio. The section is therefore revised to ISMB 550 which is checked and found satisfactory.

Concerning the problem of lateral bracing, the purlin spacing is selected as 2 m and the girt spacing as 1·67 m. The left rafter will be the most critical since it has the smallest r_y-value. A slenderness ratio of 63·1 will be adequate since the plastic hinge in the centre of the rafter will be the last to form. A preliminary check of the left-hand column ISLB 350 shows that a more refined examination is required. A consideration of the restraint coefficient improved the situation somewhat (compare the required slenderness ratio of 42·4 with the value of 52·7 that exists in the structure). The designer could either place an additional brace part way down the column or could check the hinge rotation at Section 4 to see if it was as severe as assumed in the theory.

In proportioning the diagonal stiffener for Connection 4 (ISLB 350), the member is so light that the initial choice will be based on a diagonal with thickness equal to that of the rolled section flange. In checking for local buckling of this element (Rule 3) even slightly lesser thickness is required. Therefore, a 12-mm plate is specified.

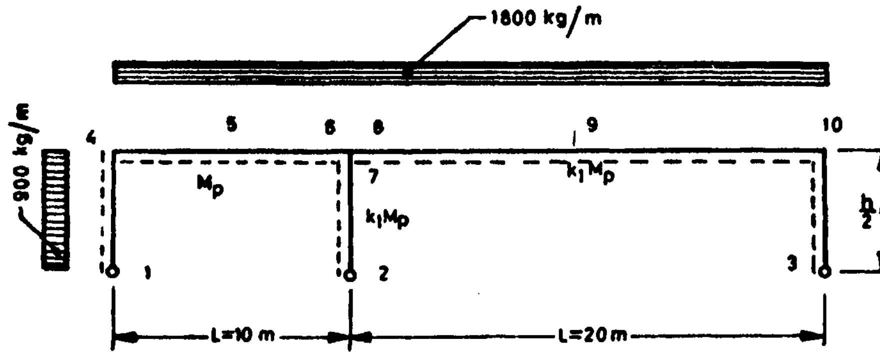
A similar situation arises for connection 10, except that the local buckling provision becomes more critical. In this case, instead of starting out with the flange thickness, the equation for t_s was used, resulting in a required value of 1·0 cm. The buckling provision requires a thickness of 1·23 cm and therefore a 1·4-mm plate is specified. Use of the flange thickness 2·08 cm would have been adequate, and the problem suggests that this rule-of-thumb guide is probably the best one to use in design where light members are involved.

With regard to interior connection 6-7-8, since the full moment capacity of the ISMB 600 member need not be transmitted into the vertical column, the existing web thickness may be adequate. Equation 56 may be employed as a check, using for S and d the values for member 2-7 (ISMB 550), the required thickness to be compared with that furnished by the ISMB 600 shape. This amounts to making sure that the joint will transmit the moment in the column top. The web is inadequate on this basis. Because of the position of the ISLB 350 with respect to the ISMB 600 (*see* Sketch *h*), the upper portion of the knee web *may* be adequate and the lower part inadequate. By considering equilibrium of forces on the top flange it may be shown that a web thickness of 1·0 cm would be adequate for the upper part. Good use of a diagonal stiffener (t_s= 15 mm) in the lower part may be made as shown and this is all that is necessary.

This completes the design.

DESIGN EXAMPLE 7 TWO-SPAN FRAME

Structure and Loading



(a)

Loading Conditions

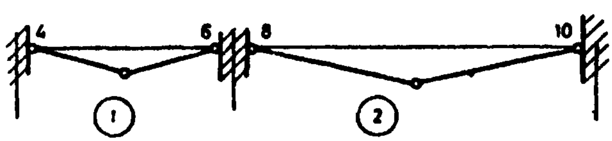
Treat problem with distributed load:

Case I — $(DL+LL)$;	$F=1{\cdot}85$	$w_u=(1{\cdot}8)(1{\cdot}85)=3{\cdot}33$ t/m
Case II — $(DL+LL+\text{Wind})$;	$F=1{\cdot}40$	$w_v=(1{\cdot}8)(1{\cdot}40)=2{\cdot}52$ t/m $=W_u$
		$w_h=(0{\cdot}9)(1{\cdot}40)=1{\cdot}26$ t/m $=W_{u/2}$

*Plastic Moment Ratios**

Consider beams as fixed-ended (sketch *b*), $M_p=\frac{W_uL_1^2}{16}$

$$\left.\begin{aligned} M_p\,(4\text{-}6) &= \frac{w_uL^2}{16}=M_p \\ M_p(8\text{-}10) &= \frac{w_u(2L)^2}{16}=k_1M_p \end{aligned}\right\} \quad \begin{aligned} k &= \frac{(2L)^2}{L^2} \\ &= 4 \end{aligned}$$



(b)

End columns provide full restraint to beams

$M_p(1\text{-}4)=M_p$, $M_p(3\text{-}10)=kM_p=4\ M_p$ $\qquad M_6=-M_p$, $M_8=-k_1M_p$

*Sign convention — $+M$ produces tension on side with dotted line.

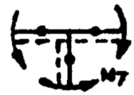
(Continue

DESIGN EXAMPLE 7 TWO-SPAN FRAME — *Contd*

Interior column to provide joint equilibrium sketch *c*

$M_6 - M_7 - M_8 = 0$

$M_7 = M_6 - M_8 = -M_p - (-k_1 M_p) = M_p(k_1 - 1) = 3M_p$ $(k_1 = 3)$



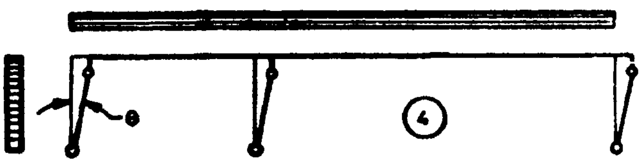
(c)

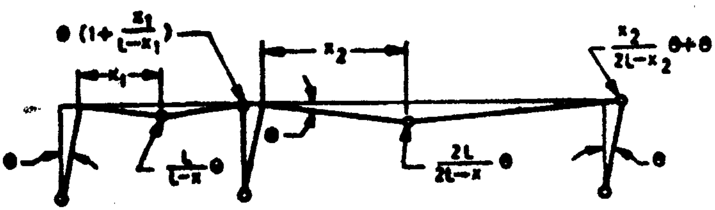
Case I — Analysis (Mechanism Method)

Possible Plastic Hinges — $N = 7$ (Sections 4, 5, 6, 7, 8, 9 and 10)

Possible Independent Mechanisms — $n = N - X = 4$

Mechanisms 1 and 2	Beam
Mechanism 3	Joint
Mechanism 4	Panel
Mechanism 5	Composite





(d)

Solution for Mechanism 1

$$M_p(\theta + 2\theta + \theta) = w\frac{L\theta}{2}\cdot\frac{L}{2}$$

$$M_p = \frac{w_u L^2}{16} = \frac{3{\cdot}33\,(10)^2}{16} = 20{\cdot}8 \text{ m.t (Member 4-6)}$$

$k_1 M_p = (4)(20{\cdot}8) = 83{\cdot}2$ m.t (Member 8-10)

(*Continued*)

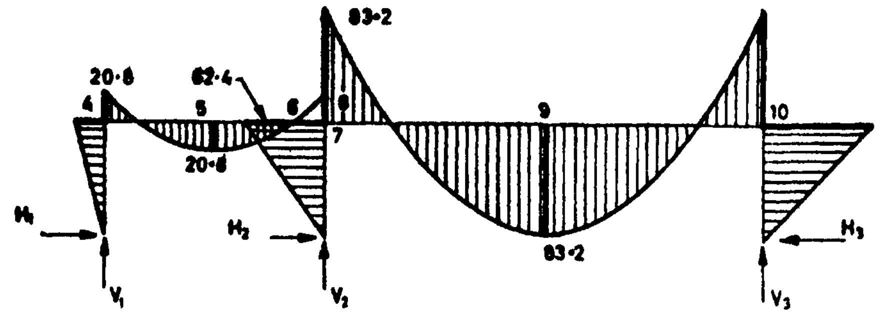
DESIGN EXAMPLE 7 TWO-SPAN FRAME — *Contd*

Moment Check — Sketch *e*

Joint equilibrium at 6-7-8 — $k_2 M_p = 62 \cdot 4$ m.t (Member 2-7)

All $M < M_p$. . . . . OK

$(M_p)_1 = M_p = 20 \cdot 8$ m.t



(e)

Reactions at Ultimate Load

$$H_1 = \frac{M_p}{h} = \frac{20 \cdot 8}{5} = 4 \cdot 16 \text{ t} \qquad V_1 = w_u L/2 = \frac{3 \cdot 33\ (10)}{2} = 16 \cdot 65 \text{ t}$$

$$H_2 = \frac{k_2 M_p}{h} = \frac{62 \cdot 4}{5} = 12 \cdot 5 \text{ t} \qquad V_2 = \frac{w_u L}{2} + \frac{w_u (2L)}{2} = 1 \cdot 5\ w_u L$$

$$= 1 \cdot 5 \times 3 \cdot 33 \times 10$$

$$= 49 \cdot 95 \text{ t}$$

$$H_3 = \frac{K_1 M_p}{h} = \frac{83 \cdot 2}{5} = 16 \cdot 7 \text{ t} \qquad V_3 = \frac{w_u (2L)}{2} = 33 \cdot 30 \text{ t}$$

Case II — Analysis

Hinges and mechanisms — See Case I

Solution for Mechanism 1 — $M_p = (20 \cdot 8) \dfrac{2 \cdot 52}{3 \cdot 33} = 15 \cdot 74$ m.t

Solution for Mechanism 4 — (Sketch *d*)

$$M_p \theta + (k_2 M_p)\theta + (k_1 M_p)\theta = w_h \theta\ h. \frac{h}{2}$$

$$M_p(1+3+4) = \frac{0 \cdot 5\ w_u}{2} \left(\frac{L^2}{4}\right) \frac{1 \cdot 26\ (100)}{8}$$

$M_p = 1 \cdot 97$ m.t $< M_p$ Case I

(*Continued*)

DESIGN EXAMPLE 7 TWO-SPAN FRAME — *Contd*

Solution for Mechanism 5 — (1+2+3+4): Sketch *d*

NOTE — Assume hinges form at mid span ($x_1 = L/2$, $x_2 = L$, $\theta_4 = \theta_6 = \theta_8 = \theta_{10} = 2\theta$)

$$M_p\theta[2+2+(2\times4)+(2\times4)] = w_u\left(\frac{L}{2}\,\theta\right)(L)\left(\frac{1}{2}\right)+W_u(\theta L)(2L)(1/2)+\frac{w_u h}{2}\left(\theta\frac{h}{2}\right)$$

$$20M_p = w_uL^2(1/4+1+1/16) = \frac{21}{16}(2\cdot52)(10)^2,\ M_p = 16\cdot5 \text{ m.t (Upper Bound Solution)}$$

∴ Check Moment

Moment Check

Beam (4-6) $-M_5 = \frac{M_4}{2}+\frac{M_6}{2}+\frac{w_uL^2}{8}$

$$M_4 = 2M_5 - M_6 - \frac{w_uL_1^2}{4} = 2M_p + M_p - \frac{w_uL^2}{4}$$

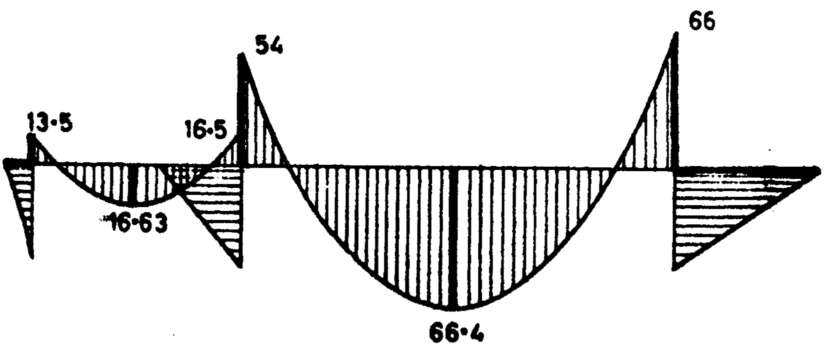
$$= 3(16\cdot5) - \frac{2\cdot52\ (100)}{4} = -13\cdot5 \text{ m.t} < 16\cdot5 \text{ ..... OK}$$

Beam (8-10) — $M_8 = 2M_9 - M_{10} - \frac{w_uL_2^2}{4} = (2)\ (4M_p) + 4M_p - \frac{w_u\,(2L)^2}{4}$

$$M_8 = 12(16\cdot5) - \frac{2\cdot52\ (400)}{4} = -54 < 66 \text{ m.t ..... OK}$$

Joint (6-7-8) — $M_7 = M_6 - M_8 = -16\cdot5 + 54 = 37\cdot5$ m.t $< 3M_p$
$= 112\cdot5$ m.t ...... OK

Case I (without wind) is critcal



(f)

Selection of Sections — Controlling moment diagram — Sketch *e*, $M_p = 20\cdot8$ m.t

Left beam }
Left column } $Z_{4-6} = \frac{M_p}{\sigma_y} = 20\cdot8\times39\cdot8 = 827\cdot8$ cm³

Use ISLB 350, Z = 831·1 cm

A = 63·01 cm²	w = 0·74
d = 350	s = 751·9
b = 6·5 mm	I = 13 158
t = 1·19	r_y = 13·17

(Continued)

DESIGN EXAMPLE 7 TWO-SPAN FRAME — *Contd*

Right beam
Right column } $Z_{8-10} = 4\ Z_{4-6} = 3\,311{\cdot}2\ cm^3$

Use ISMB 600 Z = 3 510·6 cm³

A = 156·21		w =	12 mm
d = 600 mm		s =	3 060·4 cm³
b = 210 mm		I =	91 813·0 cm⁴
t = 20·8 mm		r_y =	4·12 cm

Centre column $Z_{2-7} = 3Z_{4-6} = 2\,483{\cdot}4\ cm^3$

Use ISLB 600 Z = 2 798·6 cm³

A = 126·7 cm²		w =	10·5 mm
d = 600 cm		s =	2 428·9 cm³
b = 10 mm		I =	72 867·6 cm⁴
t = 210·5 mm		r_y =	3·79 cm

Axial Force (Rule 1)

Left column $\dfrac{P}{P_y} = \dfrac{V_1}{\sigma_y A_{1-4}} = \dfrac{16{\cdot}65}{2\,520(63{\cdot}01)} = 0{\cdot}105 < 15$ ..... OK

Interior column $\dfrac{P}{P_y} = \dfrac{V_2}{\sigma_y A_{2-7}} = \dfrac{49{\cdot}95}{(126{\cdot}7)(2\,520)} = 0{\cdot}156 > 0{\cdot}15$

(Modification Required)

$$Z_{Req} = Z_{Trial}\left(\frac{P}{P_y} + 0{\cdot}85\right)$$

$= 2\,483{\cdot}4(0{\cdot}156 + 0{\cdot}85)\ 2\,498\ cm^3 < 2\,798\ cm^3$ ...... OK

Shear Force (Rule 2)

Left beam $V_{max} = V_1 = 16{\cdot}65$
(ISLB 350)

$V_{allow.} = 1\,265\ wd = 32{\cdot}8 > 16{\cdot}65$ t ..... OK

Right beam $V_{max} V_2 = 33{\cdot}7$ t
(ISMB 600)

$V_{allow} = 1\,265\ wd = 91{\cdot}1 > 33{\cdot}7$ ..... OK

Cross-Section Proportions (Rule 3)

Shape	$b/t (<17)$	$d/w (<55)$
ISLB 350	14·5	47·30 ..... OK
ISLB 600	13·55	57·00 ..... Not good
ISMB 600	10·1	50·00 ..... OK

d/w for ISLB 600 is not adequate

∴ use ISMB 550

A	=	132·11 cm²
d	=	550 mm
b	=	190 mm
t	=	19·3 mm
w	=	11·2 mm
s	=	2 359·8 cm³
I	=	64 893·6 cm⁴

(*Continued*

DESIGN EXAMPLE 7 TWO-SPAN FRAME — *Contd*

$$r_y = 3{\cdot}73 \text{ cm}$$

$$r_x = \frac{P}{P_y} = \frac{V_8}{\sigma_A A_{8\text{-}7}} = \frac{49{\cdot}95 \times 1\,000}{2\,520 \times 132{\cdot}11} = 0{\cdot}150$$

$$= 0{\cdot}15 \ldots\ldots \text{OK}$$

$$h/t = 9{\cdot}8$$

$$d/w = 49{\cdot}1$$

(*Lateral Bracing*) (Rule 4)

Purlin spacing = 2·0 m, Girt spacing = 1·67 m

Spacing

Left Raftor (ISLB 350) $\frac{L_B}{r_y} = \frac{200}{3{\cdot}17} = 63{\cdot}1 \ldots\ldots$ OK

Loft column (ISLB 350) $\frac{L_B}{r_y} = \frac{167}{3{\cdot}17} = 5\,2{\cdot}7 \left(\frac{M}{M_p} = \frac{2/3 M_p}{M_p} = 0{\cdot}67\right)$

Eq 51 $= \left(\frac{l_B}{r_y}\right)_{cr} = 35 < 52{\cdot}7$ ∴ More refined check necessary.

Restraint coefficient

$$f = 1/2\left(\frac{L_R}{L_{R(cr)}} + \frac{L_L}{L_{L(cr)}}\right);\ L_{R(cr)} = r_y\left(60 - 40\,\frac{M}{M_p}\right)$$

$$= \left(60 + 40\,\frac{6{\cdot}64}{20{\cdot}8}\right) 3{\cdot}17 = 230{\cdot}7 \text{ cm}$$

$L_{L(LR)} = 30{\cdot}7;\ l/b = 20,$

$$1 = 20 \times 16{\cdot}5 = 335 \text{ cm}$$

$$\therefore f = 1/2\left(\frac{2{\cdot}0}{2{\cdot}31} + \frac{1{\cdot}67}{3{\cdot}35}\right) = 0{\cdot}685$$

$$\therefore C_f = 1{\cdot}21$$

$$\left(\frac{L_B}{r_y}\right)_{Cr} = C_f\,\frac{L_B}{r_y} = (1{\cdot}21)(35) = 42{\cdot}4 < 52{\cdot}7$$

NOTE — Small hinge rotation requirement would exist for this case. Therefore, assume it to be OK. Alternatively, provide additional brace.

Bracing Details

1) Provide vertical welded plates at centre purlins at both rafters.
2) At sections 4, 7 and 10 brace to inner (compression) corners.

Columns (Rule 5)(Rule 7)

Left column (ISLB 350) $\frac{P}{P_y} = 0{\cdot}105 < 0{\cdot}15$ Full M_p available

Right column (ISMB 600) $\frac{P}{P_y} = \frac{V_8}{\sigma_y A_{8\text{-}10}} = \frac{33{\cdot}3}{2\,520(132{\cdot}1)} < 0{\cdot}1 = 0{\cdot}15$

∴ Full M_p available

(Continued)

DESIGN EXAMPLE 7 TWO-SPAN FRAME — *Contd*

Interior column (ISMB 550) $\frac{P}{P_y} = 0{\cdot}15 = 0{\cdot}15$ **Simple check for axial force (Rule 1) is adequate.**

$L/P_x = \frac{500}{22{\cdot}16} = 22{\cdot}6$ ∴ original design is OK

Connection Details

(Use straight connections without haunches)

Connection 4 (ISLB 350)

Use diagonal stiffener equal to flange thickness $= 1{\cdot}14$ cm

for local buckling: $t \geqslant b/17 = \frac{16{\cdot}5}{17} = 0{\cdot}97$ cm;

See detail *h*, in sketch *h* of Design Example 4.

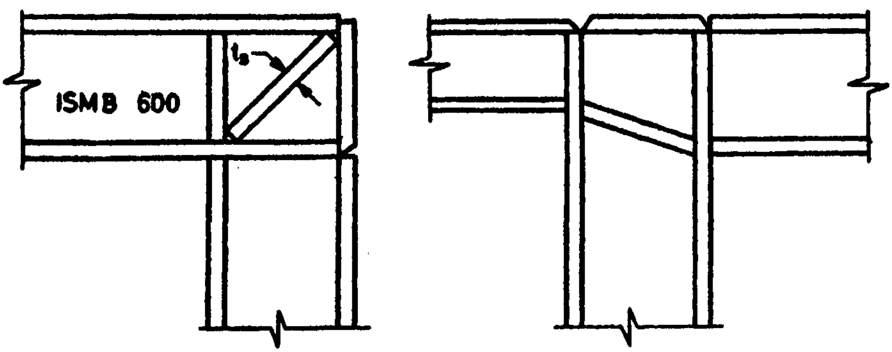
Connection 10 (ISMB 600) (Sketch *g*)

$$t_s = \frac{\sqrt{2}}{b}\left(\frac{s}{d} - \frac{w_t d}{\sqrt{3}}\right) = \frac{1{\cdot}45}{21{\cdot}0}\left(\frac{3\,060{\cdot}4}{60{\cdot}0} - \frac{1{\cdot}2 \times 60}{\sqrt{3}}\right)$$

$t_s = 0{\cdot}57$ cm

$t_s \geqslant \frac{b}{17} = \frac{21{\cdot}0}{17} = 1{\cdot}23$ cm

Use 100 × 14 mm plate



(g)

Interior connection 6-7-8 (Sketch *h*)

$$w_r = 0{\cdot}6\frac{\Delta M}{d_c d_b}$$

$$= 0{\cdot}6 \times \frac{62{\cdot}4 \times 100}{60 \times 60}$$

$$= 1{\cdot}04 \text{ cm}$$

$w_{6\text{-}7} = 1{\cdot}2 < 1{\cdot}6$ cm

Use partial diagonal stiffener (1·2 cm thick) as per sketch.

Splices — Provide as part of corner connection detail (interior column continuous).

26.5 Design Example 8 (Two-Span Frame with Gabled Roof and Fixed Column Bases)— The use of gabled roofs and the fixing of column bases adds sufficient additional complexity to a structure to justify further consideration of the mechanism method of analysis. Since most of the design rules have been illustrated in the previous problems, in this example attention will be focussed solely upon the over-all analysis and design of the structure. The examination of details is left as an exercise.

In actual design practice, problems of this type would undoubtedly be solved by the simplified procedures described in **31**. However, this example is given to illustrate basic principles. More complicated problems, for which charts are not available, could then be solved.

The methods presented here would also be helpful in the solution of problems involving three or more bays.

The two-span frame, symmetrical throughout is shown in sketch *a* of Sheet 1. The roof load, concentrated at the quarter points of the rafters, might be thought of as an approximation to a uniformly distributed load of 1·5 t/m. Similarly, the side load produces the same overturning moment about the base as that of a uniformly distributed horizontal load of 0·6 t/m acting on the vertical projection of the structure.

There are 18 possible plastic hinges, 6 redundants, and, therefore, there are 12 independent mechanisms as shown in sketch *b*. The mechanism solutions are worked in tabular form. The sketch of the various mechanisms in the table does not repeat the deformed shape, since the essence of these mechanism are shown in sketch *b*. The internal work is computed in column 3 of the table of mechanism analysis, and to facilitate checking, the work done at each hinge is listed in the same sequence as the numbering given in sketch *a*. Column 4 contains the computation of external work, and M_p in terms of P_uL is given in Col 5.

Independent Mechanisms 1 to 10 are shown first. Possible combinations follow, and these are made in such a way as to eliminate plastic hinges that appear in the independent mechanism so combined. Only in this way can the ratio M_p/P_uL be increased. Mechanism 13 is formed by combining Mechanisms 9 and 10. Hinges will be eliminated at Sections 1 and 4 only if $\theta_9 = 2\theta_{10}$. Mechanism 13 is sketched accordingly. The required plastic moment is less than for Mechanism 10 alone.

Mechanism 13(a) is the same as Mechanism 13 except that the solution is obtained by a summation of work equations for the independent mechanisms as described in **18**. The combination eliminates mechanism angles of 2θ at Sections 1 and 4 of Mechanism 10 and of 2θ at the same sections of Mechanism 9 ('Cancel $8M_p$'). The same answer is thus obtained as by the first method.

A moment check was made, next, to see if Mechanism 10 was the correct failure mode. It was found to be incorrect because $M = 1{\cdot}5M_p$ at Section 8. This suggests a combination of Mechanisms 10, 11, 2 and 4, and the resulting Mechanism 15 does, in fact, give the correct answer as shown in sketch *d*. The required value of M_p for the Case I loading is 57 m.t.

For Case II loading, Mechanisms 9 and 16 are investigated. The latter is a combination of Mechanisms 9, 10, 11, 5 and 7. The moment check for this case is shown in sketch *e*. Since the frame is determinate at failure $[I = X-(M-1) = 6-(7-1) = 0]$, a possible equilibrium moment diagram may be obtained directly. It is obtained by plotting the known M_p-values (sections 2, 3, 4, 6, 10, 14 and 18) and solving first for the moment at location 7 $(M_7 = +0{\cdot}13M_p)$. Since $M_{10} = -M_p$, the moment at location 8 equals $+M_p$, so the moment diagram may be completed for rafter 4-7-10. Using the 'trial and error' method, it is assumed that $M_{11} = 0$. Hence $M_{12} = M_p$ and the moment diagram for the right hand span would be identical to the left hand span. Making use of the sway equilibrium equation, the moment at Section 1 is $+0{\cdot}56\ M_p$. Therefore M is less than or equal to M_p throughout and the value $M_p = 0{\cdot}139PL$ is correct for the Case II loading.

Since $(M_p)_{II} < (M_p)_I$, the Case I loading controls the design and $M_p = 57$ m.t. An ISWB 500 supplies the needed plastic modulus. To complete an actual problem the appropriate design 'rules' would have to be checked.

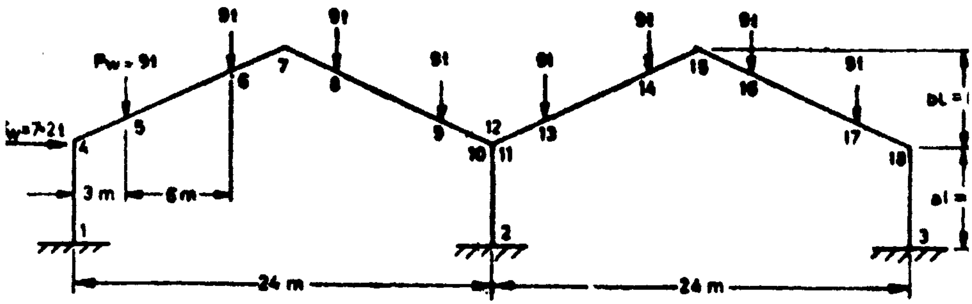
DESIGN EXAMPLE 8 TWO-SPAN GABLED FRAME

Structural and Loading

$P = 9$ t
$T = 7{\cdot}2$ t

Column bases fixed

$T_w = 7{\cdot}2$ t



(a)

Loading Conditions

Case I Load $(DL+LL)$
$F = 1{\cdot}85$

P_u (9) (1·85) = 16·65 t
T_u —

Case II $(DL+LL+$ Wind$)$
$F = 1{\cdot}40$

(9) (1·40) = 12·6 t
(7·2) (1·40) = 10·08 = 0·8 P

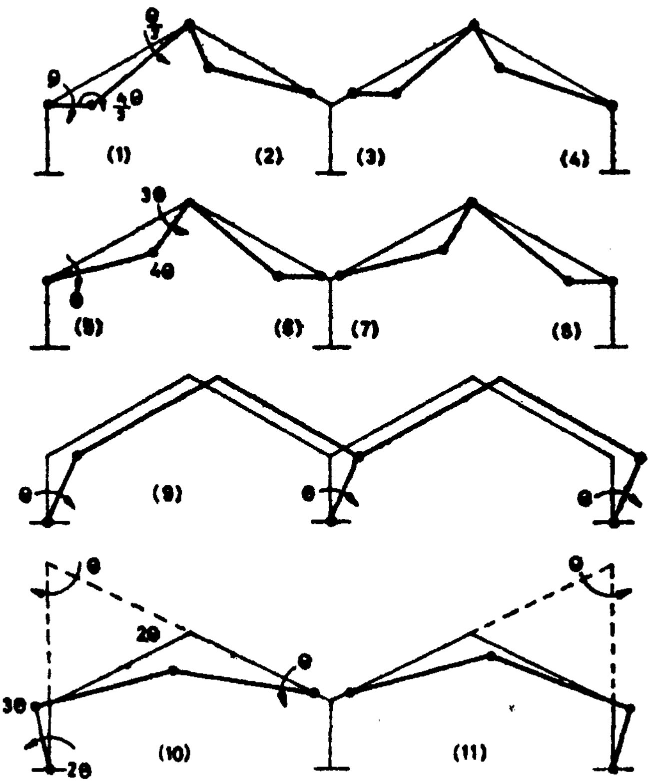
Plastic Moment Ratio — Try constant section throughout
Independent Mechanisms
Possible Plastic Hinges, $N = 18$ (Numbered section in sketch *a*)
Redundant, $X = 6$ (Remove support at Sections 2 and 3)

Number of Independent Mechanism, $n = N - X = 12$

Mechanisms 1-4:	Beam mechanisms
Mechanisms 5-8:	Beam mechanisms
Mechanism 9:	Panel mechanism
Mechanisms 10, 11:	Gable mechanisms
Mechanism 12:	Joint mechanism

(Continued)

DESIGN EXAMPLE 8 TWO-SPAN GABLED FRAME — *Contd*



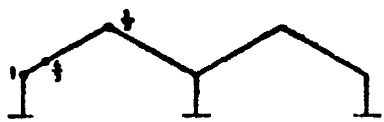
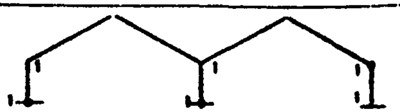
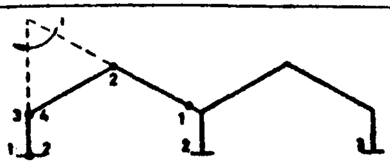
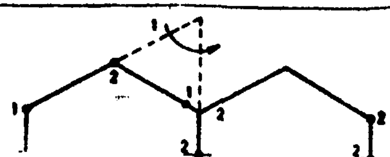
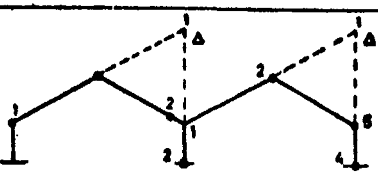
(b)

(Continu

DESIGN EXAMPLE 8 TWO-SPAN GABLED FRAME — *Contd*

Case I — Solution

Mechanism Analysis

No.	Mechanism	Internal Work ($WI/M_p\theta$)	External Work ($WE/PL\theta$)	M_p/P_uL
(1)	(2)	(3)	(4)	(5)
1-4		$1+\frac{4}{3}+\frac{1}{3}=\frac{8}{3}$	$\frac{1}{8}+\left(\frac{1}{8}\right)\left(\frac{1}{3}\right)$ $=\frac{1}{6}$	$\frac{1}{16}$
5-8	Similar (*see* sketch *b*)	$1+4+3=8$	$\frac{1}{8}+\frac{3}{8}=\frac{1}{2}$	$\frac{1}{16}$
9		$1+1+1+1$ $+1+1=6$	0	0
10 11		$2+3+2+1$ $=8$	$\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}$ $=1$	$\frac{1}{8}$
13 (9+10)		$2+2+1+2$ $+1+2+2$ $=12$	$\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}$ $=1$	$\frac{1}{12}$
13a	Solution by summation mechanism solution	Two times Mechanism 9 12 Mechanism 10 8 Cancelled −8 Total 12	0 1 — 1	$\frac{1}{12}$
4 (11 −12 +13)		$2+4+1+2+2$ $+1+2+5=19$	$\frac{1}{8}(1+3$ $+3+1)(2)$	$\frac{2}{19}$

(Continued)

DESIGN EXAMPLE 8 TWO-SPAN GABLED FRAME - - *Contd*

No.	Mechanism	Internal Work ($WI/M_p\theta$)	External Work ($WE/PL\theta$)		M_p/P_uL
(1)	(2)	(3)	(4)		(5)
14a	Solution by summation	Mechanism 11	8	1	
		Mechanism 13	12	1	
		Mechanism 12	3	0	$\frac{2}{19}$
		Cancelled (Sections 11, 12)	−4	2	
		Total	19		

Moment Check for

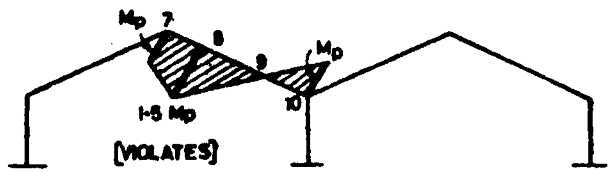
$$M_p = \frac{PL}{8} \text{ (Mechanisms 10 and 11)}$$

Beam 7-10

$$M_8 = \frac{3}{4}\,M_7 - \frac{M_{10}}{4} + \frac{PL}{8}$$

$$= +\frac{3}{4}\,M_p - \frac{M_p}{4} + M_p$$

$$= 1{\cdot}5\ M_p \text{ (Violates)}$$

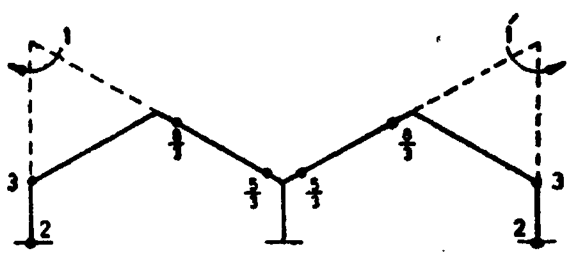


(c)

Mechanism Analysis (Additional)

15 (2+7 +10+11)	$2+3+\frac{8}{3}+\frac{5}{3}=\frac{28}{3}$	$\frac{1}{8}+\frac{3}{8}+\frac{5}{8}+\left(\frac{1}{8}\right)\left(\frac{5}{3}\right)=\frac{4}{3}$	$\frac{1}{7}$

(Due to symmetry only one-half of frame is solved)



(Continued)

DESIGN EXAMPLE 8 TWO-SPAN GABLED FRAME — *Contd*

Moment check for Mechanism 15 — $\left(M_p = \frac{1}{7} P_u L\right)$

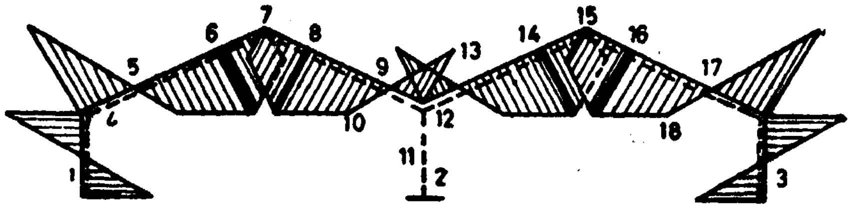
Beam 7-10

$$M_8 = \frac{3}{4}M_7 + \frac{1}{4}M_{10} + \frac{PL}{8}$$

$$M_7 = \frac{4}{3}M_8 - \frac{1}{3}M_{10} + \frac{PL}{6}$$

$$= \frac{4}{3}M_p + \frac{1}{3}M_p - \frac{7}{6}M_p$$

$$M_7 = +\frac{M_p}{2}$$



(d)

Beam 4-7

$$M_6 = \frac{1}{4}M_4 + \frac{3}{4}M_7 + \frac{PL}{8}(1)$$

$$= -\frac{1}{4}M_p + \frac{3}{8}M_p + \frac{7}{8}M_p,\ M_6 = +M_p$$

All $M \leq M_p$

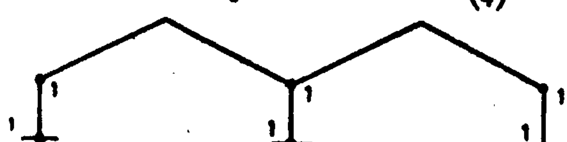
$$M_p = \frac{P_u L}{7} = \frac{16{\cdot}65 \times 24}{7}$$

$= 57$ m.t

Case II — Solution

Mechanism solutions

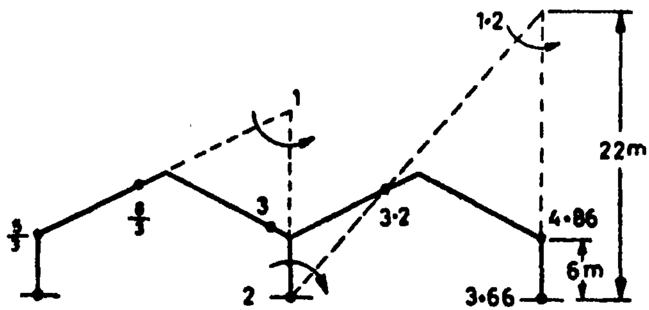
No.	Mechanism	Internal Work ($WI/M_p\theta$)	External Work ($WE/PL\theta$)	M_p/P_uL
(1)	(2)	(3)	(4)	(5)
9(II)		1+1+1+1+1+1 = 6	$(0{\cdot}8)\left(\frac{1}{4}\right)(1) = 0{\cdot}2$	$\frac{1}{30}$



(*Continued*)

DESIGN EXAMPLE 8 TWO-SPAN GABLED FRAME — *Contd*

No.	Mechanism	Internal Work ($WI/M_p\theta$)	External Work ($WE/PL\theta$)	M_p/P_uL
(1)	(2)	(3)	(4)	(5)
16 (9+10 +11+12 +5+7)		$2+3.66+\frac{5}{3}+\frac{8}{3}$ $+3+3.2+4.86$ $=21.06$	$\left(\frac{1}{8}\right)\left(\frac{5}{3}\right)+\left(\frac{3}{8}\right)\left(\frac{5}{3}\right)$ $+\frac{3}{8}+\frac{1}{8}+\left(\frac{1}{8}\right)(2)$ $\left(\frac{3}{8}\right)(2)+\left(\frac{3}{8}\right)(1.2)$ $+\left(\frac{1}{8}\right)(1.2)=2.93$	0.139



Moment check for Mechanism 16 — $(M_p)_{II} = 0.139\ P_uL$

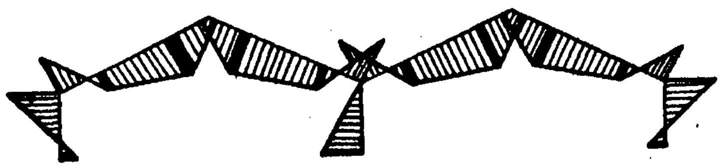
Beam 4-7

$$M_6 = \frac{M_4}{4}+\frac{3M_7}{4}+\frac{PL}{8}$$

$$M_7 = \frac{M_4}{3}+\frac{4}{3}\ M_6-\frac{PL}{6}$$

$$= +\frac{M_p}{3}+\frac{4}{3}\ M_p-\frac{M_p}{(6)(0.139)}$$

$$M_7 = +0.13\ M_p < M_p \ldots\ldots \text{OK}$$



(e)

Sway

$$T_a L + M_1 - M_2 - M_3 - M_4 + M_{11} + M_{12} = 0$$
$$T_a L + M_1 - M_p - M_p + M_p + 0 - M_p = 0$$
$$M_1 = -Tal + 2M_p = \frac{-(0{\cdot}8)\ M_p(20)}{(0{\cdot}139)(80)} + 2M_p = 0{\cdot}56M_p \ldots\ldots \text{OK}$$
$$(M_p)_{II} = 0{\cdot}139\ P_u L = 0{\cdot}139\ (12{\cdot}6)(24) = 42\ \text{m.t} < 57\ \text{m.t} \quad \textit{Case I controls}$$

Reaction for Case I

$$H_2 \times 6 = 2M_p$$
$$H_2 = \frac{M_p}{3} = \frac{57}{3}$$
$$= 19\ \text{t}$$
$$V_1 = V_3 = 2\ \text{P} = 18\ \text{t}$$
$$V_2 = 2V_1 = 4\ \text{P} = 36\ \text{t}$$

Selection of Sections

$$Z = \frac{M_p}{\sigma y} = 57 \times \frac{100\ 000}{2\ 520}$$
$$= 2\ 260\ \text{cm}^3$$

Use ISWB 500
$Z = 2\,351{\cdot}36\ \text{cm}^3$

Aerial Force

$$\text{Central column} = \frac{P}{Py} = \frac{V_2}{\sigma y A}$$
$$= \frac{36\ 000}{2\ 520 \times 121{\cdot}2} = 0{\cdot}12 < 0{\cdot}15 \ldots\ldots \text{OK}$$

27. DESIGN EXAMPLE ON MULTI-STOREY STRUCTURES

27.0 So far, our attention has been restricted to one-storey structures consisting of rectangular and gabled portal frames and to the multi-span frames that are typical of the industrial-type buildings to which plastic design can now be applied. A fair quantity of steel, however, goes into the construction of multi-storey structures, both of the tier building type and the more frequent two- or three-storey structure.

What is the reason that our attention has been restricted to the single-storey building ? First of all, a considerable tonnage of steel goes into such structures and, therefore, it is most advantageous to document the necessary provisions which will enable the engineer to apply plastic design to the industrial building. Secondly, and perhaps the most important, as the number of stories increases, the columns become more and more highly loaded. As already mentioned, the moment capacity of columns with relatively high axial load drops rapidly. The related problems are not completely solved and more research is needed before a 'complete' plastic design can be applied to all classes of tier buildings. As will be noted below, however, the outlook is heartening for at least a limited application of plastic analysis, and as Walter Weiskopf, consulting engineer, has remarked: 'it seems natural, therefore, to take the next step, that is to apply plastic design to this large class of buildings'.

In a stimulating and thought-provoking article[35], Weiskopf has discussed the application of plastic design to multi-storey structures, emphasizing the tier building type. He has suggested that the application of plastic design to such structures depends on the relative importance of horizontal forces. If horizontal forces are not a consideration (they may be so small that an ordinary masonry wall panel would carry any such small forces that might exist), then the regular connections are free of moment due to side sway and a very large savings in steel is possible when comparing a plastic design of the beams to a conventional simple beam design. In fact, for uniformly distributed load the savings theoretically could be 50 percent if it were not for other factors such as the cost of connections, etc, that tend to cancel out the potential savings due to economy in main material. When compared with rigidly connected elastic design there will, of course, be a savings in a plastic design.

The approach to design will be largely influenced by what is done about bracing against horizontal forces. Three situations may arise, as follows:

a) No horizontal load should be resisted (any minor loads taken by wall panels),
b) Horizontal forces carried by moment connections, and
c) Those cases in which the horizontal forces are carried by cross-bracing around elevator shafts or elsewhere in walls.

The application of plastic design to Cases (a) and (c) above will simply consist of a plastic analysis of continuous beams. For the second conditions (horizontal forces resisted by moment connections), the area of possible application of plastic design is dependent to the greatest extent on further research because plastic hinges might form in the columns, and as already mentioned, more needs to be known about the performance of columns under high axial load and as part of a framework.

It is for the third case in which the horizontal forces are carried by cross-bracing that a plastic design approach seems possible. When provision is made for wind bracing in wall panels, the beam and girders would be proportioned for full (plastic) continuity. The columns, on the other hand, would be proportioned according to conventional (elastic) methods. By this procedure, none of the plastic hinges participate in the resistance to side load. Such load is all carried by the diagonal bracing. The only mechanisms are the beam mechanisms.

The top one or two stories might be designed by a 'complete' plastic analysis, hinges forming both in the columns and in the beams. In those cases the vertical load in the columns would be relatively low and would be governed by considerations already described for the previous examples.

As far as the tall building is concerned, the column problem actually may not be as severe as first intimated. The most critical loading condition on a column is one which subjects it to equal end moments producing single curvature; the maximum moment then occurs at the midheight of the member. On the other hand, in tall buildings the columns will usually be bent in double curvature with a point of inflection (zero moment) near the middle of the member. The critical sections in that case are at the ends. Such columns are better able to develop plastic hinges than columns loaded in single curvature.

The problem of the connection for tier buildings also relates to the ability of these components to form plastic hinges. In riveted work it is very difficult to design a connection of strength equal to that of the beams unless large brackets are used. Therefore, if riveting were to be used to achieve continuity at connections in a plastically designed structure and without the use of these large brackets, further studies would be needed. The use of high-strength bolts offers another method of achieving continuity.

As has already been emphasized, maximum continuity with minimum added connection material can often be achieved by the use of welding. Numerous design recommendations have been made in Section E that are directly applicable to multi-storey buildings.

Naturally, no sharp dividing line exists between the form of structure that, on the one hand, may be designed by the plastic method and, on the other hand may not. An example will now be given of the plastic design of a two-storey building in which cross-bracing is not used, but any possible side sway is resisted by moment connections. After the selection of member sizes, the design of some of the connections also will be examined.

27.1 Design Example 9 (Two-Storey Building)— Using the mechanism method of analysis, a two-storey, two-span building frame is designed to support the loads shown in sketch *a*. A study of all possible loading conditions shows that Case I (dead load *plus* live gravity load) is critical. Therefore only this condition will be illustrated, the procedures for investigation of the other cases being identical.

The loads are uniformly distributed, but, for the sake of analysis are replaced by concentrated loads acting at the quarter points. The side loads, T, produce equivalent overturning moment about the bottom of the columns of the particular storey. Alternatively, it would have been possible to assume that plastic hinges formed in the centre of each beam span, to treat the load as uniformly distributed, and to revise the design (upward) to suit the precise plastic moment requirement.

Assuming that vertical load alone will control the design, the plastic moment ratio of the different members are selected such that simultaneous failure of beams A, B, C, and D will occur. If Span B has a plastic moment value of M_p, it is found that for Span A the plastic moment value should be 1·78 M_p; for Span C, 2·37 M_p; and for Span D, 1·33 M_p.

The fourteen independent mechanisms are shown in sketch *b* except that only two of the eight possible beam mechanisms are shown; the rest would be similar. The solutions for the various mechanisms are worked in tabular form. All beam mechanisms give the same answer — a check on the accuracy of the selection of M_p-ratios. The sequence of terms in the work equations follows the numbering sequence of sketch *a*.

Although for a frame of this type, one could be reasonably sure that the correct answer had been obtained already, Mechanism 15 is also investigated. It is found to require a smaller plastic modulus and therefore the critical case selected for the moment check is the simultaneous formation of the beam mechanisms.

In making the moment check, the diagram for the beams may be drawn without difficulty and this is shown in sketch *c*. Quite evidently the plastic moment condition is not violated in any beam.

A possible equilibrium moment diagram for the columns is shown in sketch *d*. Since the frame is still indeterminate at failure, it is not 'exact'; but it shows that the plastic moment condition is not violated. The method used is the 'trial and error' one. M_{18}, M_{22}, and M_{26} are first obtained by the joint equilibrium equations. The left and right columns are selected as having a moment strength equal to that of the beams which they restrain, and the same moment capacity is assumed for the full column height at this stage. In order to obtain an idea of the magnitude of moment at Section 16, since the horizontal reaction at 3 would act to the left, it is assumed that M_{17} is at the full plastic value 25·35 m.t in the direction indicated. The joint equilibrium equation gives the magnitude of M_{16}. M_{10} is assumed equal to $-M_{22}$; this is a completely arbitrary assumption, but since there is no side load, any small value would be reasonable at this stage. M_5 is then obtained by the panel (sway) equilibrium equation and is found to be 28·13 m.t. Since all moments are less than kM_p, the upper storey is satisfactory thus far.

The moments at Sections 4 and 11 may now be determined by joint equilibrium. Subsequently, the sway equation is used to check the bottom storey, a calculation that is made on the basis that the moments are zero at the column bases. Equilibrium is satisfied, and, therefore, the moment check is complete.

M_p for the Case I loading is thus equal to 25·35 m.t. Cases II and III loading are not shown here, but are found to require a smaller plastic modulus. Therefore, sections would be selected on the basis of the Case I solution, care being taken to modify the sections used for the columns to account for the influence of axial force.

A result of the moment check given above and shown in sketches *c* and *d* is that the fixed bases are not required for this problem. Pinned bases would have been just as satisfactory and would not have resulted in an increase in member sizes.

DESIGN EXAMPLE 9 TWO-STOREY, TWO-SPAN BUILDING

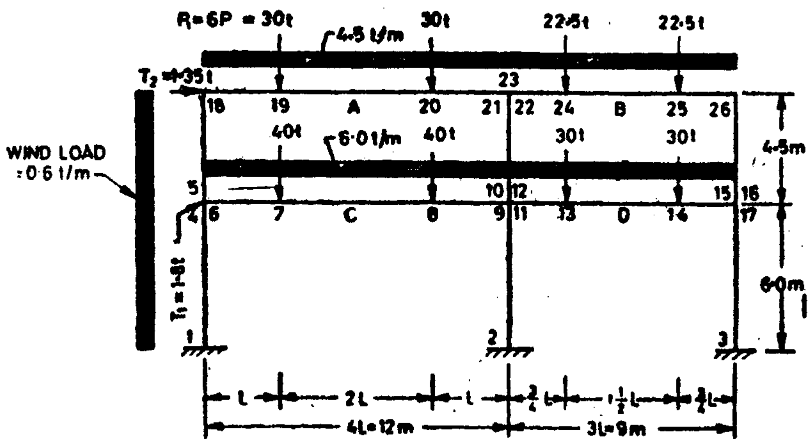
Structure Loading

Roof load $= 4{\cdot}5$ t/m
Floor load $= 6{\cdot}0$ t/m
Wind load $= 0{\cdot}6$ t/m

Replace distributed load by concentrated loads at quarter points.
Replace horizontal load by concentrated loads at roof and floor lives.

$$T_1 = \frac{0{\cdot}6 \times 6 \times 3}{6} = 1{\cdot}8 \text{ t}$$

$$T_2 = \frac{0{\cdot}6 \times 4{\cdot}5 \times 2{\cdot}25}{4{\cdot}5} = 1{\cdot}35 \text{ t}$$



(a)

Loading Conditions

Case I — $DL+LL$	$F = 1{\cdot}85$	$P_u = 9{\cdot}25$ t
Case II — $DL+LL+$Wind from left	$F = 1{\cdot}40$	$P_u = 7{\cdot}0$ t
Case III — $DL+LL+$Wind from right	$F = 1{\cdot}40$	$P_u = 7{\cdot}0$ t

Plastic Moment Ratios

For simultaneous failure of spans A and B under Case I loading use plastic moment in ratio of square of spans. For equal spans, the ratio is to vary as the load.

$K_B = 1{\cdot}00$ Left column: Use $k = K_A = 1{\cdot}78$

$$K_A = \left(\frac{12}{9}\right)^2 K_B = 1{\cdot}78$$

$$K_D = \left(\frac{18}{13{\cdot}5}\right) K_B = 1{\cdot}33$$ Centre column: Use $k = K_B = 1{\cdot}00$

$$K_C = \left(\frac{12}{9}\right)^2 K_D = 2{\cdot}37$$ Right column: Use $k = K_B = 1{\cdot}00$

(Continued)

DESIGN EXAMPLE 9 TWO-STOREY, TWO-SPAN BUILDING — *Contd*

Case I — Solution

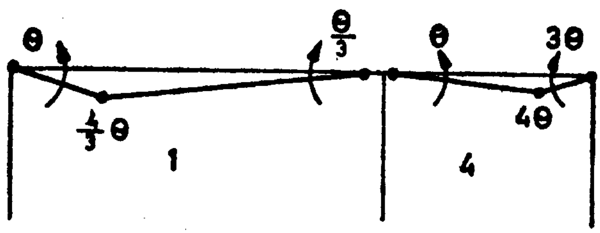
Mechanisms

Possible Plastic Hinges — $N = 26$ (At all neutral sections in Sketch *a* except 1 and 3
Redundant, $X = 9$ (Cut beams A and B, remove reaction at 2, remove fixity at 1 and supply roller at 3)

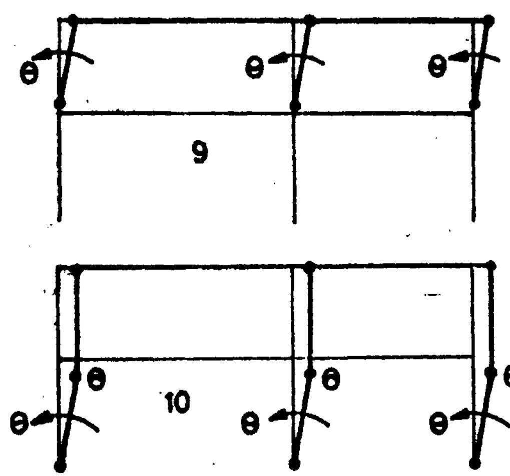
Independent Mechanisms — $n = N - X = 14$

Beam mechanism 1-8

Mechanisms 2, 3, 5-8 similar



Panel Mechanisms 9 and 10

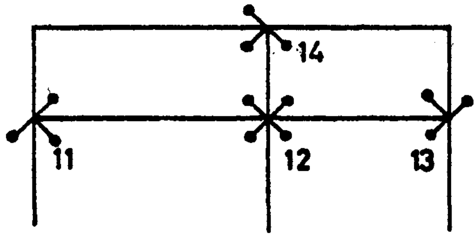


(b)

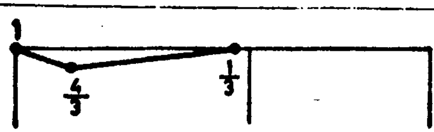
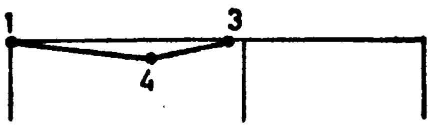
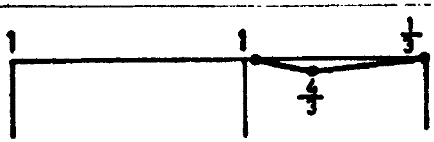
(Continued)

DESIGN EXAMPLE 9 TWO-STOREY, TWO-SPAN BUILDING — *Contd*

Joint Mechanism 11-12-13-14



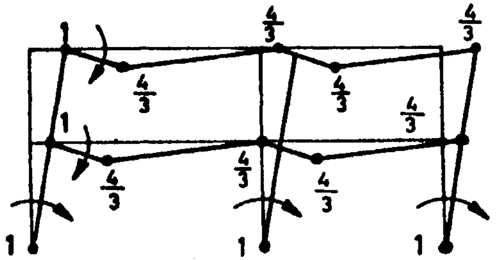
Mechanism Solutions

No.	Mechanism	Internal Work ($W_I/M_p\theta$)	External Work ($W_E/PL\theta$)	M_p/P_uL
(1)	(2)	(3)	(4)	(5)
1		$K_A\left(1+\frac{4}{3}+\frac{1}{3}\right)$ $=\frac{8}{3}(1\cdot78)=4\cdot74$	$6(1)(1)$ $+6\left(\frac{1}{3}\right)(1)=8$	1·69
2		NOTE— 2 is identical with 1.		
3,4		$K_B\left(1+\frac{4}{3}+\frac{1}{3}\right)$ $=\frac{8}{3}(1\cdot00)=2\cdot67$	$4\cdot5(1)\left(\frac{3}{4}\right)$ $+4\cdot5\left(\frac{1}{3}\right)\left(\frac{3}{4}\right)$ $=4\cdot5$	1·69
5-8	NOTE — Mechanisms and vertical work equations are similar.			1·69
15 (2+4 +6+8 +9+10 +11+12 +13+14)		$K_C+K_D+K_A+K_B$ $K_C\left(\frac{4}{3}+\frac{4}{3}\right)+K_D\left(\frac{4}{3}+\frac{4}{3}\right)$ $+K_A\left(\frac{4}{3}+\frac{4}{3}\right)$ $+K_B\left(\frac{4}{3}+\frac{4}{3}\right)$ $=\frac{11}{3}K_A+\frac{14}{3}K_B+\frac{8}{3}$ (K_C+K_D)	$8\left(1+\frac{1}{3}\right)+6\left(1+\frac{1}{3}\right)$ $\left(\frac{3}{4}\right)+6\left(1+\frac{1}{3}\right)+4\cdot5$ $-\left(1+\frac{1}{3}\right)\left(\frac{3}{4}\right)$ $=\frac{32}{3}+6+8+4\cdot5$ $=29\cdot17$	1·38

(Continued)

DESIGN EXAMPLE 9 TWO-STOREY, TWO-SPAN BUILDING — *Contd*

$$=\frac{11}{3}(1\cdot78)+\frac{14}{3}(1\cdot00)$$
$$+\frac{8}{3}(2\cdot37+1\cdot33)$$
$$=21\cdot07$$

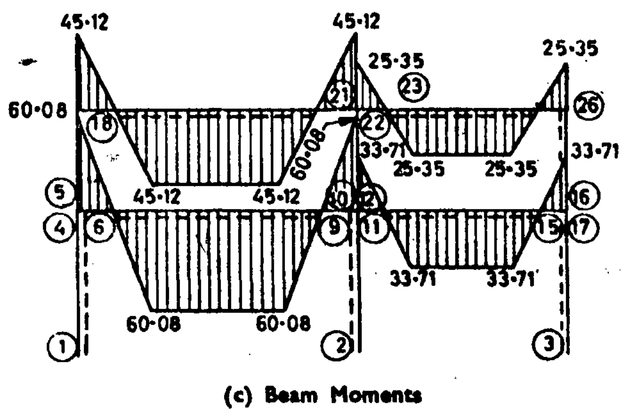


Moment Check for Beam Mechanisms (*See* Sketch *c*)

$M_P = 1\cdot69\ P_uL = 1\cdot69(5)(3) = 25\cdot35$ m.t

$M_{PA} = 1\cdot78\ M_p = 45\cdot1$ m.t
$M_{PB} = 1\cdot0\ M_p = 25\cdot35$ m.t
$M_{PC} = 2\cdot37\ M_p = 60\cdot08$ m.t
$M_{PD} = 1\cdot33\ M_p = 33\cdot71$ m.t

Joint 21-23: $M_{22} = -M_{21} - M_{23}$
$M_{22} = -45\cdot12 + 25\cdot35 = 19\cdot77$ m.t
Assume $M_{17} = -M_p$



(c) Beam Moments

(Continued)

DESIGN EXAMPLE 9 TWO-STOREY, TWO-SPAN BUILDING — *Contd*

Joint 15-17: $M_{16} = -M_{15} + M_{17}$
$M_{16} = 33 \cdot 71 - 25 \cdot 35 = 8 \cdot 36$ m.t

Assume $M_{10} = -M_{22} = +19 \cdot 77$ m.t

Sway of top storey

$$M_5 + M_{10} + M_{16} + M_{18} + M_{22} + M_{26} = 0$$
$$M_5 = 19 \cdot 77 + 8 \cdot 36 - 45 \cdot 12 + 19 \cdot 77 + 25 \cdot 35 = 28 \cdot 13 \text{ m.t}$$

Joint 4-6: $M_4 = M_5 + M_6 = +28 \cdot 13 - 60 \cdot 08 = -31 \cdot 95$ m.t

Sway of bottom storey

$$M_1 - M_2 - M_3 - M_4 + M_{11} + M_{17} = 0$$
$$0 + 0 + 0 + 31 \cdot 95 - 6 \cdot 6 - 25 \cdot 35 = 0$$

Allowable $M \leqslant kM_p$
$\therefore (M_p)_l = 25 \cdot 35$ **m.t**

Use the following sections as different members of the frame

Left column, $k = K_A = 1 \cdot 78$, $M_p = 25 \cdot 35 \times 1 \cdot 78 = 45 \cdot 12$ m.t

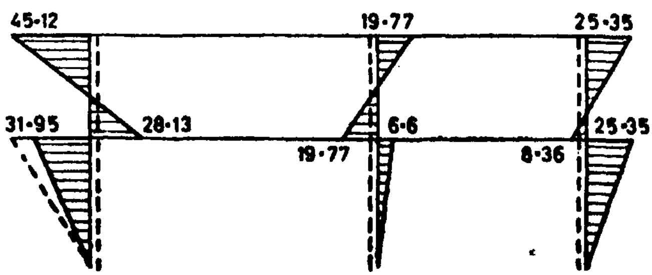
Use ISLB 550

Beam $C = K = K_c = 2 \cdot 37$, $M_p = 2 \cdot 37 \times 25 \cdot 35 = 60 \cdot 08$ m.t

Use ISLB 600

Similarly other sections may be decided.

These sections should be checked to see if they satisfy other secondary considerations.



(d) Column Moments

SECTION G

SIMPLIFIED PROCEDURES

28. INTRODUCTION

28.1 One of the advantages of plastic design is that the engineer is able to complete the analysis in less time than required by conventional (elastic) procedures. It is possible, however, to shorten the design time even further, by taking advantage of the same technique that is used in conventional design and one that is frequently used whenever a procedure becomes time-consuming. The solution of frequently-encountered standard cases may be given as a formula or in chart form.

Such an opportunity is open to the engineer interested in plastic design. In this section some techniques will be described and certain of them illustrated. The presentation is by no means a complete one. Indeed, the ingenuity of engineers will undoubtedly lead them to develop many other such design aids.

Two words of caution:

a) Since superposition does not hold in plastic analysis, generally it is not possible to combine two separate solutions as is done so commonly in elastic design. Any 'formula' or 'chart' can only assist in the solution of the particular loading and geometry for which it was developed.

b) Even though the formulas and charts are correct in themselves, it is a good rule to check the plastic moment condition by drawing the moment diagram. In this way one is assured of the correct answer.

The simplified procedures which apply to continuous beams are discussed in **29**. This will include a tabulation of solutions for various loading conditions. Formulas for the rapid determination of the required plastic moment for single-span frames with pinned bases are given in **30**. The use of charts for the same purpose is also described there. Finally, in **31** the solution of problems involving multi-span frames are discussed.

29. CONTINUOUS BEAMS

29.1 Although the analysis of continuous beams for maximum strength represents the simplest possible application of the plastic method, the engineer may wish to avail himself of tables and charts for the rapid solution of continuous beam problems.

In Fig. 66 (*see* Appendix C) are given 'beam diagrams and formulas' for certain loading conditions on beams. The table is patterned after similar tables contained in Ref 13. In addition to the reaction and M_p-values for these standard cases, the position of plastic hinges and points of inflection are indicated. Eventually values for deflection at ultimate load (δ_u) and deflection at working load could be added to such a table.

30. SINGLE-SPAN FRAMES (SINGLE STOREY)

30.1 Two approaches are possible in simplifying the procedure for the solution of single-span frames. The virtual work equations can be expressed as a formula which would reflect both the frame geometry and the loading conditions. Alternatively curves may be prepared which present the solution in chart form. At Lehigh University, Dr Robert L. Ketter has made an outstanding contribution[45] that enables the engineer to determine with the aid of charts the required plastic moment of a single-span frame in a fraction of the time required in a 'routine' plastic analysis. The method of derivation and some examples are contained in Ref 46. Ref 47 makes use of both the 'formula' and the 'chart' approach and in this aspect is based substantially on Ketter's work. It is cited here for reference (when available) for additional examples. A few illustrations will be given.

Restricting ourselves to single storey-structures of uniform plastic moment throughout, Fig. 67 (*see* Appendix C) shows a gabled frame with uniformly distributed vertical and horizontal loading. For simplicity the horizontal distributed load is replaced by a concentrated load, acting at the eaves, such that it produces the same overturning moment about the base at Location 1.

Since

$$M = \frac{w_h\,(a+b)^2L^2}{2}$$

then

$$T = \frac{w_h(a+b)^2L}{2a} \qquad \ldots(71)$$

In order to simplify the form of the solution a parameter C is introduced which is a function of the magnitude of the overturning moment. It is determined from:

$$M = C\,\frac{w_v L^2}{2}$$

and thus

$$C = \frac{2T_e}{w_u L} = \frac{w_h}{w_u}(a+b)^2 \qquad \ldots \qquad \ldots \qquad \ldots(72)$$

Consider, now, the mechanism shown in Fig. 68 (*see* Appendix C). (Of course there are other possible mechanisms but in most practical cases, this will be the one to form.) Using instantaneous centres, the rotations at each of the plastic hinges may be computed and then, by use of the mechanism method, the required plastic moment may be determined in terms of the variables w_u, L, Q, C and x.

The following equation results:

$$M_p = \frac{wL^2}{4}\left[\frac{\left(1-\frac{x}{L}\right)\left(C+\frac{x}{L}\right)}{\sqrt{(1+Q)(1-QC)}}\right] \qquad \ldots \qquad \ldots \qquad \ldots(73)$$

where x is given by:

$$X = \frac{L}{Q}\left[\sqrt{(1+Q)(1-QC)}-1\right] \quad (Q>0)$$

$$X = L\left(\frac{1-C}{2}\right) \qquad (Q=0) \qquad \ldots \qquad \ldots(74)$$

and is computed by the methods already discussed.

The only remaining problem is to determine the range of variables for which the mechanism shown in Fig. 68 (*see* Appendix C) is in fact the correct solution. Figure 69 (*see* Appendix C) summarizes the applicable formulas for the pinned base single-span, single-storey frame.

Similar solutions may be developed for other loading conditions and for fixed bases.

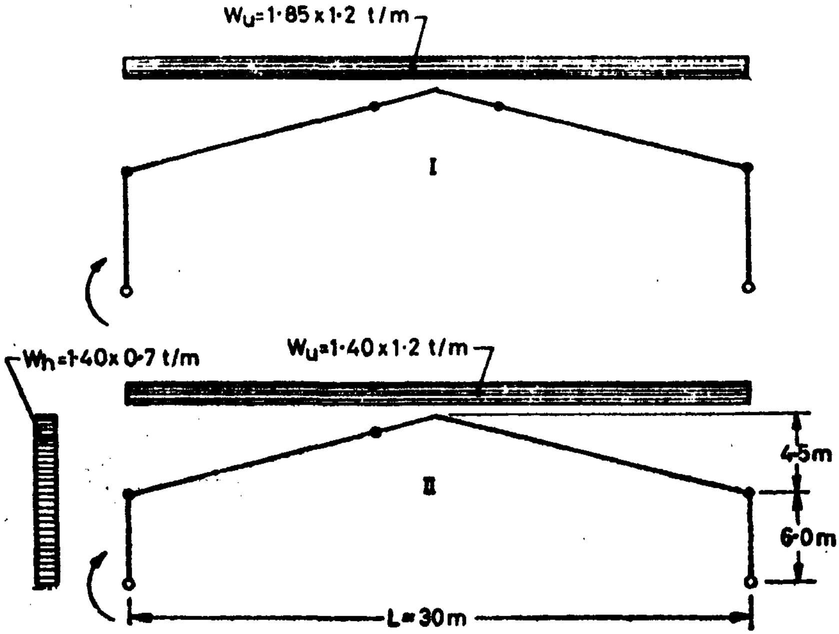
Ketter[46] has presented all possible solutions to the single-span, single-storey frame in the form of two charts — one which gives the value of M_p/wL^2 as influenced by C and Q, and one which gives the distance, x, to the plastic hinge in the rafter (also a function of C and Q). These two charts are indicated in Fig. 69 (*see* Appendix C) and for the major range of variables, they are simply representations of Eq 73 and 74. Their use will be indicated by the example which follows:

Example 8 — *Single-Span Rigid Frame Without Haunched Corners*

This example is the same as that of Design Example 6 except that no haunch is to be used. The two loading conditions are as shown at the top of Fig. 70 (*see* Appendix C). The distributed load acting horizontally on the frame produces an overturning moment from which C may be computed (Eq 72). The values of C are thus determined as zero for Case I and 0·074 4 for Case II. Knowing that $Q = a/b = 0{\cdot}75$, all the needed information is available for entering the chart of Fig. 69(a).

For Case I, M_p/wl^2 is equal to 0·046 and for Case II it is 0·055. To determine the critical or controlling case, it is sufficient to compare M_p/L^2 ratios since L is the same in both cases. On this basis, Case I is found to be critical. A ISWB 600 member is specified.

The moment check shows that the plasticity condition is not violated and thus the answer is correct. The secondary design conditions would next be checked.



Case I

$F(DL+LL) = 1{\cdot}85$

$w_u = (1{\cdot}85)(1{\cdot}50) = 2{\cdot}22$ t/m

$C = 0$

$Q = b/a = 0{\cdot}75$

Analysis from chart in Fig. 69(a)

$M_p/wL^2 = 0{\cdot}046$

$M_p/L^2 = (0{\cdot}046)(2{\cdot}22) = 0{\cdot}102$ t/m

Case II

$F(DL+LL+\text{Wind}) = 1{\cdot}40$

$w_u = (1{\cdot}40)(1{\cdot}2) = 1{\cdot}68$ t/m

$C = \dfrac{w_h(a+b)^2}{W_u} = 0{\cdot}073\ 5$

$M_p/wL^2 = 0{\cdot}055$

$M_p/L^2 = (0{\cdot}055)(1{\cdot}68) = 0{\cdot}092$ t/m

$M_p = 91{\cdot}8$ m.t

Case I (without wind) is critical

Selection of Section

Use ISWB 600

$\mathbf{Z = 3\ 986{\cdot}7\ cm^3}$

Moment Check

$$M_4 = \frac{wL^2}{8} - \frac{M_p}{aL}(10{\cdot}5) = \frac{2{\cdot}22 \times 30^2}{8} - \frac{91{\cdot}8}{6} \times 10{\cdot}5$$
$$= 89{\cdot}3 \text{ m.t} < M_p \qquad \ldots\ldots\ldots \text{OK}$$

31. MULTI-SPAN FRAMES

31.1 When multi-span single storey frames are considered, Ref 45 makes possible an even more dramatic savings of design time. Again, graphical representation of the equilibrium equations may be used to facilitate the solution of these problems.

Example 9

Consider the problem shown in Fig. 70(a) (*see* Appendix C) below, that of a two-span flat roof frame. It is the same structure, in fact, that was studied in Design Example 7. As the frame fails, the usual mode of failure will be that shown in sketch *b*. Now actually we can consider the behaviour of two separate structures as shown in sketch *c* without changing the total internal and external work. (The work done by the moments and forces as the two separate structures move through the virtual displacement becomes zero when 'continuity' is restored at the cut section.) The problem may be simplified still further by replacing all overturning forces and moments by imaginary moments acting about the column bases. The resulting separate structures which are equivalent to the original structure are shown in sketch *d*.

Charts may then be prepared for the general case shown in sketch *d* of Fig. 71 (*see* Appendix C) just as described before. Panel *A* is given a virtual displacement and the work equation is then written. It takes the following form[45]:

$$\frac{M_p}{wL^2} = \frac{1}{4}\left[\frac{\left(1-\frac{x}{L}\right)\left(\frac{x}{L}+C-D\right)-2DQ\,\frac{x}{L}}{1+Q\,\frac{x}{L}}\right] \qquad \ldots \qquad \ldots(75)$$

with $X = \frac{L}{Q}\left[\sqrt{1-Q\{C(1+Q)-D(1-Q)-1\}}\right] \qquad (Q>0)$

$$X = L\left(\frac{1-C+D}{2}\right) \qquad (Q=0) \qquad \ldots(76)$$

Whereas D was zero in the single-span problem (*see* 30), for the multi-span frame D becomes an additional parameter. Therefore it is necessary to prepare one set of charts for each value of Q for which a solution is desired. Figure 71 represents the solution for the flat roof frame in chart form. The left hand portion represents Eq 75 with $Q=0$. The right is the second form of Eq 76. Notice that the lower cut-off line on the chart is a beam mechanism in which $M_p/wL^2 = 1/16$ (Appendix C, C-5).

Now, to solve a problem we note from the loading [Fig. 70(d)] that $C_1 = w_h/w_u a^2$ and that $D_3 = 0$. The correct answer will be determined when the overturning moments at 2 are equated. Thus, $D_2 \frac{wL_1^2}{2} = G_2 \frac{wL_2^2}{2}$; so we will use the chart of Fig. 71(a) twice one for structure A and once for structure B and will eventually obtain an answer in terms of M_p/wL^2 for which the overturning moments at Section 2 will just cancel. The following example will help to explain this.

Example 10 — *Two-Span Flat Roof Frame*

The case for vertical load alone will result in beam mechanisms and need not be considered here further to illustrate the use of the charts. [The problem is the same as Design Example 7, for which Case I (without wind) was critical.]

In the first portion of the calculation, the known quantities are indicated. The value of C_1 is found to be 0·012 5. The value of D_3 is 0 since there is no external overturning moment applied to member 3-10. The only unknown values at this stage are D_2 and C_2, both of which may be found at the same time the value M_p/wL^2 is determined from the condition that $D_2 = 4C_2$.

Although it would not be possible to pick at the outset the value of M_p/wL^2 that satisfies this condition, by use of the chart of Fig. 71(a) in Appendix C one can determine *possible solutions* for each panel and find the *correct* answer graphically. A table is thus prepared with the aid of the chart. Panel A is first analyzed for $C_1 = 0·125$ and for various values of D. (Linear interpolation will be satisfactory if the range of $C-D$-values is small when compared with that of the chart; therefore two points will be sufficient, and $D_2 = 0$ and $D_2 = 0·10$ were selected.) The same thing is done for panel B except that now D_3 is known and C_2 is unknown. So, values of M_p/wL^2 are determined for two values of C_2 (0 and 0·50). The sketch c shows how this is done.

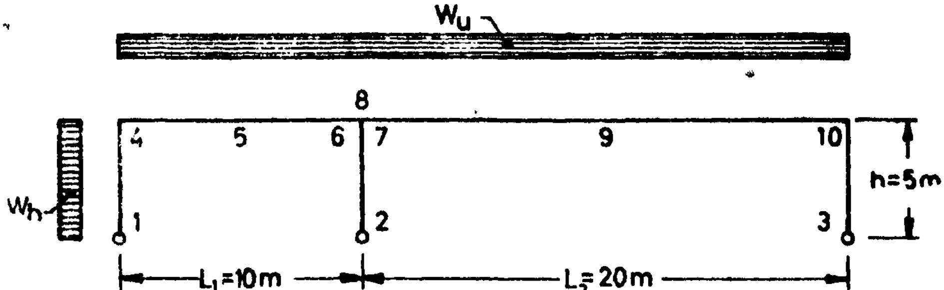
Now on a separate graph may be plotted the information contained in the table in the calculation sheet, it is shown in sketch d. Where the two curves intersect, $D_2 = 4C_2 = 0·102$, $M_p/wL^2 = 0·006\,52$, and the problem is solved. Note that the value of M_p for member 4-6 [M_p = (16·43 m.t)] agrees with the value determined for this same problem by direct use of the mechanism method. [*See* Design Example 7, Case II (M_p = 16·5 m.t).]

These charts and others were developed in Ref 45 covering both flat-roof and gabled frames.

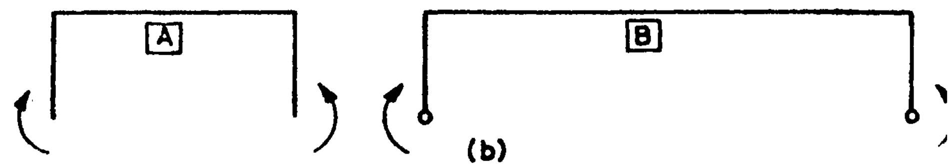
In all of these procedures, the final step in the analysis will be to draw the moment diagram with the aid of charts such as Fig. 71(b). Finally the secondary design considerations must be examined.

DESIGN EXAMPLE 10 TWO-SPAN FRAME

Case II — Vertical load plus wind (loads same as in Design Example 7)



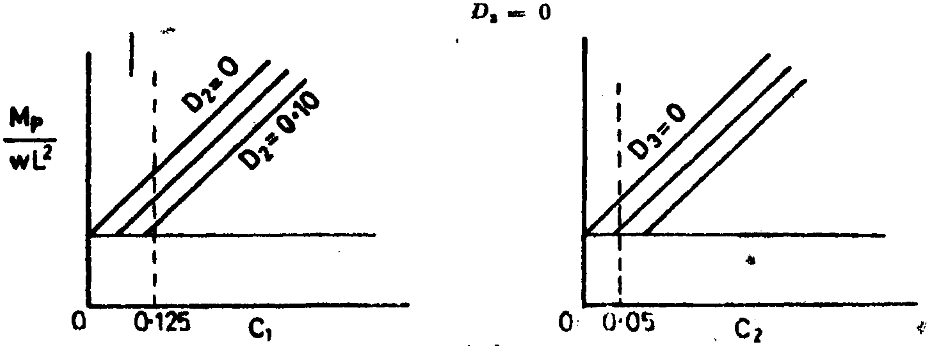
(a)



(b)

$C_1\frac{w_u L_1^2}{2}$ $D_2\frac{w_h L_1^2}{2}$ $C_2\frac{w_u L_2^2}{2}$ $D_3\frac{w_h L_2^2}{2}$

$$F\ (DL+LL+\text{Wind}) = 1{\cdot}40$$
$$w_u = (1{\cdot}40)(1{\cdot}8) = 2{\cdot}52 \text{ t/m}$$
$$w_h = (1{\cdot}40)(0{\cdot}9) = 1{\cdot}26 \text{ t/m}$$
$$Q = 0$$
$$C_1 = \frac{w_h}{w_u}\left(\frac{aL}{L_1}\right)^2 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = 0{\cdot}125$$
$$D_2 = C_2\left(\frac{L_2}{L_1}\right)^2 = 4C_2$$
$$D_3 = 0$$



(c)

(Continued)

DESIGN EXAMPLE 10 TWO-SPAN FRAME — *Contd*

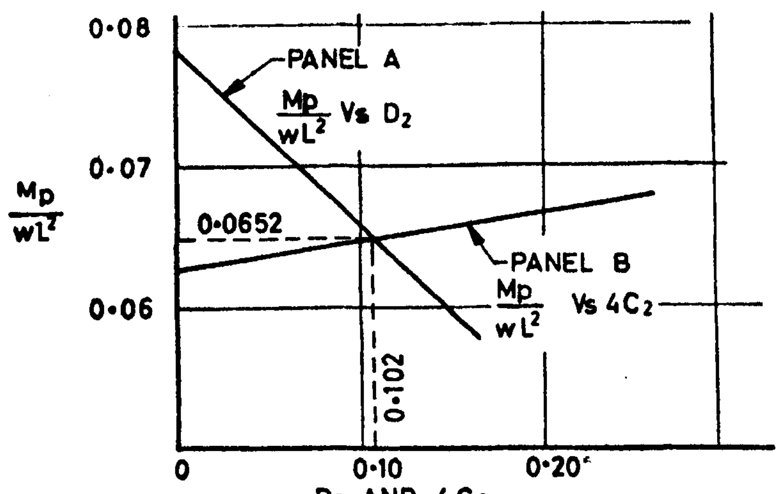
Analysis from chart in Fig. 71

Analysis of Panel A for $C_1 = 0{\cdot}125$		Analysis of Panel B for $D_2 = 0$		
D_2	Mp/wL_1^2	C_2	Mp/wL_2^2	$4C_2$
0	0·078 7	0	0·062 5	0
0·10	0·065 5	0·05	0·068 2	0·20

M_p (4-6) $= 0{\cdot}065\,2\ w_u L_1^2$

$= 0{\cdot}065\,2\ (2{\cdot}52)(10)^2$

$M_p = 16{\cdot}43$



(d)

APPENDIX A

(*See* Foreword)

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APPENDIX B

(*Clause* 23.4)

SPACING OF LATERAL BRACING

B-1. Equation 51 not only assures that the cross-section will be able to plastify (develop the full plastic moment) but also be able to rotate through a sufficient inelastic angle change to assure that all necessary plastic hinges will develop. In deriving this equation, the basic lateral buckling equation[48] has been used, the analysis being based on an idealized cross-section that consists of only two flanges separated by the web-distance. Therefore it already reflects and, in fact, makes use of the parameters l/h and d/t. Using the elastic constants of the material, and considering idealized behaviour as shown in Fig. 17, it may be shown that this procedure leads to a critical slenderness ratio of about 100.

While this might be reasonable for a section that was only called upon to support M_p, it is unlikely that the resulting critical bracing would allow much inelastic rotation — a rotation that is ordinarily required at the first plastic hinge. Reference 10 suggests that it will be adequate to require only that plastic yield penetrate through the flange. It is quite evident from Fig. 18, however, that the resulting further inelastic hinge rotation thus available is relatively small. One of the important contributions of Ref 18 was that it developed methods of correlating the critical length for lateral buckling with the magnitude of required hinge rotation.

B-2. This appendix is to outline the procedure for checking the adequacy of the spacing of bracing to prevent lateral buckling. It is the procedure that was used in the examples of Section VI. The problem is a two-fold one: First of all, what is the lateral buckling strength of an elastic-plastic segment of a member that has been called upon to absorb varying amounts of rotation at the plastic hinge? Secondly, what is the necessary hinge rotation, namely, the required rotation of a given plastic hinge to assure that the total structure reaches the computed ultimate load?

B-3. There will be considered first the matter of lateral buckling of an elastic-plastic beam segment. Figure 64 represents an approximation to the work of Ref 18. In preparing the figure, assumptions were made with regard to various factors that influence lateral buckling strength. Commencing with a beam that is deformed until the point of strain-hardening has been reached throughout, the resulting critical length

$(L/r_y)_{cr} = 18$ may be revised upward to account for the influence of moment gradient, St. Venant's torsion, the extent of yielding (partial

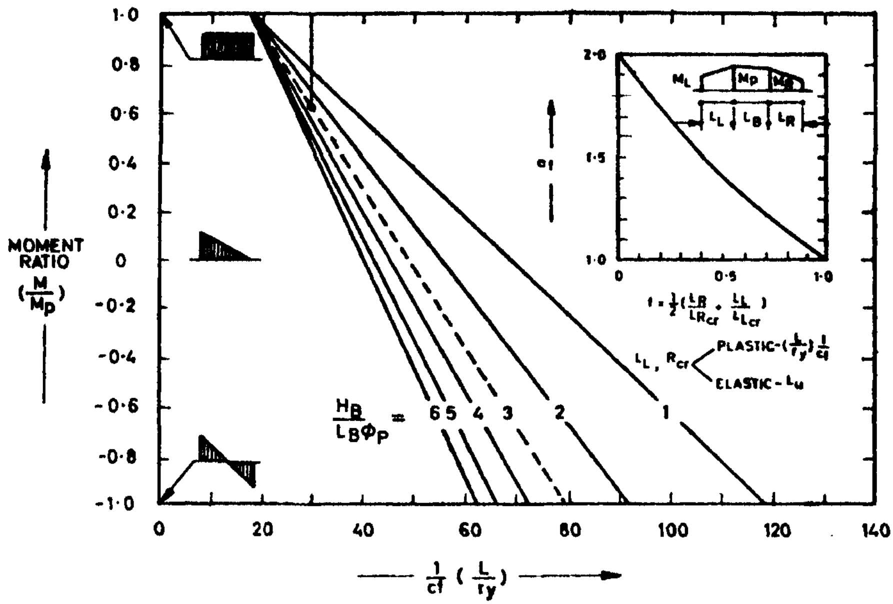


FIG. 64 COMPARISON OF SLENDERNESS AND MOMENT RATIOS

yielding), and the effect of end restraint. Reference 18 considered the influence of each of these factors and Fig. 64 is an approximation to these results, presented in terms of the moment ratio. The equation $L/r_y = 18+30\ (1-M/M_p)$ is, in fact, the equation of the heavy dashed line shown in the figure with a 'cut-off' at $M/M_p = 0.6$. The significance of various parts of Fig. 4 should appear in the course of the following description. The procedure for using Fig. 64 is as follows:

a) Assume a purlin bracing (usually dictated by available roofing materials); Compute L/r_y.

b) Examine the structure to see which segment (or segments) will be the most critical. For equal purlin spacing it will be the one near each hinge with the largest moment ratio. Referring to the insert sketch, call this the braced span, L_B.

c) Compute the precise moment ratio for the span being considered (length $= L_B$). This moment ratio is the ratio of the smaller moment to the plastic moment (M_R/M_p).

d) Compare the slenderness ratio existing in the structure with that which would be permitted for the particular moment ratio according to the dashed line in Fig. 64 (or the equation noted above), neglecting for the time being the parameter $H_B/L_B\phi_p$ which relates to hinge rotation. The selected purlin spacing is adequate if its slenderness ratio is less than permitted according to the figure. Otherwise, further refinements are required as follows.

1) As a first step in evaluating the end fixity correction, compute the value f which gives the 'fixing' influence of the adjoining spans. In the equation for the abscissa of the inset sketch on Fig. 64 the values L_L and L_R are the lengths to left and right of the braced span, and the subscripts 'cr' indicate the corresponding critical lengths of those members. These latter values may be determined roughly, as follows:

If the member is partly plastic (L_L, for example) then the critical value to use would be that obtained from the chart — a value L_B that could, itself, be refined to account for C_f. Thus, if the adjacent span, L_L, had a moment ratio of zero, then the value L_{Lcr} would be taken as 48. This value could either be computed from Eq 51 or from Fig. 64 (for $H_B/L_B\phi_p = 3 \cdot 0$). In other words, in order to obtain the critical length of a partially plastic adjacent span for use in determining f, it is assumed that $C_f = 1 \cdot 0$ and the critical length is obtained as if it were a 'buckling' segment. If the number is elastic (like span I_R) then the elastic critical lateral buckling length would be used and as a conservative approximation one could take L_u as given in the AISC Handbook of steel construction.

2) The resulting value of f enables one to compute C_f from the insert chart. Multiplying the allowable slenderness ratio by C_f then gives a value which can be compared to the ratio existing in the structure.

If the selected spacing is still too great and a closer spacing is undesirable, the rotation requirement may next be checked. The principles and general methods for computing hinge rotations have already been described. However, the calculations are tedious and, if required, would tend to obviate one of the advantages of plastic design. Alternatively, charts may be prepared which would enable the rapid determination of the magnitude of hinge rotations and the sequence of formation of plastic hinges. (The latter question assumes some importance because a 'last hinge' would require a very small rotation*.)

---

*As discussed in Ref 9 a slenderness ratio of 100 could be permitted for this case, a value incidentally, which agrees with the British recommendation (one that is intended to cover all cases).

Figure 65 presents some of the limiting values obtained as a result of a study by G. C. Driscoll in which such charts are being prepared. It shows that the last hinge occurs in the rafter in most cases until the columns become relatively high with respect to the frame span.

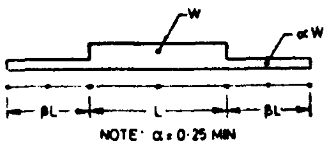
As regards to the hinge rotation, the value $H/L\phi_p$ in Fig. 64 and 65 is a non-dimensional function in which H is the calculated hinge rotation, L is the frame or beam span, and ϕ_p is the curvature at $M_p(\phi_p = M_p/EI)$. Before it can be used in Fig. 64 it should be corrected to L_B (the length of the braced segment) and H_B (the hinge rotation *within the braced-segment*). It was suggested in Ref 18 that value the H_B may be determined from the gradient of the moment diagram. The following equation may, therefore, be used to compute H_B:

$$H_B = \frac{H}{1+\frac{gr}{gl}} = \frac{H}{1+\frac{M_p - M_R}{M_p - M_L}\frac{L_L}{L_B}} \qquad \ldots (A1)$$

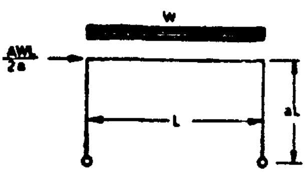
where the values are as indicated in Fig. 64.

Thus the final step in the procedure would be:

f) Determine the value $H/L\phi_p$ either from a deformation analysis or from charts (Fig. 65 summarizes a portion of the pertinent information), compute $H_B/L_B\phi_p$, and revise the allowable slenderness ratio according to Fig. 64.

Sl No.	*Structure*	*Location of Last Hinge to Form*	*Maximum 'Angles' for Given Geometry*
(1)		At midspan in all cases except the following: (1) $0.7 < \beta < 1.0$ $0.25 < \alpha < 0.30$ (2) $1.3 < \beta < 2.9$ $0.6 < \alpha < 1.0$	$H/L\phi_p$ First hinge at support 0·425 ($\alpha_{min} = 0.25$) First hinge at midspan 0·05 ($\alpha_{min} = 0.25$) First hinge inside span 0·186

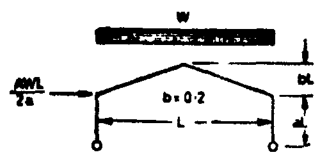
(2)



In the rafter for the following cases: a		First hinge at the corner: a	
0·2	$0 < A < 1.0$	0·2	0·475
0·4	$0 < A < 1.0$	0·4	0·455
0·5	$0 < A < 1.0$	0·6	
0·6	$0.024\,1 < A < 1.0$	0·6	0·450
0·8	$0.067\,4 < A < 1.0$	0·8	0·440
1·0	$0.106 < A < 1.0$	1·0	0·425

Otherwise at column

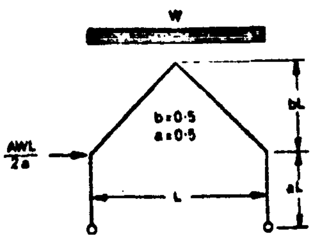
(3)



In the rafter for the following cases: a		First hinge at the corner: a	
0·2	$0 < A < 0.5$	0·2	0·85
0·4	$0 < A < 0.5$	0·4	0·64
0·6	$0 < A < 0.5$	0·6	0·57
0·8	$0.02 < A < 0.5$	0·8	0·53
1·0	$0.05 < A < 0.5$	1·0	0·50

Otherwise at column

(4)



In the rafter for:

$0 < A < 0.5$ 1·05

FIG. 65 LOCATION AND ANGLES OF PLASTIC HINGES

APPENDIX C

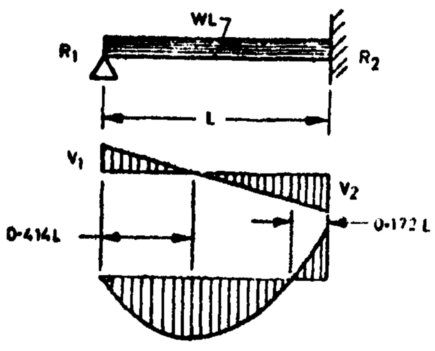
(*Clauses* 29, 30 *and* 31)

CHARTS AND FORMULAS FOR BEAMS

C-1. SIMPLE BEAMS

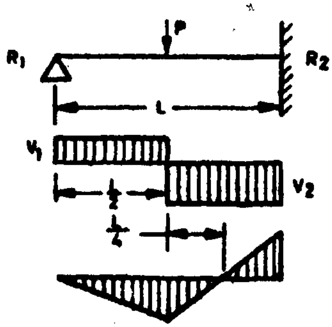
$$M_p = M_{Max}$$
$$W_u = W$$
$$\delta_u = \Delta_{Max}$$

C-2. BEAM FIXED AT ONE END, SUPPORTED AT THE OTHER — UNIFORMLY DISTRIBUTED LOAD



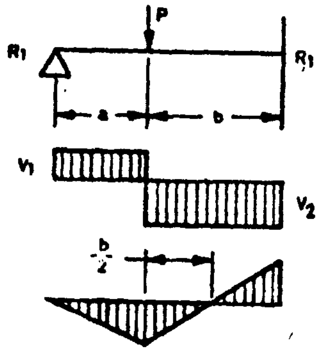
$$R_1 = V_1 = 0{\cdot}414\ WL$$
$$R_2 = V_2 = 0{\cdot}586\ WL$$
$$M_p = 0{\cdot}085\,8\ WL^2$$

C-3. BEAM FIXED AT ONE END, SUPPORTED AT THE OTHER — CONCENTRATED LOAD AT THE CENTRE



$$R_1 = V_1 = \frac{P}{3}$$
$$R_2 = V_2 = \frac{2P}{3}$$
$$M_p = \frac{PL}{6}$$

C-4. BEAM FIXED AT ONE END, SUPPORTED AT THE OTHER — CONCENTRATED LOAD AT ANY POINT

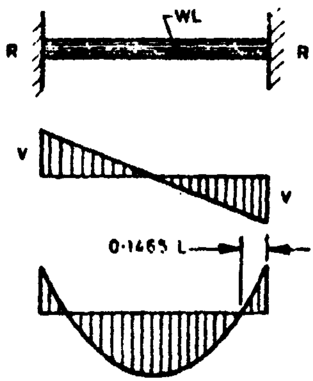


$$R_1 = V_1 = \frac{Pb}{a+L}$$

$$R_2 = V_2 = \frac{2Pa}{a+L}$$

$$M_p = \frac{Pab}{a+L}$$

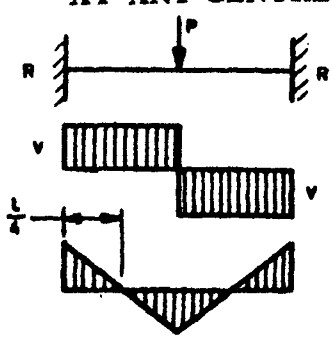
C-5. BEAM FIXED AT BOTH ENDS — UNIFORMLY DISTRIBUTED LOAD



$$R = V\frac{WL}{2}$$

$$M_p = \frac{WL^2}{16}$$

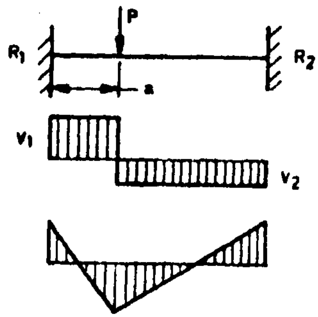
C-6. BEAM FIXED AT BOTH ENDS — CONCENTRATED LOAD AT ANY CENTRE



$$R = V = \frac{P}{2}$$

$$M_p = \frac{PL}{8}$$

C-7. BEAM FIXED AT BOTH ENDS — CONCENTRATED LOAD AT ANY POINT

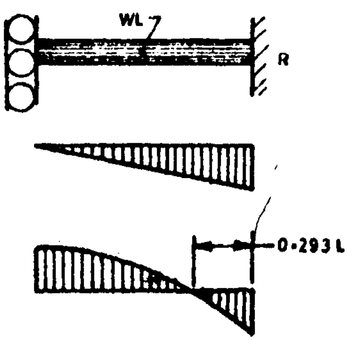


$$R_1 = V_1 = \frac{Pb}{L}$$

$$R_2 = V_2 = \frac{Pa}{L}$$

$$M_p = \frac{Pab}{2L}$$

C-8. BEAM FIXED AT ONE END, FREE BUT GUIDED AT THE OTHER — UNIFORMLY DISTRIBUTED LOAD



$$R = V = WL$$

$$M_p = \frac{WL^2}{4}$$

$$Q = \frac{b}{a}$$

$$M = W_h \frac{(a+b)^2 L^2}{2} = C \frac{W_u L^2}{2}$$

$$T = W_h \frac{(a+b)^2 L}{2a}$$

$$C = \frac{2T_a}{W_u L} = \frac{W_h (a+b)^2}{W_u}$$

Z = constant

TABLE 3 FORMULAS FOR THE SOLUTION OF PINNED BASE FRAMES

VERTICAL LOAD ALONE

$$V_1 = V_7 = \frac{W_u L}{2}$$

$$H_1 = H_7 = \frac{M_p}{aL}$$

For $Q = 0: M_p = \frac{W_u L^2}{16},\ x = \frac{L}{2}$

For $Q > 0: M_p = \frac{W_u L^2}{4}\left[\frac{\frac{x}{L}\left(1 - \frac{x}{L}\right)}{\sqrt{1+\theta}}\right]$

$$x = \frac{L}{\theta}\left[\sqrt{1+\theta} - 1\right]$$

VERTICAL AND HORIZONTAL LOAD

$$V_1 = \frac{W_u L}{2}(1-C);\ V_7 = \frac{W_u L}{2}(1+C)$$

$$H_1 = W_h(a+b)tH_7;\ H_7 = H_p/aL$$

For $C > \frac{1}{1+\theta}$ (Panel Mech):

$$M_p = \frac{W_u L^2}{4}C,\ x = 0$$

For $C < \frac{1}{1+\theta}$ (Combined Mech):

$$\theta = 0:\ M_p = \frac{W_u L^2}{16}(1+C)^2\ \ x = L\frac{(1-C)}{2}$$

$$\theta > 0:\ M_p = \frac{W_u L^2}{4}\left[\frac{\left(1 - \frac{x}{L}\right)\left(C + \frac{x}{L}\right)}{\sqrt{(1+\theta)(1-\theta C)}}\right]$$

$$x = \frac{L}{\theta}\left[\sqrt{(1+\theta)(1-\theta C)} - 1\right]$$

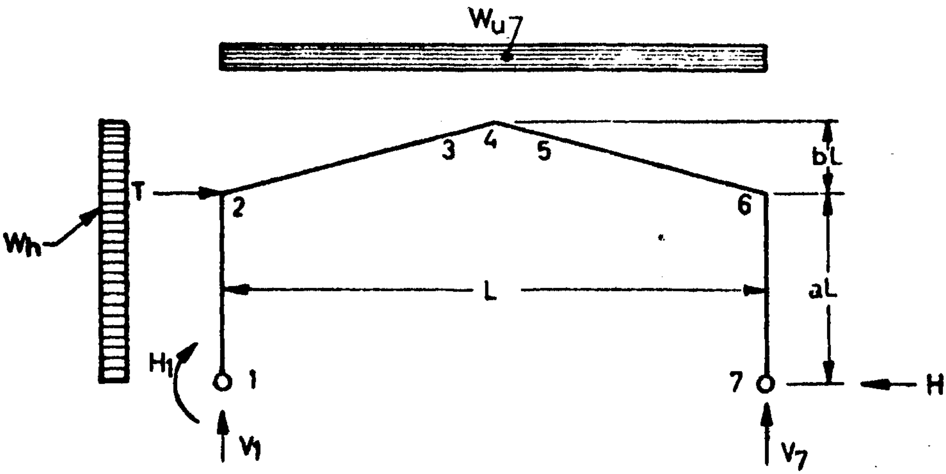


FIG. 67 SINGLE-STOREY STRUCTURE WITH UNIFORM PLASTIC MOMENT

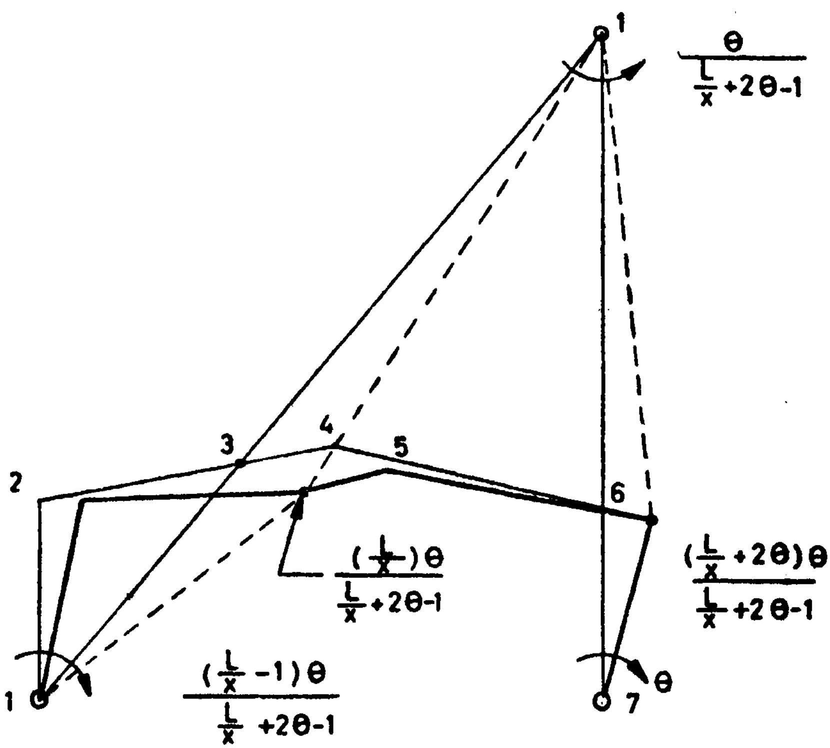


FIG. 68 PLASTIC HINGE MECHANISM

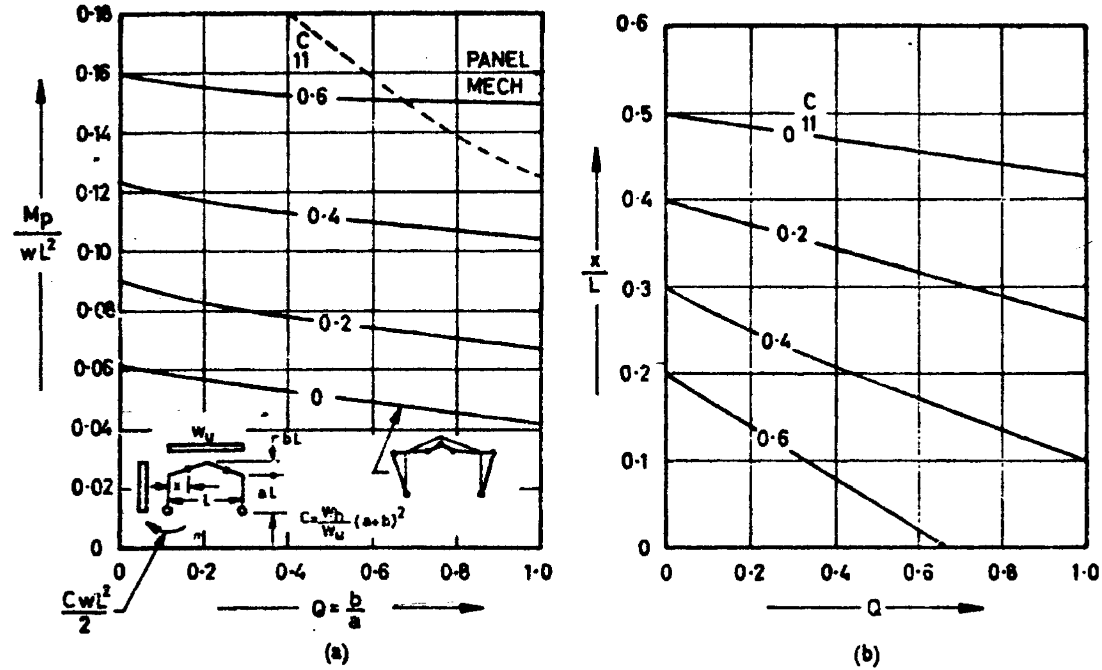


FIG. 69 SUMMARY OF MECHANISM FORMULAE FOR PINNED BASE SINGLE SPAN, SINGLE-STOREY FRAME

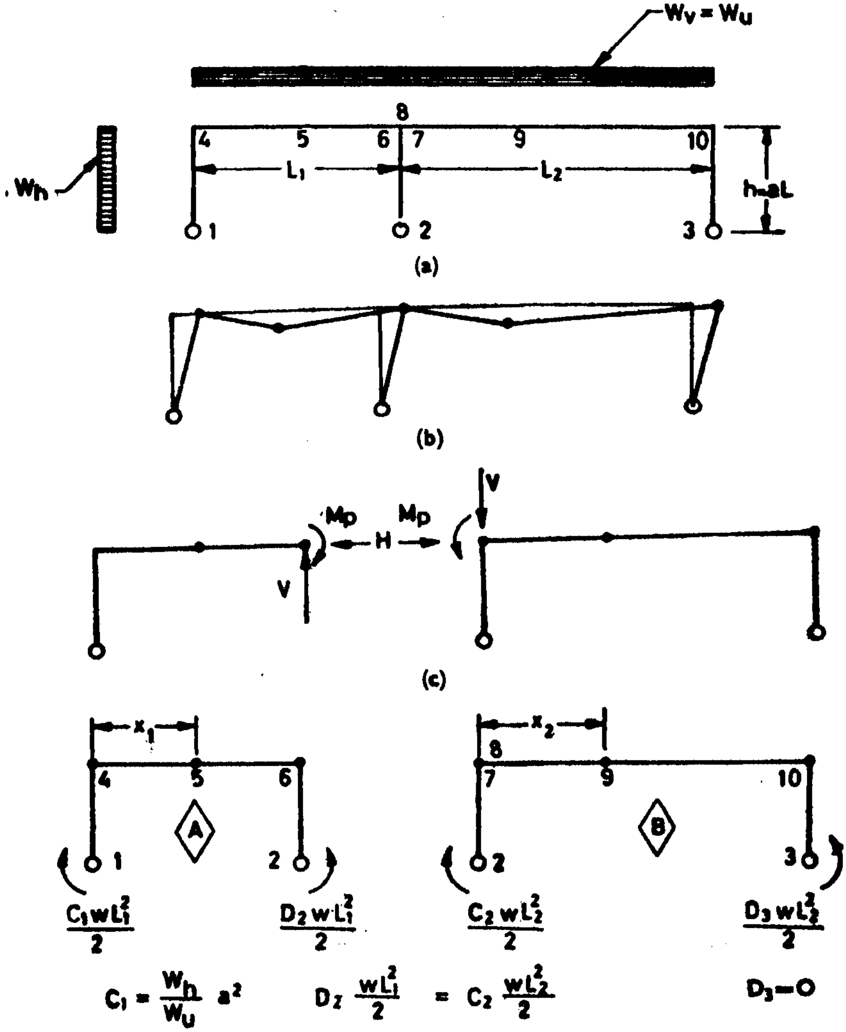


FIG. 70 ANALYSIS OF TWO-SPAN FLAT ROOF FRAME

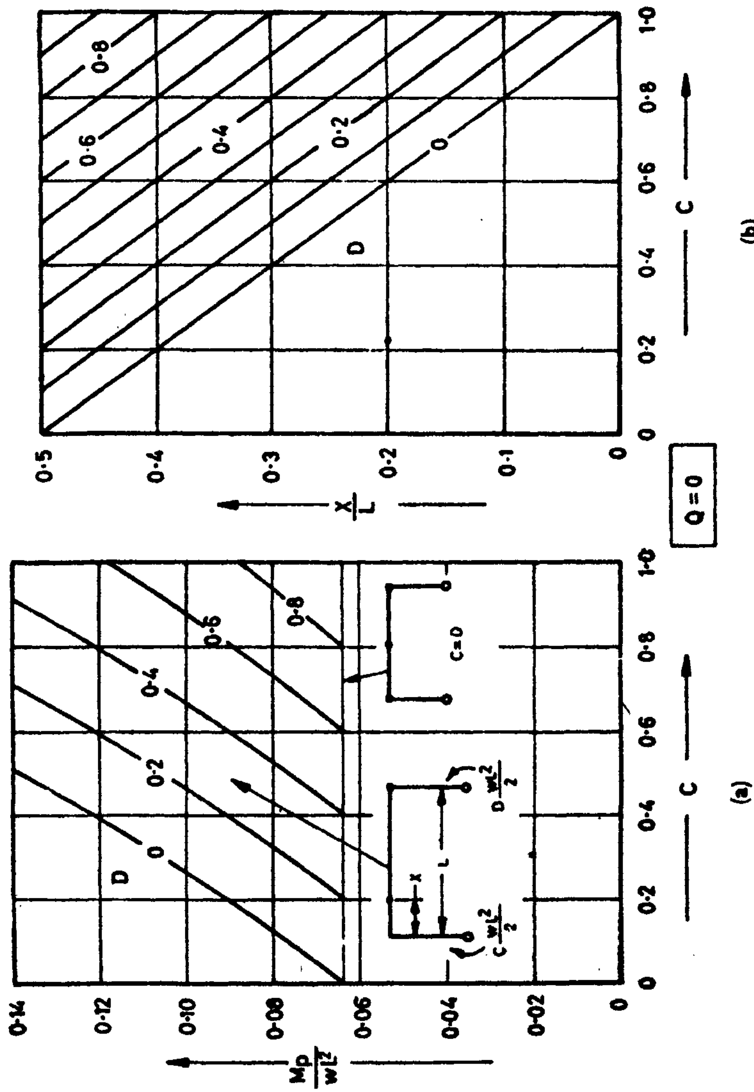


FIG. 71 GRAPHS SHOWING ANALYSIS OF MULTI-SPAN FRAME

TABLE 4 PLASTIC MOMENT

(*Clause* 24)

PLASTIC MODULUS IN cm³ Z_{xx}	DESIGNATION	WEIGHT PER METRE kg/m	SECTION MODULUS IN cm³ S_{xx}	SHAPE FACTOR Z/S	M_p *Cap* IN m.t $\frac{2\,520 \times 2}{100\,000}$	SHEAR CARRYING CAPACITY V_c INT. (1 265 *wd*)	P_y IN t $\frac{(A \times 2\,520)}{100\,000}$	d/w	r_x cm	r_y cm
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
4 341·631 8	ISWB 600	*145·1	3 854·2	1·126 5	109·409 1	89·6	465·847 2	50·847 5	25·01	5·35
3 986·662 9	ISWB 600	*133·7	3 540·0	1·126 2	100·463 9	85·0	429·357 6	53·571 4	24·97	5·25
3 510·633 9	MB 600	*122·6	3 060·4	1·147 1	88·468 0	91·1	393·649 2	50·000 0	24·24	4·12
3 066·298 9	ISWB 550	*112·5	2 723·9	1·125 7	77·270 7	73·1	361·216 8	52·381 0	22·86	5·11
2 798·561 8	ISLB 600	*99·5	2 428·9	1·152·2	70·523 8	79·7	319·258 8	57·142 9	23·98	3·79
2 711·984 0	MB 550	103·7	2 359·8	1·149 2	68·342 0	77·9	332·917 2	49·107 1	22·16	3·73
2 351·355 8	ISWB 500	*95·2	2 091·6	1·124 2	59·254 2	62·6	305·474 4	50·505 0	20·77	4·96
2 228·167 1	ISLB 550	*86·3	1 933·2	1·152 6	56·149 8	68·9	277·124 4	55·555 6	21·99	3·48
2 074·673 4	MB 500	86·9	1 808·7	1·147 1	52·281 8	64·5	279·064 8	49·019 6	20·21	3·52
2 030·946 1	ISHB 450	92·5	1 793·3	1·132 5	51·179 8	64·3	297·082 8	39·823 0	18·50	5·08
1 955·031 1	ISHB 450	87·2	1 742 7	1·121 8	49·266 8	55·8	280·072 8	45·918 4	18·78	5·18
1 773·671 7	ISLB 500	*75·0	1 543·2	1·149 3	44·696 5	58·2	240·660 0	54·347 8	20·10	3·34
1 760·586 4	ISWB 450	79·4	1 558·1	1·130 0	44·366 8	52·4	254·898 0	48·913 0	18·63	4·11
1 626·356 4	ISHB 400	82·2	1 444·2	1·126 1	40·984 2	53·6	263·743 2	37·735 8	16·61	5·16
1 566·356 4	ISHB 400	77·4	1 404·2	1·115 5	39·472 2	46·0	248·623 2	43·956 0	16·87	5·26
1 533·363 6	MB 450	*72·4	1 350·7	1·150 0	39·144 8	53·5	232·520 4	47·872 3	18·15	3·01
1 401·354 7	ISLB 450	*65·3	1 223·8	1·145 1	35·314 1	49·0	209·512 8	52·325 6	18·20	3·20
1 290·185 5	ISWB 400	66·7	1 171·3	1·127 1	33·268 7	43·5	214·225 2	46·511 6	16·60	4·04
1 268·694 8	ISHB 350	72·4	1 131·6	1·121 2	31·971 1	44·7	232·369 2	34·653 5	14·65	5·22
1 213·534 8	ISHB 350	67·4	1 094·8	1·108 5	30·581 1	36·7	216·493 2	42·168 7	14·93	5·34
1 176·176 0	MB 400	*61·6	1 022·9	1·149 8	29·639 6	45·0	197·719 2	44·943 8	16·15	2·82
1 099·459 7	ISLB 400	*56·9	965·3	1·139 0	27·706 4	40·5	182·523 6	50·000 0	16·33	3·15
995·490 3	ISWB 350	56·9	887·0	1·122 3	25·086 4	35·4	182·700 0	43·750 0	14·63	4·03
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

921·681 0	ISHB 300	58·8	836·3	1·102 1	23·226 4	28·8	188·622 0	39·473 7	12·95	5·41
891·027 3	ISMC 400	*49·4	754·1	1·181 6	22·453 9	43·5	158·583 6	46·511 6	15·48	2·83
889·571 3	MB 350	52·4	778·9	1·142 1	22·417 2	35·9	168·109 2	43·209 9	14·29	2·84
851·114 3	ISLB 350	49·5	751·9	1·132 0	21·448 1	32·8	158·785 2	47·297 3	14·45	3·17
825·023 3	ISLC 400	*45·7	699·5	1·179 4	20·790 6	40·5	146·790 0	50·000 0	15·50	2·81
731·211 1	ISWB 300	48·1	654·8	1·116 7	18·426 5	28·1	154·551 6	40·540 5	12·66	4·02
708·432 4	ISHB 250	54·7	638·7	1·109 2	17·852 5	27·8	175·669 2	28·409 1	10·70	5·37
687·759 6	ISLB 325	*43·1	607·7	1·131 7	17·331 5	28·8	138·348 0	46·428 6	13·41	3·05
678·732 4	ISHB 250	51·0	618·9	1·096 7	17·104 1	21·8	163·699 2	36·231 9	10·91	5·49
672·195 1	ISMC 350	*42·1	571·9	1·175 4	16·939 3	35·9	135·223 2	43·209 9	13·66	2·83
651·741 3	MB 300	*44·2	573·6	1·136 2	16·423 9	28·5	141·775 2	40·000 0	12·37	2·84
622·954 5	ISLC 350	*38·8	532·1	1·170 7	15·698 4	32·8	124·664 4	47·297 3	13·72	2·82
554·318 7	ISLB 300	*37·7	488·9	1·133 8	13·968 8	25·4	121·161 6	44·776 1	12·35	2·80
542·218 9	ISHB 225	46·8	487·0	1·113 4	13·663 9	24·5	150·343 2	26·162 8	9·58	4·84
527·573 9	ISWB 250	40·9	475·4	1·109 7	13·294 9	21·2	131·166 0	37·313 4	10·69	4·06
515·823 9	ISHB 225	43·1	469·3	1·098 7	12·993 7	18·5	138·448 8	34·615 4	9·80	4·96
496·771 1	ISMC 300	*35·8	424·2	1·171 1	12·518 6	28·8	115·012 8	39·473 7	11·81	2·61
465·709 5	MB 250	37·3	410·5	1·134 5	11·735 9	21·8	119·826 0	36·231 9	10·39	2·65
466·725 8	ISLC 300	*33·1	403·2	1·157 6	11·761 5	25·4	106·117 2	44·776 1	11·98	2·87
443·097 8	ISLB 275	*33·0	392·4	1·130 5	11·178 7	22·3	105·890 4	42·968 8	11·31	2·61
414·234 1	ISHB 200	40·0	372·2	1·112 9	10·438 7	19·7	128·368 8	25·641 0	8·55	4·42
397·230 1	ISHB 200	37·3	360·8	1·101 0	10·010 2	15·4	119·800 8	32·786 9	8·71	4·51
389·932 5	ISWB 225	33·9	348·5	1·118 9	9·826 3	18·2	108·964 8	35·156 2	9·52	3·22
356·722 2	ISMC 250	*30·4	305·3	1·168 4	8·989 4	22·5	97·448 4	35·211 3	9·94	2·38
348·273 8	MB 225	31·2	305·9	1·138 5	8·776 5	18·5	100·094 4	34·615 4	9·31	2·34
338·690 5	ISLB 250	*27·9	297·4	1·138 8	8·535 0	19·3	89·535 6	40·983 6	10·23	2·33
338·115 1	ISLC 250	28·0	295·0	1·146 2	8·520 5	19·3	89·838 0	40·983 6	10·17	2·89
293·988 1	ISHB 250	28·8	262·5	1·120 0	7·408 5	15·4	92·509 2	32·786 9	8·46	2·99
277·932 5	ISHC 225	*25·9	239·5	1·160 5	7·003 9	18·2	83·185 2	35·156 2	9·03	2·38
260·131 0	ISLC 225	*24·0	226·5	1·148·5	6·555 3	16·5	76·935 6	38·793 1	9·34	2·62
254·719 5	ISLB 225	*23·5	222·4	1·145 3	6·418 9	16·5	75·398 4	38·793 1	9·15	1·94
253·861 2	MB 200	25·4	223·5	1·135 8	6·397 3	14·4	81·471 6	35·087 7	8·32	2·15
251·642 8	ISHB 150	34·6	218·1	1·153 8	6·341 4	22·4	111·081 6	12·711 9	6·09	3·35
232·517 8	ISHB 150	30·6	205·3	1·132 6	5·859 5	15·9	98·229 6	17·857 1	6·29	3·44
215·642 8	ISHB 150	27·1	194·1	1·111 0	5·434 2	10·2	86·889 6	27·777 8	6·50	3·54
211·255 7	ISMC 200	*22·1	181·9	1·161 4	5·323 6	15·4	71·089 2	32·786 9	8·03	2·23

(Continued)

TABLE 4 PLASTIC MOMENT — *Contd*

(*Clause* 24)

PLASTIC MODULUS IN cm³ Z_{xx}	DESIGNATION	WEIGHT PER METRE kg/m	SECTION MODULUS IN cm³ S_{xx}	SHAPE FACTOR Z/S	M_p *Cap* IN m.t $\frac{2\,520 \times 2}{100\,000}$	SHEAR CARRYING CAPACITY V_c INT. (1 265 *wd*)	P_y IN t $\frac{(A \times 2\,520)}{100\,000}$	d/w	r_x cm	r_y cm
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
198·772 0	ISLC 200	*20·6	172·6	1·151 6	5·009 1	13·9	66·074 4	36·363 6	8·11	2·37
194·200 1	ISWB 175	22·1	172·5	1·125 8	4·893 8	12·8	70·837 2	30·172 4	7·33	2·59
184·345 1	ISLB 200	*19·8	169·7	1·137 0	4·645 5	13·7	63·680 4	37·037 0	8·19	2·13
166·077 0	MB 175	*19·5	144·3	1·142 2	4·185 1	12·2	62·042 4	31·818 2	7·19	1·86
161·646 3	ISMC 175	*19·1	139·8	1·156 3	4·073 5	12·6	61·437 6	30·701 8	7·08	2·23
150·359 9	ISLC 175	*17·6	131·3	1·145 2	3·789 1	11·3	56·448 0	34·313 7	7·16	2·38
143·301 4	ISLB 175	*16·7	125·3	1·143 7	3·611 2	11·3	53·676 0	34·313 7	7·17	1·93
134·153 2	ISJB 225	*12·8	116·3	1·153 5	3·380 7	10·5	41·025 6	60·810 8	8·97	1·58
133·106 3	ISJC 200	13·9	116·1	1·146 5	3·354 3	10·4	44·780 4	48·780 5	8·08	2·18
126·857 6	ISWB 150	17·0	111·9	1·133 7	3·196 8	10·2	54·608 4	27·777 8	6·22	2·09
119·824 6	ISMC 150	16·4	103·9	1·153 3	3·019 6	10·2	52·617 6	27·777 8	6·11	2·21
110·480 3	MB 150	15·0	95·7	1·140 1	2·784 1	9·1	47·880 0	31·250 0	6·18	1·66
106·171 7	ISLC 150	14·4	93·0	1·141 6	2·675 5	9·1	46·267 2	31·250 0	6·16	2·37
104·505 7	ISLB 150	14·2	91·8	1·138 4	2·633 5	9·1	45·561 6	31·250 0	6·17	1·75
94·223 5	ISJC 175	*11·2	82·3	1·144 9	2·374 4	8·0	35·884 8	48·611 1	7·11	1·88
90·897 7	ISJB 200	*9·9	78·1	1·163 9	2·290 6	8·6	31·852 8	58·823 5	7·86	1·17
81·848 3	MB 125	13·2	70·9	1·139 9	2·062 6	7·9	41·832 0	28·409 1	5·20	1·62
77·154 0	ISMC 125	12·7	66·6	1·158 5	1·944 3	7·9	40·798 8	25·000 0	5·07	1·92
73·930 5	ISLB 125	11·9	65·1	1·135 6	1·863 0	7·0	38·102 4	28·409 1	5·19	1·69
72·047 0	ISJC 150	9·9	62·8	1·147 2	1·815 6	6·8	31·878 0	41·666 7	6·10	1·73

64·222 4	ISJB 175	*8·1	54·8	1·179 9	1·618 4	7·1	25·905 6	54·687 5	6·83	0·97
58·652 9	MB 100	11·5	50·4	1·138 9	1·478 0	5·1	36·792 0	25·000 0	4·20	1·67
49·573 6	ISJB 150	*7·1	42·9	1·155 6	1·249 2	5·7	22·705 2	50·000 0	5·98	1·01
49·082 3	ISJC 125	7·9	43·2	1·136 2	1·236 9	4·7	25·376 4	41·666 7	5·18	1·60
43·826 1	ISMC 100	9·2	37·3	1·175 0	1·104 4	5·9	29·484 0	21·276 6	4·00	1·49
38·885 6	ISLB 100	8·0	33·6	1·157 3	0·979 9	5·1	25·729 2	25·000 0	4·06	1·12
38·084 9	ISLC 100	7·9	32·9	1·157 6	0·959 7	5·1	25·250 4	25·000 0	4·06	1·57
28·375 4	ISJC 100	*5·8	24·8	1·144 2	0·715 1	3·8	18·673 2	33·333 3	4·09	1·42
24·165 7	ISMC 75	6·8	20·8	1·190 4	0·609 0	4·2	21·848 4	17·045 5	2·96	1·21
22·353 0	ISLB 75	6·1	19·4	1·152 2	0·563 3	3·5	19·429 2	20·270 3	3·07	1·14
20·609 3	ISLC 75	*5·7	17·6	1·171 0	0·519 4	3·5	18·295 2	20·270 3	3·02	1·26

NOTE — For using this table, proceed as follows:

i) Find, in the column 'Plastic Modulus' or '*Mp Cap*' the value equal to, or, failing that, the value next higher to the required value of Z_{xx} or *Mp Cap*.

ii) If the section opposite this selected value in the column 'Weight per Metre' bears an asterisk (*), choose it, as it is the lightest section in the series to serve the requirement. Otherwise, proceed higher up and choose the first section bearing the asterisk as all sections above, the section opposite to the selected value also satisfy the requirement with regard to Z_{xx} or *Mp Cap*.

iii) If conditions require that the section must not exceed a certain depth, proceed up to the column until the section with the required depth is reached. Check up to see that no lighter beam with an asterisk, of the same depth, appears higher up.

iv) Check up the selected section for shear carrying capacity. Also, make proper provisions in cases of eccentric loading or any other special conditions of loading.

v) Check further if the finally selected section satisfies the design requirements specified in section V of the Handbook.

APPENDIX D

COMPOSITION OF STRUCTURAL ENGINEERING SECTIONAL COMMITTEE, SMBDC 7

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APPENDIX E

(*See* Foreword)

INDIAN STANDARDS RELATING TO STRUCTURAL ENGINEERING

General

IS: 800-1962 Code of practice for use of structural steel in general building construction (*revised*)
IS: 801-1958 Code of practice for use of cold formed light gauge steel structural members in general building construction
IS: 802 (Part I)-1967 Code of practice for use of structural steel in overhead transmission-line towers: Part I Loads and permissible stresses
IS: 803-1962 Code of practice for design, fabrication and erection of vertical mild steel cylindrical welded oil storage tanks
IS: 804-1967 Specification for rectangular pressed steel tanks (*first revision*)
IS: 806-1968 Code of practice for use of steel tubes in general building construction (*first revision*)
IS: 807-1963 Code of practice for design, manufacture, erection and testing (structural portion) of cranes and hoists
IS: 808-1964 Specification for rolled steel beams, channel and angle sections (*revised*)
IS: 811-1965 Specification for cold formed light gauge structural steel sections (*revised*)
IS: 1173-1967 Specification for hot rolled and slit steel, tee bars (*first revision*)
IS: 1252-1958 Specification for rolled steel sections, bulb angles
IS: 1730-1961 Dimensions for steel plate, sheet and strip for structural and general engineering purposes
IS: 1731-1961 Dimensions for steel flats for structural and general engineering purposes
IS: 1732-1961 Dimensions for round and square steels bars for structural and general engineering purposes
IS: 1852-1967 Specification for rolling and cutting tolerances for hot-rolled steel products (*first revision*)
IS: 1863-1961 Dimensions for rolled steel bulb plates
IS: 1864-1963 Dimensions for angle sections with legs of unequal width and thickness
IS: 2314-1963 Specification for steel sheet pilling sections
IS: 2713-1969 Specification for tubular steel poles for overhead power lines (*first revision*)

IS: 3177-1965 Code of practice for design of overhead travelling cranes and gantry cranes other than steel work cranes
IS: 3443-1966 Specification for crane rail sections
IS: 3908-1966 Specification for aluminium equal leg angles
IS: 3909-1966 Specification for aluminium unequal leg angles
IS: 3921-1966 Specification for aluminium channels
IS: 3954-1966 Specification for hot rolled steel channel sections for general engineering purposes
IS: 3964-1967 Specification for light rails
IS: 4000-1967 Code of practice for assembly of structural joints using high tensile friction grip fasteners
IS: 4014 (Part I)-1967 Code of practice for steel tubular scaffolding: Part I Definitions and materials
IS: 4014 (Part II)-1967 Code of practice for steel tubular scaffolding: Part II Safety regulations for scaffolding
IS: 4137-1967 Code of practice for heavy duty electric overhead travelling cranes including special service machines for use in steel works

Handbooks

SP: 6 ISI Handbook for structural engineers:

SP: 6(1)-1966 Structural steel sections
SP: 6(2) Steel beams and plate girders
SP: 6(3) Steel columns and struts